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LOVELL'S SERIES OF SCHOOL BOOKS.

# KEY

TO THE

## NATIONAL ARITHMETIC;

CONTAINING

*Full Solutions to nearly all the Problems.*

DESIGNED FOR THE

USE OF TEACHERS AND PRIVATE STUDENTS.

BY JOHN HERBERT SANGSTER, M.A.,

MATHEMATICAL MASTER AND LECTURER IN CHEMISTRY AND NATURAL  
PHILOSOPHY IN THE NORMAL SCHOOL FOR UPPER CANADA.

.....  
SECOND EDITION—CAREFULLY REVISED.  
.....

Montreal:

PRINTED AND PUBLISHED BY JOHN LOVELL,

SOLD BY ROBERT MILLER, MONTREAL; ADAM MILLER, TORONTO  
AND BY ALL BOOKSELLERS.

1864.

Entered, according to the Act of the Provincial Parliament, in  
the year one thousand eight hundred and sixty-one, by  
JOHN LOVELL, in the Office of the Registrar of the Pro-  
vince of Canada.

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## P R E F A C E .

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It was the original intention of the Author to give, in the Key, merely a series of brief hints upon the Solutions of the more difficult Problems. He was led to modify this plan, and to issue the work in its present form, chiefly from the consideration that as there are in the country many young persons who, from various causes, are unable to avail themselves of the advice and assistance of a teacher, it would be a great boon to these to have access to a book to which they might refer with the certainty of having every doubt removed as to the correctness of their work and methods of solution. He offers the work to his fellow-teachers with the hope that they will accord it the same favorable reception that they have so kindly given to the National Arithmetic.

TORONTO, *May*, 1861.



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1888

The following is a list of the names of the persons who have been admitted to the membership of the Society since the last meeting of the Council.

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# KEY TO NATIONAL ARITHMETIC.

## EXERCISE 5—Page 50.

(1)	(2)	(3)	(4)	(5)
d	£	£ s.	£ s.	£ s.
23328	348	38 10	58 13	58 13
4	20	20	20	20
<u>93312 f.</u>	<u>6960 s.</u>	<u>770 s.</u>	<u>1173 s.</u>	<u>1173 s.</u>
		12	12	12
		<u>9240 d.</u>	<u>14076 d.</u>	<u>14076 d.</u>
				4
				<u>56304 f.</u>
(6)	(7)	(8)	(9)	
£ s. d.	£ s. d.	cwt. qrs. lbs.	cwt. qrs. lbs.	
59 13 6½	63 0 9	16 2 16	14 3 16	
20	20	4	4	
<u>1193 s.</u>	<u>1260 s.</u>	<u>66 qrs.</u>	<u>59 qrs.</u>	
12	12	25	25	
<u>14322 d.</u>	<u>15129 d.</u>	<u>346</u>	<u>311</u>	
4		132	118	
<u>57291 f.</u>		<u>1666 lbs.</u>	<u>1491 lbs.</u>	

## KEY.

[NAT. ARITH.]

(10)	(11)	(12)	(13)	(14)
lbs. oz. dwt. grs.	lbs. oz. dwt. grs.	grs. yrs.	mile.	yrs. d. h. m.
3 5 12 16	7 11 15 14	20	1	46 21 8 56
12	12	365	8	365
41 oz.	95	7300 dys.	8 fur.	251
20	20	24	40	276
832 dwts.	1915 dwts.	29200	320 per.	138
24	24	14600	5½	16811 days.
3344	7674	175200 hrs	1600	24
1664	3830		160	67252
			1760 yds.	33622
19984 grs.	45974 grs.		3	403472 hrs.
			5280 ft.	60
				24208376 min.

(15)	(16)	(17)	(18)	(19)	(20)
sq per.	a. r. per.	sq. miles.	cub. ft.	pks.	pks.
74	46 3 12	767	767	767	797
30½	4	640	1728	2	2
2220	187 r.	30680	6136	1534 gals	1594 gals
18½	40	4602	1534	4	4
2238½ sq. yds.	7492 per.	490880 sq. a.	5369	6136 qts.	6376 qts.
	30½		767		2
224760			1325376 cub. in.		12752 pts.
1873					
226633 sq. yds.					

## EXERCISE 6—Page 51.

(1)	(2)	(3)
f.	grs.	yds.
4)32756	24)23547	5½)397024
		2 2
12)8189 d.	20)981 dwt. 3 grs.	11)794048
20)682s. 5d.	12)49 oz. 1 dwt. 3 grs.	40)72186r. 2hf-yds.=1yd.
£34 2s. 5d.	4 lbs. 1 oz. 1 dt. 3 g.	8)1804 fur. 26 r. 1 yd.
		225 m. 4 f. 26 r. 1 y.

(14)  
yrs. d. h. m.  
46 21 8 56  
365

251  
76  
8

811 days.  
24  
352  
22

172 hrs.  
60  
76 min.

(20)  
pks.  
797  
2  
1594 gals  
4

6376 qts.  
2  
12752 pts.

ds.=1yd.  
r. 1 yd.  
26 r. 1 y.

(4)  
sec.  
60)28635

60)477 m. 15 sec.

7 hrs. 57 m. 15 sec. 16 cwt. 2 q. 16 lbs. 14 cwt. 3 q. 16 lbs.

(7)  
grs.  
24)115200

20)4800 dwt.

12)240 oz.

20 lbs.

(5)  
lbs.  
25)1666

4)86 qrs. 16 lbs.

16)107520

6720 lbs.

(12)

cub. in.

1728)138297

27)80 ft. 57 in.

2 c. yds. 26 c. ft. 57 c. in.

(6)  
lbs.  
25)1491

4)59 qrs. 16 lbs.

(10)

Fl. e.

767

3

4)2301 qrs.

575 yds. 1 qr.

(11)  
ft.  
3)183810

51)61270 yds.  
2) 2

11)122540

40)11140 per.

8)278 fur. 20 per.

3)34 m. 6 fur. 20 per.

11 lea. 1 m. 6 fur. 20 per.

(14)  
sec.  
60)3561829

60)59363 m. 49 sec.

(15)  
qts.  
4)1597

2)399 gals. 1 qt.

24)989 h. 23 m. 49 s. 4)199 pks. 1 gal. 1 qt.

7)41 d. 5 h. 23 m. 49 s. 49 bush. 3 pecks 1 gal. 1 qt.

5 wks. 6 days. 5 hrs. 23 min. 49 sec.

(16)  
c. ft.  
8)1000

125 cords.



$$\begin{array}{r} (17) \\ \text{seconds.} \\ 60 \overline{) 10000} \end{array}$$

$$\text{'60) 166' 40''}$$

$$2^{\circ} 46' 40''$$

$$\begin{array}{r} (18) \\ \text{sq. links.} \\ 10000 \overline{) 70000} \end{array}$$

$$7 \text{ sq. ch.}$$

$$\begin{array}{r} (19) \\ \text{grs.} \\ 30 \overline{) 11521} \end{array}$$

$$3) 576 \text{ scr. 1 gr.}$$

$$8) 192 \text{ dr. 1 gr.}$$

$$12) 24 \text{ oz. 1 gr.}$$

$$2 \text{ lbs. 1 gr.}$$

$$\begin{array}{r} (20) \\ \text{sq. ft.} \\ 9 \overline{) 26025} \end{array}$$

$$\begin{array}{r} 30\frac{1}{4} \overline{) 2891} \text{ yds. 6 ft.} \\ 4 \quad ) \quad 4 \end{array}$$

$$121) 11564 \text{ quarter yards.}$$

$$95 \text{ per. 69 quar. yds. 6 ft. =}$$

$$40) 95 \text{ per. 17 yds. 8 ft. 36 in.}$$

$$2 \text{ r. 15 sq. p. 17 sq. y. 8 sq. ft. 36 sq. in.}$$

## EXERCISE 7—Page 53.

(1)

$$\text{£}3 \times 400 = 1200 \text{ cents.}$$

$$7\text{s.} \times 20 = 140 \text{ "}$$

$$1\frac{1}{4}\text{d.} = 5 \text{ far.} \times 5 \div 12 =$$

$$2\frac{1}{2} \text{ "}$$

$$3\frac{1}{4}\text{d.} = 14 \text{ far.} \times 5 \div 12 = .05\frac{1}{2}$$

$$\text{£}3 \text{ 7s. } 1\frac{1}{4}\text{d.} = 1342\frac{1}{2} \text{ cts.}$$

$$\text{£}29 \text{ 18s. } 3\frac{1}{4}\text{d.} = \$119.65\frac{1}{2}$$

(3)

$$11\frac{1}{4}\text{d.} = 45 \text{ far.} \times 5 \div 12 = 18\frac{3}{4} \text{ cts.}$$

(4)

$$\text{£}89 \times 400 = \$278.00$$

$$15\text{s.} \times 20 = 3.00$$

$$6\text{d.} = 24 \text{ far.} \times 5 \div 12 = .10$$

$$\text{£}89 \text{ 15s. } 6\text{d.} = \$279.10$$

(2)

$$\text{£}29 \times 400 = \$116.00$$

$$18\text{s.} \times 20 = 3.60$$

$$14 \text{ far.} \times 5 \div 12 = .05\frac{1}{2}$$

$$\text{£}29 \text{ 18s. } 3\frac{1}{4}\text{d.} = \$119.65\frac{1}{2}$$

(5)

$$18\text{s.} \times 20 = \$3.60$$

$$8\frac{1}{4}\text{d.} = 34 \text{ far.} \times 5 \div 12 = .14\frac{1}{2}$$

$$18\text{s. } 8\frac{1}{4}\text{d.} = \$3.74\frac{1}{2}$$

(6)

$$£17 \times 400 = \$68.00$$

$$16s. \times 20 = 3.20$$

$$5\frac{1}{2}d. = 23 \text{ far.} \times 5 \div 12 = .09\frac{1}{2}$$

$$£17 \text{ } 16s. \text{ } 5\frac{1}{2}d. = \$71.29\frac{1}{2}$$

(7)

$$£87 \times 400 = \$348.00$$

(8)

$$15s. \times 20 = \$3.00$$

$$11\frac{1}{2}d. = 47 \text{ far.} \times 5 \div 12 = .19\frac{1}{2}$$

$$15s. \text{ } 11\frac{1}{2}d. = \$3.19\frac{1}{2}$$

(9)

$$£16 \times 400 = \$64.00$$

$$6s. \times 20 = 1.20$$

$$2d. = 8 \text{ far.} \times 5 \div 12 = .03\frac{1}{3}$$

$$£16 \text{ } 6s. \text{ } 2d. = \$65.23\frac{1}{3}$$

(10)

$$£2 \times 400 = \$8.00$$

$$9s. \times 20 = 1.80$$

$$11d. = 44 \text{ far.} \times 5 \div 12 = .18\frac{1}{3}$$

$$£2 \text{ } 9s. \text{ } 11d. = \$9.98\frac{1}{3}$$

## EXERCISE 13—Page 90.

(1)

$$36 = 12 \times 3$$

$$\$169.78$$

$$12$$

$$2037.36$$

$$3$$

$$\$6112.08$$

(2)

$$121 = 11 \times 11$$

$$796342.3$$

$$11$$

$$8759765.3$$

$$11$$

$$96357418.3$$

(3)

$$144 = 12 \times 12$$

$$\$33460$$

$$12$$

$$401520$$

$$12$$

$$\$4818240$$

(4)

$$648 = 12 \times 9 \times 6$$

$$735$$

$$12$$

$$8820$$

$$9$$

$$79380$$

$$6$$

$$476280$$

(5)

$$18 = 6 \times 3$$

$$£ \text{ s. d.}$$

$$3 \text{ } 7 \text{ } 6$$

$$6$$

$$20 \text{ } 5 \text{ } 0$$

$$3$$

$$60 \text{ } 15 \text{ } 0$$

(6)

$$22 = 11 \times 2$$

$$£ \text{ s. d.}$$

$$5 \text{ } 14 \text{ } 6\frac{1}{2}$$

$$11$$

$$62 \text{ } 19 \text{ } 11\frac{1}{2}$$

$$2$$

$$125 \text{ } 19 \text{ } 11$$

(7)

$$810 = 10 \times 9 \times 9$$

$$£ \text{ s. d.}$$

$$3 \text{ } 4 \text{ } 7$$

$$10$$

$$32 \text{ } 5 \text{ } 10$$

$$9$$

$$290 \text{ } 12 \text{ } 6$$

$$9$$

$$2615 \text{ } 12 \text{ } 6$$

(8)

$$54 = 9 \times 6$$

$$\text{cwt. qrs. lbs. oz.}$$

$$11 \text{ } 3 \text{ } 14 \text{ } 7$$

$$9$$

$$107 \text{ } 0 \text{ } 4 \text{ } 15$$

$$6$$

$$642 \text{ } 1 \text{ } 4 \text{ } 10$$

(9)

$$49 = 7 \times 7$$

bush.	pks.	gal.	qt.	pt.
26	3	1	1	1
				7

188	1	1	2	1
				7

1319	0	1	1	1
------	---	---	---	---

(10)

$$63 = 9 \times 7$$

yds.	qrs.	na.	in.
2	2	2	2
			9

24	0	2	0
			7

168	3	2	0
-----	---	---	---

(11)

$$288 = 12 \times 12 \times 2$$

dys.	hrs.	min.	sec.
5	17	33	11
			12

68	18	38	12
			12

825	7	38	24
			2

1650	15	16	48
------	----	----	----

## EXERCISE 14—Page 92.

(1)

$$83 = 3 + 10 \times 8$$

£	s.	d.	£	s.	d.
12	2	4	36	7	0
		10			

121	3	4	8	969	6	8
-----	---	---	---	-----	---	---

1005	13	8
------	----	---

(2)

$$999 = 10 \times 10 \times 10 - 1$$

£	s.	d.
963	0	0½
		10

9630	0	7½
		10

96300	6	3
		10

963003	2	6
963	0	0½

962040	2	5½
--------	---	----

(3)

$$3178 = 8 + 10 \times 7 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 3$$

£	s.	d.	£	s.	d.
3	6	5½	26	11	6
		10			

33	4	4½	7	232	10	7½
		10				

332	3	9	1	332	3	9
		10				

3321	17	6	3	9965	12	6
------	----	---	---	------	----	---

10556	18	4½
-------	----	----

(4)

$$678 = 8 + 10 \times 7 + 10 \times 10 \times 6$$

bush.	pk.	gal.	bush.	pk.	gal.
16	3	1	135	0	0
		10			

168	3	0	7	1181	1	0
		10				

1687	2	0	6	10125	0	0
------	---	---	---	-------	---	---

11441	1	0
-------	---	---

(11)

$$= 12 \times 12 \times 2$$

rs.	hrs.	min.	sec.
5	17	33	11
			12

8	18	38	12
			12

5	7	38	24
			2

0	15	16	48
---	----	----	----

(2)

$$\times 10 \times 10 - 1$$

£	s.	d.
963	0	0½
		10

9630	0	7½
		10

6300	6	3
		10

3003	2	6
963	0	0½

2040	2	5½
------	---	----

(4)

$$7+10 \times 10 \times 6$$

bush.	pk.	gal.
135	0	0

= 1181	1	0
--------	---	---

= 10125	0	0
---------	---	---

11441	1	0
-------	---	---

(5)

$$247 = 7 + 10 \times 4 + 10 \times 10 \times 2$$

m.	fur.	rds.	yds.		m.	fur.	rds.	yds.
23	6	33	4	$\times 7 =$	166	7	36	0½
			10					

238	4	17	1½	$\times 4 =$	954	1	29	0½
			10					

2385	4	12	4	$\times 2 =$	4771	0	25	2½
					5892	2	10	3½

(6)

$$721 = 1 + 10 \times 2 + 10 \times 10 \times 7$$

S.	deg.	min.	sec.		S.	deg.	min.	sec.
3	16	30	45	$\times 1 =$	3	16	30	45
			10					

35	15	7	30	$\times 2 =$	71	0	15	0
			10					

355	1	15	0	$\times 7 =$	2485	8	45	0
					2559	25	30	45

## EXERCISE 15—Page 93.

(6)	(7)	(8)	(9)
7071	15607	39948123	2778588
556	3094	6007	9867
42426	62428	279636861	19450116
35355	140463	23968873800	16671528
35355	468210	239968374861	22228704
3931476	48288058		25007292
			27416327796

## EXERCISE 16—Page 95.

(4)	(5)	(6)	(7)	(8)
3.2517	64.001	482000	3782.4	87.96
.023	340	.37	.00917	220
97551	2560040	3374000	264768	175920
65034	192003	1446000	37824	17592
.0747891	21760.340	178340.00	340416	19351.20
			34.684608	

## EXERCISE 17—Page 100.

(1)	(2)	(3)	(4)
$216 = 6 \times 6 \times 6$	\$61135.37	255226	$176 = 11 \times 8 \times 2$
\$83469	229	143	
6	55021833	765678	203736
500814	12227074	1020904	11
6	12227074	255226	2241096
3004884	\$13999999.73	36497318	8
6			17928768
\$18029304			2
			35857536

(5)	(6)	(7)	(8)
116700	3721	$297 = 11 \times 9 \times 3$	$35 = 7 \times 5$
235	73	32000	9344000
583500	11163	11	7
350100	26047	352000	65408000
233400	271633	9	5
27424500		3168000	327040000
		3	
		9504000	

(9)	(10)
$749 = 9 + 10 \times 4 + 10 \times 10 \times 7$	$999998 = 1000000 - 2$
lbs. oz. drs. scr. gr.	lbs. oz. drs. scr. grs.
123 4 7 2 $17 \times 9 =$	1110 8 7 1 13
10	1698732
1234 1 7 1 $10 \times 4 =$	1000000
10	1698732000000
12341 7 3 0 $0 \times 7 =$	3397464
86391 3 5 0 0	1698728602536
92438 8 2 1 13	

(4)

$$176 = 11 \times 8 \times 2$$

203736

11

2241096

8

17928768

2

35857536

(11)

$$640 = 10 \times 8 \times 8$$

bush. pk. gal. qt. pt.

123 1 1 1 1

10

1234 0 1 3 0

8

9873 3 0 0 0

8

78990 0 0 0 0

(12)

89

.73

267

623

\$64.97

(8)

$$35 = 7 \times 5$$

9344000

7

65408000

5

327040000

(13)

$$1143 = 3 + 10 \times 4 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 1$$

yds. qrs. na. in.

7 3 2 1  $\times 3 =$  23 2 3 01

10

$$79 \ 0 \ 0 \ 1 \times 4 = 316 \ 0 \ 1 \ 11$$

10

$$790 \ 1 \ 0 \ 1 \times 1 = 790 \ 1 \ 0 \ 1$$

10

$$7902 \ 3 \ 0 \ 1 \times 1 = 7902 \ 3 \ 0 \ 1$$

9032 3 2 0

(14)

1634.5789

635000

81728945000

49037367

98074734

1037957601.5

(10)

$$8 = 1000000 - 2$$

1698732

1000000

698732000000

3397464

698728602536

(15)

\$968.49

3.4

387396

290547

\$3292.866

\$12183.6042

3292.866

968.49

\$16444.9602

\$3292.866

3.7

23050062

9878598

\$12183.6042



## EXERCISE 18—Page 110.

(9)

6423)798965(1248413

6423

15666

12846

28205

25692

2513

(10)

£ s. d.

12)176 14 6

14 14 6

(11)

741)56789(76417

5187

4919

4446

473

(12)

7894)6785158(8594813

63152

46995

39470

75258

71046

4212

(13)

£ s. d. £ s. d.  
317)4728 16 2(14 18 417

317

1558

1268

290

20

5816

317

2646

2536

110

12

1322

1268

54

(15)

6)970763

161793·8333+

(16)

9)71234

79148

(14)

429)\$97896·64(\$228·19113

858

—

1209

858

—

3516

3432

—

84·6

42·9

—

41·74

38·61

—

3·13

(17)

1209 47600)977076(2045878

858

—

3516

3432

—

84·6

42·9

—

41·74

38·61

—

3·13

(10)

$$\begin{array}{r} \text{£ s. d.} \\ 8 \ 14 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \ 14 \ 6\frac{1}{2} \\ \hline \end{array}$$

(12)

$$58(859\frac{1}{2}\frac{1}{2})$$

5

9

8

6

2

$$28 \cdot 19\frac{1}{2}$$

(17)

$$977076(20\frac{1}{2}\frac{1}{2})$$

95200

25076

(18)

lbs.	oz.	drs.	scr.	grs.	lbs.	oz.	drs.	scr.	grs.
498)7289	6	4	2	13	14	7	5	0	12
498									
2309									
1992									

(19)

£	s.	d.	s.	d.
487)157	16	7	6	5
12				
20				

3810

3486

324

8

2596

2490

106

3

320

20

6413

5976

437

3156

2922

234

12

2815

2435

380

4

1520

1461

59

(20)

9712)7867674(810<sup>244</sup>

77696

9807

9712

954

(21)

m.	fur.	rds.	m.	fur.	rds.
37)422	3	38	11	3	14

407

15

8

123

111

12

40

518

37

148

148

## EXERCISE 19—Page 112.

(1)

25=5×5

5)3766

5)753... 1

150... 3

3×5+1=16

150 $\frac{1}{5}$ 

(2)

42=7×6

7)26406

6)3772... 2

628... 4

4×7+2=30

628 $\frac{1}{7}$ 

(3)

96=12×8

12)25431

8)2119... 3

264... 7

7×12+3=87

264 $\frac{1}{6}$ 

(4)

24=12×2

£	s.	d.
12)24	17	6

2)2 1 5 $\frac{1}{2}$ 1 0 8 $\frac{1}{2}$ 

B

(5)

$49 = 7 \times 7$

	£	s.	d.
7)740	13	4	
<hr/>			
7)105	16	2½	...1
<hr/>			
	15	2	3½...1

(6)

$56 = 8 \times 7$

	£	s.	d.
8)	547	12	4
<hr/>			
7)	68	9	0½
<hr/>			
	9	15	6½... $\frac{49}{88}$

$$\begin{array}{r} (7) \\ 35 = 7 \times 5 \\ 7)6789436 \\ \underline{\phantom{0000000}} \\ 5)969919\dots3 \\ \underline{\phantom{0000000}} \\ 193983\dots4 \\ 4 \times 7 + 3 = 31 \\ 1939833! \end{array}$$
$$\begin{array}{r}
 (8) \\
 147 = 7 \times 7 \times 3 \\
 \underline{7) 753293} \\
 7) 107613 \dots 2 \\
 \underline{\phantom{7) 107613} 7) 15373 \dots 2} \\
 \underline{\phantom{7) 107613} \phantom{7) 15373} 5124 \dots 1} \\
 1 \times 7 \times 7 + 2 \times 7 + 2 = 65 \\
 5124_{65}
 \end{array}$$

(9)

$81 = 9 \times 9$

lbs. oz. dwt. grs.

9)1798 6 11 9

---

9)199 10 1 6...3

---

22 2 9 0...6

$6 \times 9 + 3 = 57$

22 lbs. 2 oz. 9 dwt. 0.57 grs.

**EXERCISE 20—Page 114.**

(1)				(2)			
£	s.	d.	£	s.	d.	m. fur. rds.	m. fur. rds.
491	12	0½	) 8968	13	7½	17 5 27	) 1027 1 6
20			20			8	8
9832			179373			141	8217
12			12			40	40
117984			2152483			5667	) 328686 (58
4			4				28335
471937	)		8609934	(18115068			45336
			471937	471937			45336
			3890564				
			3775496				
			115068				

(7)

$$35 = 7 \times 5$$

$$7)6789436$$

$$5)969919...3$$

$$193988...4$$

$$4 \times 7 + 3 = 31$$

$$193983\frac{3}{8}$$

(9)

$$81 = 9 \times 9$$

$$\text{oz. dwt. grs.}$$

$$6 \ 11 \ 9$$

$$10 \ 1 \ 6...3$$

$$2 \ 9 \ 0...6$$

$$+ 3 = 57$$

$$9 \text{ dwt. } 0\frac{4}{7} \text{ grs.}$$

(2)

$$\text{s. m. fur. rds.}$$

$$) 1027 \ 1 \ 6$$

$$8$$

$$8217$$

$$40$$

$$328686 \ (58)$$

$$28335$$

$$45336$$

$$45336$$

(3)

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 57 \quad 0 \quad 7\frac{1}{2} \end{array} ) \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 171 \quad 1 \quad 10\frac{1}{2} \end{array}$$

$$20 \quad 20$$

$$1140 \quad 3421$$

$$12 \quad 12$$

$$13687 \quad 41062$$

$$4 \quad 4$$

$$54750 \ ) \quad 164250 \ (3)$$

$$164250$$

(4)

$$\begin{array}{r} \text{dwt. grs.} \quad \text{lbs} \quad \text{oz.} \quad \text{dwt. grs.} \\ 5 \ 9 \ ) \quad 9 \ 9 \ 3 \ 12 \end{array}$$

$$24 \quad 12$$

$$129 \quad 117$$

$$24 \quad 20$$

$$2343$$

$$24$$

$$9384$$

$$4686$$

$$129)56244(436$$

$$516$$

$$464$$

$$387$$

$$774$$

$$774$$

## EXERCISE 21—Page 115.

(5)

$$\begin{array}{r} \text{a.} \quad \text{r.} \quad \text{per.} \\ 91 \quad 0 \quad 6 \end{array} ) \begin{array}{r} \text{a.} \quad \text{r.} \quad \text{per.} \\ 2366 \quad 3 \quad 36 \end{array}$$

$$4 \quad 4$$

$$364 \quad 9467$$

$$40 \quad 40$$

$$14566 \ ) \quad 378716 \ (26)$$

$$29132$$

$$87396$$

$$87396$$

(6)

$$47 \cdot 655 \div 4 \cdot 5 =$$

$$45)476 \cdot 55(10 \cdot 59$$

$$45$$

$$26 \cdot 5$$

$$22 \cdot 5$$

$$4 \cdot 05$$

$$4 \cdot 05$$

(8)

$$47 \cdot 5782975 \div 26 \cdot 175 =$$

$$26175)47578 \cdot 2975(1 \cdot 8177$$

$$26175$$

$$21403 \cdot 2$$

$$20940 \cdot 0$$

$$463 \cdot 29$$

$$261 \cdot 75$$

$$201 \cdot 547$$

$$183 \cdot 225$$

$$18 \cdot 3225$$

$$18 \cdot 3225$$

(7)

$$756 \cdot 98 \div 76 \cdot 73612 =$$

$$7673612)75698000 \ (9 \cdot 864 +$$

$$69062508$$

$$6635492 \cdot 0$$

$$6138889 \cdot 6$$

$$496602 \cdot 40$$

$$460416 \cdot 72$$

$$36185 \cdot 680$$

$$30694 \cdot 448$$

$$5491 \cdot 232$$

$$\begin{array}{r}
 (9) \\
 1 \div 7.8345 = \\
 76345 \overline{) 10000.0(0.1309 +} \\
 \underline{7634.5} \\
 2365.50 \\
 \underline{2290.35} \\
 75.1500 \\
 68.7105
 \end{array}$$

$$\begin{array}{r}
 (10) \\
 75.347 \div 0.3829 = \\
 3829 \overline{) 753470(196.7798 +} \\
 \underline{3829} \\
 37057 \\
 \underline{34461} \\
 25960 \\
 \underline{22974} \\
 2986.0 \\
 \underline{2680.3} \\
 305.70 \\
 \underline{268.03} \\
 37.670 \\
 \underline{34.461} \\
 3.2090 \\
 \underline{3.0632} \\
 .1458
 \end{array}$$

$$\begin{array}{r}
 (11) \\
 .0002 \div 0.00000008 = \\
 8) 200000 \\
 \underline{25000}
 \end{array}$$

## EXERCISE 22—Page 116.

(1)		(2)
95) \$3300000 (\$34736.8421		126) \$3860000 (\$30634.9206
285	(3) dys.	378
450	28800) 95270400 (3308	800
380	86400	756
700	88704	440
665	86400	378
350	230400	620
285	230400	504
650	days.	116.0
570	365½) 3308	113.4
80.0	4 4	
76.0	1461) 13232 (9 20½	2.60
4.00	13149	2.52
3.80	4) 83	.800
.200	20½	.756
.190		.044
.100		(4)
.095		35781628) \$1145012096 (\$32
.005		107344884
		71563256
		71563256

$$\begin{array}{r} = \\ + \\ (11) \\ \div 000000008 = \end{array}$$

$$\begin{array}{r} 8)200000 \\ \hline 25000 \end{array}$$

$$\begin{array}{r} (2) \\ 0(\$30634 \cdot 9206 \end{array}$$

$$\begin{array}{r} .0 \\ .4 \\ .60 \\ .52 \\ .800 \\ .756 \\ .044 \end{array}$$

$$\begin{array}{r} 2096(\$32 \\ 384 \\ 3256 \\ 3256 \end{array}$$

(5)	(6)	(7)
27475271)	33764112127(\$137	9)\$972
27475271		108)\$972(\$9
		972
	\$108	
101658502	(9)	(10)
82425813	792)340480(429	1728)1000(578 oz.
	3168	864-0
192326897		
192326897		136-00
	2368	120-96
(8)	1584	
294)\$8526(\$29		(12)
588	7840	19)4750(250lbs. 15-040
m. fur.	38	13-824
2646 33 2	7128	
2646 8		95
	712	95
266	712=88	
40		(14)
10640	bush. pk. gal. qt. pt. bush. pk. gal. qt. pt.	
51	297)729 1 1 1 1 (2 1 1 2 1 1	
	594	
53200		
5320	135	
	4	
1155)58520(50 <sup>770</sup> <sub>1166</sub>	541	
5775	297	
770	244	
50 <sup>770</sup> <sub>1166</sub> =50 <sup>3</sup>	2	
(13)	489	
978-634÷96-34762 =	297	
9634762)97863400(10-157		
9634762	192	
	4	
1515780-0		
963476-2	769	
	594	
552303-80		
481738-10	175	
	2	
70565-700		
67443-334	351	
	297	
3122-366		
	54	
	54=11	



(15)				(16)			
lbs. oz. dr.	swt. gr. lbs. oz. dr.			m. fur. rds.	m.	dys.	hrs.
9 7 8 )	170 3 4 16 0			93 4 7	25000		
16	4			8			
151	719						
16	25						
914	3599			748	200000		
151	1438			40	40		
2424	17979	29927	29927)	8000000			
	16			59854			
	107890			201460			
	17979			179562			
	287680			218980			
	16			209489			
	1726080			9491			
	287680			24			
2424)	4602880	(1898303		37964			
	2424			18982			
	21788			227784			
	19392			209489			
	23968			18295			
	21816						
	21520						
	19392						
	2128	2128 = 256.					

EXERCISE 23—Page 118.

(3)

DOCIX, M̄VCCCLXXV, M̄XCMXCIX, LXXXVMIV,  
 MMMCMXLVMDCCVI.

(4)

$$72 = 8 \times 9$$

lbs. oz.

$$749 \text{ } 10$$

8

$$\underline{5997 \text{ } 0}$$

9

$$\underline{53973 \text{ } 0}$$

(5)

$$17 = 7 + 10 \times 1$$

s. d. £ s. d.

$$4 \text{ } 7\frac{1}{2} \times 7 = 1 \text{ } 12 \text{ } 6\frac{1}{2}$$

10

$$\underline{\text{£} 2 \text{ } 6 \text{ } 5\frac{1}{2} + 1 = 2 \text{ } 6 \text{ } 5\frac{1}{2}}$$

$$\underline{3 \text{ } 18 \text{ } 11\frac{1}{2}}$$

(6)

$$30)2850000000$$

dys. hrs.

$$24)95000000(3958333 \text{ } 8$$

72

230

216

3654

4

ysr.

$$3958333(10837$$

4

140

120

$$1461)15833332$$

$$\underline{1461}$$

200

192

$$\underline{12233}$$

$$\underline{11688}$$

80

72

$$\underline{5453}$$

$$\underline{4383}$$

80

72

$$\underline{10702}$$

$$\underline{10227}$$

80

72

$$4)475 \text{ quarter days.}$$

$$\underline{\hspace{1cm}} \text{ days. hrs.}$$

$$118\frac{1}{2} = 118 \text{ } 18$$

8 rem.

Add

8

$$\underline{119 \text{ } 2}$$

$$10837 \text{ yrs. } 119 \text{ days, } 2 \text{ hrs.}$$

(7)

$$\text{£} 729 \times 400$$

$$= \$2916 \cdot 00$$

$$17\text{s.} \times 20$$

$$= 3 \cdot 40$$

$$6\frac{1}{2}\text{d.} = 25 \text{ far.} \times 5 \div 12 =$$

$$10\frac{5}{12}$$

$$\underline{\text{\$} 2919 \cdot 50\frac{5}{12}}$$

(8)

$$\text{\$} 10000$$

$$\underline{9876 \cdot 23}$$

$$\text{\$} 123 \cdot 77$$

(10)

$$\begin{array}{r}
 \text{in.} \\
 12)7964327 \\
 \hline
 12)663693-11 \left. \vphantom{12)663693-11} \right\} 119 \text{ in.} \\
 \hline
 55307-9 \\
 \hline
 9)55307 \text{ ft. } 119 \text{ in.} \\
 \hline
 304) 6145 \text{ yds. } 2 \text{ ft. } 119 \text{ in.} \\
 \begin{array}{r} 4 \quad 4 \end{array}
 \end{array}$$

(11)

$$\begin{array}{r}
 \$729.43 \\
 18.79 \\
 976.81 \\
 9987.17 \\
 429.00 \\
 129.19 \\
 \hline
 \$12268.30
 \end{array}$$

$$\begin{array}{r}
 121)24580 \\
 11)24580
 \end{array}$$

$$\begin{array}{l}
 203 \text{ p. } 4\frac{1}{2} \text{ y.} = 203 \text{ p. } 4 \text{ yds. } 2 \text{ ft. } 36 \text{ in.} \\
 \text{Add} \qquad \qquad \qquad 2 \text{ ft } 119 \text{ in}
 \end{array}$$

$$\begin{array}{r}
 11)2234-6 \\
 \hline
 203-1 \left. \vphantom{11)2234-6} \right\} 17 \text{ qr. yds.}
 \end{array}$$

$$40)203 \text{ p. } 4 \text{ yds. } 5 \text{ ft. } 11 \text{ in.}$$

$$4)5 \text{ rd. } 3 \text{ p. } 4 \text{ yds. } 5 \text{ ft. } 11 \text{ in.}$$

$$1 \text{ a. } 1 \text{ r. } 3 \text{ p. } 4 \text{ yds. } 5 \text{ ft. } 11 \text{ in.}$$

(12)

$$429 = 9 + 10 \times 2 + 10 \times 10 \times 4$$

wks.	dys.	hrs.	min.		wks.	dys.	hrs.	min.
6	4	3	17	$\times 9 =$	59	2	5	33
			10					

65	6	8	50	$\times 2 =$	131	5	17	40
			10					

659	0	16	20	$\times 4 =$	2636	2	17	20
					2827	3	16	33

wks.

$$52)2827(54 \text{ yrs. } 19 \text{ wks. } 3 \text{ dys. } 16 \text{ hrs. } 33 \text{ min.}$$

260

227

208

19 wks.

(15)

tons.			
324			
20			
cwt. qr. lbs.			
13	2	14	6480
4			4
54			25920
25			25

284 129600  
108 51840

1364 ) 648000 (475  $\frac{100}{1364}$  =  
5456 475  $\frac{25}{34}$  hds.

(14)  
78.96 10240  
000042 9548

15792 6920  
31584 6820  
0331632 100

(16)

tons. \$136  
\$136  $\times$  4 = 544 — 95 = 449  
1902  
2487

\$9237 — \$2487 = \$6750

(17)

yds. qrs. na. yds. qrs. na.  
3 1 2 ) 39 2 3

4 4

13 158

4 4

54 ) 635 (11  $\frac{11}{54}$

54

95

54

41

(18)

(19)

(21)

a. a. a. r. per.  
25 732 96 3 17  
197 674 4  
156 — —  
97 58 387  
199 40  
674 15497 ) \$7764.0 (\$0.501

7748.5

15.500

15.497

3

(20)

\$  
20 \$312  
75 275  
97 —  
83 \$ 37

275

lbs. oz. dwt. grs.  
12) 36 8 14 16

3 0 14 13  $\frac{1}{3}$

(22)

a. r. per.  
6 3 12  
7 2 0  
9 0 13  
5 2 36  
29 0 21

26

KEY.

[NAT. ARITH.]

(23)

(24)

(25)

5	lbs. oz. dwt. grs.	$\pounds 972 \times 400 = \$3888.00$
7	5 9 8 0	$11s. \times 20 = 2.20$
9	3 2 16 16	$11\frac{1}{2}d. = 45 \text{ far.} \times 5 \div 12 = .18\frac{1}{2}$
—	4 6 17 0	
21)294(14	1 8 19 22	<u>\$3890.38\frac{1}{2}</u>
21		
—	15 4 1 14	
84		
84		

(26)

(27)

(28)

lbs. oz. drs. scr. grs.	56	ewt. qr. lbs.
179 3 3 1 14	25	6 2 11
12	—	5 3 16
—	280	8 0 7
2151 oz.	112	3 1 17
8	—	lbs.
—	1400	24 0 1 = 2401
17211 drs.	2	.15
3	—	
• 51634 scr.	2800 sq. ft. in roof.	12005
20	6	2401
—	—	<u>\$360.15</u>
1032694 grs.	16800	

(29)

(30)

29	\$	
57	139468	370129
—	98579	238047
203	—	—
145	\$238047	\$132082
—		
1653		
.15		
—		
8265		
1653		
—		
\$247.95		

$$\begin{array}{r}
 400 = \$3888.00 \\
 20 = 2.20 \\
 \div 12 = .18\frac{1}{3} \\
 \hline
 \$3890.38\frac{1}{3}
 \end{array}$$

(28)

$$\begin{array}{r}
 \text{qr. lbs.} \\
 2 \ 11 \\
 3 \ 16 \\
 0 \ 7 \\
 1 \ 17 \\
 \hline
 \text{lbs.} \\
 0 \ 1 = 2401 \\
 \hline
 .15 \\
 \hline
 12005 \\
 2401 \\
 \hline
 \$360.15
 \end{array}$$

$$\begin{array}{r}
 (31) \\
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 9 \ 19 \ 11\frac{1}{4} \\
 20 \\
 \hline
 199 \\
 12 \\
 \hline
 2399 \\
 84 \\
 \hline
 9659 \\
 19192 \\
 \hline
 201579
 \end{array}
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 1694 \ 16 \ 0\frac{1}{4} \\
 20 \\
 \hline
 33896 \\
 12 \\
 \hline
 406752 \\
 84 \\
 \hline
 1627030 \\
 3254016 \\
 \hline
 64167190 \\
 201579 \\
 \hline
 1400929 \\
 1209474 \\
 \hline
 1914550 \\
 1814211 \\
 \hline
 100339.0 \\
 80631.6 \\
 \hline
 19707.40 \\
 18142.11 \\
 \hline
 1565.29
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (34) \\
 \begin{array}{r}
 \text{cwt. qr. lbs.} \\
 2 \ 0 \ 17 \\
 3 \ 2 \ 15 \\
 2 \ 1 \ 20 \\
 5 \ 3 \ 17 \\
 \hline
 \text{lbs.} \\
 14 \ 0 \ 19 = 1419 \\
 \hline
 .37\frac{1}{2} \\
 \hline
 9933 \\
 4257 \\
 709\frac{1}{2} \\
 \hline
 \$532.12\frac{1}{2}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (32) \\
 \begin{array}{r}
 \text{£} 19 \times 400 = \$76.00 \\
 19\text{s.} \times 20 = 3.80 \\
 11\frac{1}{2}\text{d} = 47\text{far} \times 5 \div 12 = .19\frac{1}{2}\text{f} \\
 \hline
 \$79.99\frac{1}{2}\text{f}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (33) \\
 \begin{array}{r}
 \text{cwt. qr. lbs.} \quad \text{cwt. qr. lbs.} \\
 3 \ 2 \ 11 \quad 12 \ 0 \ 0 \\
 4 \ 1 \ 15 \quad 8 \ 0 \ 1 \\
 \hline
 \text{lbs.} \\
 8 \ 0 \ 1 \quad 3 \ 3 \ 24 = 399 \\
 \hline
 .15 \\
 \hline
 1995 \\
 399 \\
 \hline
 \$59.85
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (36) \\
 43.2 \div 76.8437 = \\
 768437)432000.0(0.562 \\
 384218.5 \\
 \hline
 47781.50 \\
 46106.22 \\
 \hline
 1675.280 \\
 1536.874 \\
 \hline
 138.406
 \end{array}$$



(37)

$$\begin{array}{r}
 123 \cdot 4 \div 000000066 = \\
 123400000000 \div 66 \\
 6) 123400000000 \\
 \hline
 11) 205666666666 \cdot 666 \\
 \hline
 1869696969 \cdot 69
 \end{array}$$

(38)

$$\begin{array}{r}
 \$63 \cdot 29 \quad \$2789 \cdot 27 \\
 \underline{17} \quad \underline{1075 \cdot 93} \\
 44303 \\
 \underline{6329} \\
 \$1075 \cdot 93
 \end{array}$$

(39)

$$\begin{array}{r}
 £29 \times 400 = \$116 \cdot 00 \quad \$278 \cdot 43 \\
 6s. \times 20 = 1 \cdot 20 \quad 417 \cdot 16 \\
 11 \frac{1}{2} d. = 47 far. \times 5 \div 12 = 19 \frac{1}{2} \quad 11 \cdot 27 \\
 \hline
 117 \cdot 39 \frac{1}{2} \quad 2110 \cdot 40 \\
 \hline
 723 \cdot 15 \\
 117 \cdot 39 \frac{1}{2} \\
 \hline
 173) 3657 \cdot 80 \frac{1}{2} \\
 \underline{12} \quad \underline{12} \\
 2076) 43893 \cdot 67 (\$21 \cdot 1433
 \end{array}$$

(40)

$$\begin{array}{r}
 2076) 491544 \cdot 2361 \frac{1}{2} \\
 \underline{4152} \\
 7634 \\
 \underline{6228} \\
 14064 \\
 \underline{12456} \\
 1608 \\
 \hline
 1898 = 198 \\
 2076) 491544 \cdot 2361 \frac{1}{2} \\
 \hline
 2373 \\
 2076 \\
 \hline
 297 \cdot 6 \\
 207 \cdot 6 \\
 \hline
 90 \cdot 07 \\
 83 \cdot 04 \\
 \hline
 7 \cdot 030 \\
 6 \cdot 228 \\
 \hline
 8020 \\
 6228 \\
 \hline
 1792
 \end{array}$$

## Exercise 24.—Page 127.

(1)	(2)	(3)	(4)
2)11368	2)2934	3)1011	2)1000
2)5684	3)1467	337	2)500
2)2842	3)489	3×337	2)250
7)1421	163		5)125
7)203	2×3 <sup>2</sup> ×163		5)25
29			5
2 <sup>3</sup> ×7 <sup>2</sup> ×29			2 <sup>3</sup> ×5 <sup>3</sup>
(5)	(6)	(7)	(8)
2)1024	2)32320	7)707	2)1118
2)512	2)16160	101	13)559
2)256	2)8080	7×101	43
2)128	2)4040		2×13×43
2)64	2)2020		
2)32	2)1010		
2)16	5)505		
2)8	101		
2)4	2 <sup>5</sup> ×5×101		
2			
2 <sup>10</sup>			

## EXERCISE 25—Page 128.

(1)

$$100 = 2^3 \times 5^2$$

1..2..4

1..5..25

1..2..4..5..10..20..25..50..100

(2)

$$810 = 3^4 \times 2 \times 5.$$

1..3..9..27..81

1..2

1..3..9..27..81..2..6..18..54..162

1..5

$$1..3..9..27..81..2..6..18..54..162..5..15..45..135..405..10..30..90..270..810 =$$

$$1..2..3..5..6..9..10..15..18..27..30..45..54..81..90..135..162..270..405..810.$$

(3)

$$920 = 2^3 \times 5 \times 23.$$

1..2..4..8

1..5

1..2..4..8..5..10..20..40

1..23

$$1..2..4..8..5..10..20..40..23..46..92..184..115..230..460..920 =$$

$$1..2..4..5..8..10..20..23..40..46..92..115..184..230..460..920.$$

(4)

$$25000 = 5^5 \times 2^3$$

1..5..25..125..625..3125

1..2..4..8

$$1..5..25..125..625..3125..2..10..50..250..1250..6250..4..20..100..500..2500..12500..8..40..200..1000..5000..25000 =$$

$$1..2..4..5..8..10..20..25..40..50..100..125..200..250..500..625..1000..1250..2500..3125..5000..6250..12500..25000.$$

## EXERCISE 26—Page 128.

(1)

$$88200 = 2^3 \times 3^2 \times 5^2 \times 7^2$$

$$3+1=4$$

$$2+1=3$$

$$2+1=3$$

$$2+1=3$$

$$4 \times 3 \times 3 \times 3 = 108$$

(2)

$$3500 = 2^2 \times 5^3 \times 7$$

$$2+1=3$$

$$3+1=4$$

$$1+1=2$$

$$3 \times 4 \times 2 = 24$$

(3)

$$6336=2^6 \times 3^2 \times 11$$

$$6+1=7$$

$$2+1=3$$

$$1+1=2$$

$$7 \times 3 \times 2=42$$

..45..135..405..

..54..81..90..135..

(5)

$$49000=2^3 \times 5^3 \times 7^2$$

$$3+1=4$$

$$3+1=4$$

$$2+1=3$$

$$4 \times 4 \times 3=48$$

..230..460..920 =

34..230..460..920.

(7)

$$75600=2^4 \times 3^3 \times 5^2 \times 7$$

$$4+1=5$$

$$3+1=4$$

$$2+1=3$$

$$1+1=2$$

$$5 \times 4 \times 3 \times 2=120$$

(4)

$$824=2^3 \times 103$$

$$3+1=4$$

$$1+1=2$$

$$4 \times 2=8$$

(6)

$$81000=2^3 \times 3^4 \times 5^3$$

$$3+1=4$$

$$4+1=5$$

$$3+1=4$$

$$4 \times 5 \times 4=80$$

(8)

$$25600=2^{10} \times 5^2$$

$$10+1=11$$

$$2+1=3$$

$$11 \times 3=33$$

3250..4..20..100..

000..25000 =

0..250..500..625..

.25000.

## EXERCISE 27—Page 129.

(1)

$$21=7 \times 3$$

$$18=2 \times 3 \times 3$$

$$27=3 \times 3 \times 3$$

$$36=4 \times 3 \times 3$$

(2)

$$21=3 \times 7$$

$$77=11 \times 7$$

$$42=2 \times 3 \times 7$$

$$35=5 \times 7$$

(3)

$$26=2 \times 13$$

$$52=2 \times 2 \times 13$$

$$91=7 \times 13$$

$$143=11 \times 13$$

3 is common to all. 7 is common to all. 13 is common to all.

(2)

$$=2^2 \times 5^3 \times 7$$

$$-1=3$$

$$+1=4$$

$$-1=2$$

$$\times 2=24$$

(4)

$$82=41 \times 2$$

$$118=59 \times 2$$

$$146=73 \times 2$$

2 is common to all.

## EXERCISE 28—Page 130.

(1)

$$\begin{array}{r} 296)407(1 \\ \underline{296} \end{array}$$

$$\begin{array}{r} 111)296(2 \\ \underline{222} \end{array}$$

$$\begin{array}{r} 74)111(1 \\ \underline{74} \end{array}$$

$$\begin{array}{r} 37)74(2 \\ \underline{74} \end{array}$$

G. C. M. = 37.

(2)

$$\begin{array}{r} 308)506(1 \\ \underline{308} \end{array}$$

$$\begin{array}{r} 198)308(1 \\ \underline{198} \end{array}$$

$$\begin{array}{r} 110)198(1 \\ \underline{110} \end{array}$$

$$\begin{array}{r} 88)110(1 \\ \underline{88} \end{array}$$

$$\begin{array}{r} 22)88(4 \\ \underline{88} \end{array}$$

G. C. M. = 22.

(3)

$$\begin{array}{r} 74)84(1 \\ \underline{74} \end{array}$$

$$\begin{array}{r} 10)74(7 \\ \underline{70} \end{array}$$

$$\begin{array}{r} 4)10(2 \\ \underline{8} \end{array}$$

$$\begin{array}{r} 2)4 \\ \underline{-} \end{array}$$

2

G. C. M. = 2.

(4)

$$\begin{array}{r} 1825)2555(1 \\ \underline{1825} \end{array}$$

$$\begin{array}{r} 730)1825(2 \\ \underline{1460} \end{array}$$

$$\begin{array}{r} 365)730(2 \\ \underline{730} \end{array}$$

G. C. M. = 365.

(5)

$$\begin{array}{r} 556)672(1 \\ \underline{556} \end{array}$$

$$\begin{array}{r} 116)556(4 \\ \underline{464} \end{array}$$

$$\begin{array}{r} 92)116(1 \\ \underline{92} \end{array}$$

$$\begin{array}{r} 24)92(3 \\ \underline{72} \end{array}$$

$$\begin{array}{r} 20)24(1 \\ \underline{20} \end{array}$$

$$\begin{array}{r} 4)20(5 \\ \underline{20} \end{array}$$

G. C. M. = 4.

## EXERCISE 29—Page 131.

(1)

$$\begin{array}{r} 110)140(1 \\ \underline{110} \end{array}$$

$$\begin{array}{r} 30)110(3 \\ \underline{90} \end{array}$$

$$\begin{array}{r} 20)30(1 \\ \underline{20} \end{array}$$

$$\begin{array}{r} 10)680 \\ \underline{68} \end{array} \quad \begin{array}{r} 10)20 \\ \underline{2} \end{array}$$

Therefore 10 is their G. C. M.

(3)

$$\begin{array}{r} 468)922(1 \\ \underline{468} \end{array}$$

$$\begin{array}{r} 454)468(1 \\ \underline{454} \end{array}$$

$$\begin{array}{r} 14)454(32 \\ \underline{42} \end{array}$$

$$\begin{array}{r} 34 \\ \underline{28} \end{array}$$

$$\begin{array}{r} 6)14(2 \\ \underline{12} \end{array}$$

$$\begin{array}{r} 2)6 \\ \underline{6} \end{array}$$

3

375 is not divisible by 2,  
and therefore their G. C. M. is 1.

(2)

$$\begin{array}{r} 1326)3094(2 \\ \underline{2652} \end{array}$$

$$\begin{array}{r} 442)1326(3 \\ \underline{1326} \end{array}$$

Also 4420 is divisible by 442;  
therefore it is their G. C. M.

(4)

$$\begin{array}{r} 204)1190(5 \\ \underline{1020} \end{array}$$

$$\begin{array}{r} 170)204(1 \\ \underline{170} \end{array}$$

$$\begin{array}{r} 34)170(5 \\ \underline{170} \end{array}$$

$$\begin{array}{r} 17)2006(118 \\ \underline{17} \end{array}$$

$$\begin{array}{r} 30 \\ \underline{17} \end{array}$$

$$\begin{array}{r} 136 \\ \underline{136} \end{array}$$

G. C. M.=17.

$$\begin{array}{r} 34)1445(42 \\ \underline{136} \end{array}$$

$$\begin{array}{r} 85 \\ \underline{68} \end{array}$$

$$\begin{array}{r} 17)34(2 \\ \underline{34} \end{array}$$

## EXERCISE 30—Page 132.

(2)

$$56=2^3 \times 7$$

$$84=2^2 \times 3 \times 7$$

$$140=2^2 \times 5 \times 7$$

$$168=2^3 \times 3 \times 7$$

The greatest factors which are common are  $2^2$  and 7;  
therefore the G. C. M.= $2^2 \times 7=28$ .

C

(3)

$$\begin{array}{r} 74)84(1 \\ \underline{74} \end{array}$$

$$\begin{array}{r} 10)74(7 \\ \underline{70} \end{array}$$

$$\begin{array}{r} 4)10(2 \\ \underline{8} \end{array}$$

$$\begin{array}{r} 2)4 \\ \underline{2} \end{array}$$

G. C. M. = 2.

22.

16(1

92

24)92(3

$$\begin{array}{r} 72 \\ \underline{72} \end{array}$$

$$\begin{array}{r} 20)24(1 \\ \underline{20} \end{array}$$

$$\begin{array}{r} 4)20(5 \\ \underline{20} \end{array}$$

G. C. M. = 4.

(3)

$$241920 = 2^5 \times 3^3 \times 5 \times 7$$

$$380160 = 2^5 \times 3^3 \times 5 \times 11$$

$$69120 = 2^9 \times 3^3 \times 5$$

$$103680 = 2^8 \times 3^4 \times 5$$

The greatest factors which are common are  $2^3$ ,  $3^3$  and 5;  
therefore the G. C. M.  $= 2^3 \times 3^3 \times 5 = 34560$ .

(4)

$$10800 = 2^4 \times 3^3 \times 5^2$$

$$28040 = 2^3 \times 5 \times 701$$

$$2160 = 2^4 \times 3^3 \times 5$$

The greatest factors which are common are  $2^3$  and 5;  
therefore the G. C. M.  $= 2^3 \times 5 = 40$ .

---

EXERCISE 31—Page 133.

(2)

$$6 = 2 \times 3$$

$$7 = 7$$

$$42 = 2 \times 3 \times 7$$

$$9 = 3^2$$

$$10 = 2 \times 5$$

$$630 = 2 \times 3^2 \times 5 \times 7$$

$$2 \times 3^2 \times 5 \times 7 = 630.$$

(3)

$$1 = 1$$

$$2 = 2$$

$$3 = 3$$

$$4 = 2^2$$

$$5 = 5$$

$$6 = 2 \times 3$$

$$7 = 7$$

$$8 = 2^3$$

$$9 = 3^2$$

$$3^2 \times 2^3 \times 5 \times 7 = 2520.$$

(4)

$$6 = 2 \times 3$$

$$9 = 3^2$$

$$12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$18 = 2 \times 3^2$$

$$21 = 3 \times 7$$

$$30 = 2 \times 3 \times 5$$

$$2^2 \times 3^3 \times 5 \times 7 = 1260.$$

(5)

$$670 = 2 \times 5 \times 67$$

$$100 = 2^2 \times 5^2$$

$$335 = 5 \times 67$$

$$25 = 5^2$$

$$2^2 \times 5^2 \times 67 = 6700$$

(6)

$$8 = 2^3$$

$$10 = 2 \times 5$$

$$18 = 2 \times 3^2$$

$$27 = 3^3$$

$$36 = 2^2 \times 3^2$$

$$44 = 2^2 \times 11$$

$$396 = 2^2 \times 3^2 \times 11$$

$$2^3 \times 3^3 \times 5 \times 11 = 11880.$$

## EXERCISE 32—Page 134.

(1)	(2)	(3)
2)12..10..24	2)14..21..3..2..63	2)18..12..39..216..234
2) 6.. 5..12	3) 7..21..3..1..63	2) 9.. 6..39..108..117
3) 3.. 5.. 6	7) 7.. 7..1..1..21	3) 9.. 3..39.. 54..117
1.. 5.. 2	1.. 1..1..1.. 3	3) 3.. 1..13.. 18.. 39
<hr/>		
2×2×3×5×2=120	2×3×7×3=126	13) 1.. 1..13.. 6.. 13
<hr/>		
1.. 1..1 .. 6.. 1		
2×2×3×3×13×6=2808		

are 2<sup>3</sup> and 5;  
=40.

(4)

6=2×3

9=3<sup>2</sup>

12=2<sup>2</sup>×3

15=3×5

18=2×3<sup>2</sup>

21=3×7

30=2×3×5

2×3<sup>3</sup>×5×7=1260.

(6)

=2<sup>3</sup>

=2×5

=2×3<sup>2</sup>

=3<sup>3</sup>

=2<sup>2</sup>×3<sup>2</sup>

=2<sup>2</sup>×11

=2<sup>2</sup>×3<sup>2</sup>×11

×5×11=11880.

(4)

2)8..18..15..20..70

2)4.. 9..15..10..35

3)2.. 9..15.. 5..35

5)2.. 3.. 5.. 5..35

2.. 3.. 1.. 1.. 7

2×2×3×5×2×3×7=2520

(6)

2)60..50..144..35..18

2)30..25.. 72..35.. 9

3)15..25.. 36..35.. 9

3) 5..25.. 12..35.. 3

5) 5..25.. 4..35.. 1

1.. 5.. 4.. 7.. 1

2×2×3×3×5×5×4×7=25200.

(5)

2)24..16..18..20

2)12.. 8.. 9..10

2) 6.. 4.. 9.. 5

3) 3.. 2.. 9.. 5

1.. 2.. 3.. 5

2×2×2×3×2×3×5=720.

(7)

2)27..54..81..14..63

3)27..27..81.. 7..63

3) 9.. 9..27.. 7..21

3) 3.. 3.. 9.. 7.. 7

7) 1.. 1.. 3.. 7.. 7

1.. 1.. 3.. 1.. 1

2×3×3×3×7×3=1134.

## EXERCISE 33—Page 136.

(1)	(2)
300 300..200..150..50..50..75..125 165	20..60..15..165..210..63..27
10  2	5 21 4.. 4 14..21.. 9
	12 4.. 4 4 4
300×10=3000.	165×21×12=41580.



(3)

$$\begin{array}{r|l}
 144 & 12 \dots 132 \dots 144 \dots 60 \dots 96 \dots 1728 \\
 12 & 11 \qquad \qquad 5 \quad 2 \quad 12 \\
 55 & 11 \qquad \qquad 5
 \end{array}$$

$$144 \times 12 \times 55 = 95040.$$

## EXERCISE 34—Page 138.

(1)	(2)	(3)	(4)
12)592835	5)3700	11)10000	6)1000000
12)49402.. <i>e</i>	5)740..0	11)909..1	6)166666.. <i>4</i>
12)4116.. <i>t</i>	5)148..0	11)82..7	6)27777.. <i>4</i>
12)343..0	5)29..3	7..5	6)4629..3
12)28..7	5)5..4	7571.	6)771..3
2..4	1..0		6)128..3
2470 <i>te</i>	104300.		6)21..2
			3..3
			33233344
(5)	(6)	(7)	(8)
8)10000	12)12345654321	9)10000	2)300
8)1250..0	12)1028804526.. <i>9</i>	9)1111..1	2)150..0
8)156..2	12)85733710.. <i>6</i>	9)123..4	2)75..0
8)19..4	12)7144475.. <i>t</i>	9)13..6	2)37..1
2..3	12)595372.. <i>e</i>	1..4	2)18..1
23420.	12)49614.. <i>4</i>	14641.	2)9..0
	12)4134.. <i>6</i>		2)4..1
	12)344.. <i>6</i>		2)2..0
	12)28.. <i>8</i>		1..0
	2..4		
	248664 <i>et</i> 69.		100101100

## EXERCISE 35—Page 139.

(1)	(2)	(3)
IX	V	IV
8)37704	7)444	9)1212201
8)4311..5	7)32..5	9)23121..0
6)168666..4	2..3	9)1101..0
6)27777..4	235.	9)21..0
6)4629..3	8)54..4	1..0
6)771..3	6..1	10000.
6)128..3	61415.	
6)21..2		
3..3		
33233344		

00  
11..1  
23..4  
13..6  
1..4  
4641.  
2)300  
2)150..0  
2)75..0  
2)37..1  
2)18..1  
2)9..0  
2)4..1  
2)2..0  
1..0  
100101100

## EXERCISE 36—Page 140.

(1)	(2)	(3)	(4)
IV	III	IX	VI
20212331	101202220	1522365	33233344
4	3	9	6
8	3	14	21
4	3	9	6
34	10	128	128
4	3	9	6
137	32	1154	771
4	3	9	6
550	96	10389	4629
4	3	9	6
2203	290	93507	27777
4	3	9	6
8815	872	841568	166666
4	3		6
35261	2618		1000000
	3		
	7854		

(6)

$$\begin{array}{r}
 \text{IX} \\
 3)132713 \\
 \hline
 3)40834..0 \\
 \hline
 3)13271..1 \\
 \hline
 3)4083..1 \\
 \hline
 3)1327..0 \\
 \hline
 3)408..1 \\
 \hline
 3)132..2 \\
 \hline
 3)40..2 \\
 \hline
 3)13..0 \\
 \hline
 3)4..0 \\
 \hline
 1..1
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 12)132713 \\
 \hline
 12)10207..9 \\
 \hline
 12)682..t \\
 \hline
 12)51..8 \\
 \hline
 3..t
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 8)132713 \\
 \hline
 8)14757..1 \\
 \hline
 8)1652..0 \\
 \hline
 8)184..6 \\
 \hline
 8)21..5 \\
 \hline
 2..3
 \end{array}$$

IX	III	XII	VIII
132713 =	11002210110 =	3t8t9 =	235601
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
12	4	46	19
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
110	12	560	157
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
997	36	6730	1262
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
8974	110	80769 den.	10096
<u>9</u>	<u>3</u>		<u>8</u>
80769 denary.	332	26923	80769 denary.
		<u>3</u>	
		80769 denary.	

(7)

IX

8)132713  
8  
 8)14757...1  
8  
 8)1652...0  
8  
 8)184...6  
8  
 8)21...5  
8  
 2...3

VIII

= 235601  
8  
 19  
8  
 157  
8  
 1262  
8  
 10096  
8  
 80769 denary.

XII

9)121290  
9  
 9)117978...0  
9  
 9)16241...2  
9  
 9)2032...4  
9  
 9)284...2  
9  
 9)37...1  
9  
 4...7

XII

6)121290  
6  
 6)185856...0  
6  
 6)34e4c...0  
6  
 6)6919...5  
6  
 6)1179...3  
6  
 6)233...3  
6  
 6)46...3  
6  
 6)9...0  
6  
 1...3

XII

4)121290  
4  
 4)268683...0  
4  
 4)78180...3  
4  
 4)1e050...0  
4  
 4)5913...0  
4  
 4)1533...3  
4  
 4)439...3  
4  
 4)10e...1  
4  
 4)32...3  
4  
 4)9...2  
4  
 2...1

XII

2)121290  
2  
 2)515146...0  
2  
 2)268683...0  
2  
 2)134341...1  
2  
 2)78180...1  
2  
 2)31010...0  
2  
 2)1e050...0  
2  
 2)e626...0  
2  
 2)5913...0  
2  
 2)2167...1  
2  
 2)1533...1  
2  
 2)877...1  
2  
 2)439...1  
2  
 2)211...1  
2  
 2)10e...0  
2  
 2)65...1  
2  
 2)32...1  
2  
 2)17...0  
2  
 2)9...1  
2  
 2)4...1  
2  
 2)2...0  
2  
 1...0

(Continued on next page.)

(7 continued.)

XII	IX	VI	IV	II	
<del>22290 = 4712420 = 130833500 = 21231330030 = 1001101101111100001100</del>					
12	9	6	4	2	
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	
122	43	9	9	2	1243
12	9	6	4	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
1474	388	54	38	4	2487
12	9	6	4	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
17690	3494	327	155	9	4975
12	9	6	4	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
312289	31450	1965	621	19	9951
12	9	6	4	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
2547468	283052	11793	2487	38	19902
	9	6	4	2	2
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
2547468	70763	9951	77	39804	
	6	4	2	2	
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
	424578	39804	155	79608	
	6	4	2	2	
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
2547468	159216	310	159216		
	4	2	2		
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
	636867	621	318433		
	4	2	2		
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
2547468	1243	636867			
		2			
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
	1273734				
	2				
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
	2547468				

## EXERCISE 37—Page 142.

II	(1)	(2)	(3)	(4)
1101101111100001100	VI	XII	III	VIII
	252	62te)32e75721(62te	201210	57264
	252	31556	102221	675
	544	161e7	21212	354604
1243	2224	1059t		513354
2	544	58192		434070
	122024	52512		51117344
2487		58801		
2		58801		
4975	(5)	(6)	(7)	(8)
2	II	VII	VII	XII
9951	101	2143)142613(50-5254+	65432	7t348
2	1001	14111	43210	5e6t4
19902	1111		1444	
2	1011	1503-0	65001	18664
	1000	1411-1	54321	
39804	1111			
2	10101	61-60	326041	
	1010100	43-16		
79608		15-410		
2		14-111		
		1-2660		
159216		1-1635		
2		-1022		
			(10)	
318433	(9)		II	
2	XII		100101)1010100001(10010-111	100101
	34t7		100101	
636867	6666			
2	18536		101000	
	18536		100101	
1273734	18536			
2	18536		111	
2547468	1136e296			

## EXERCISE 38—Page 146.

(1)							(2)			
4 ft.	7'	6"	10'''				19 ft.	10'	3"	
9	7	11	11				11	2	7	
<hr/>							<hr/>			
	4	2	11'''	3'''	2'''			11	6	11''' 9'''
2	4	2	11	3	2		3	3	8	6
41	8	4	11	10			218	4	9	
<hr/>							<hr/>			
44	9	1	8	0	5	2	222	8	0	5 9
<hr/>							<hr/>			
(3)							(4)			
9"	7'''	4'''					9½ in.	=	9'	
7	3	11'''					9'	9''		
<hr/>							<hr/>			
			8'''	9'''	8'''	8'''				
	2	4	10	0				3	3'''	0'''
5	7	3	4					5	8	3
<hr/>							<hr/>			
5	10	4	11	8	8		4	0	9	
<hr/>							<hr/>			
							4	6	8	6

(5)				
7 ft.	4'	11''		
3	2	2		
<hr/>				
	1	2	9'''	10'''
1	2	9	10	
22	2	9		
<hr/>				
23	6	9	7	10

## EXERCISE 32—Page 147.

(1)		(2)		(3)	
15 ft.				10 ft.	
1	2'	XII		5	
<hr/>		<hr/>		<hr/>	
2	6	45·6		50 sq.ft.	
15		t·3		7	
<hr/>		<hr/>		<hr/>	
17	6	1146		—c'rds.c.ft.	
8		3870		128)350(2	94
<hr/>		<hr/>		<hr/>	
11	8 0''=	398·46		256	
<hr/>		<hr/>		<hr/>	
11½ cub.ft.	=11 cub.ft 1152 cub.in.	2		94 cub.ft.	
		774·90=1096 9'			
		XII			
		774=1096 com. scale.			

0'	3"		
2	7		
<hr/>			
11	6	11'''	9'''
3	8	6	
4	9		
<hr/>			
8	0	5	9
<hr/>			
(4)			
in.	=9'	9"	
9"			
7	4'''		
<hr/>			
3	3'''	0'''	
5	8	3	
0	9		
<hr/>			
6	8	6	

(3)	
10 ft.	
5	
<hr/>	
50 sq.ft.	
7	
<hr/>	
—c'rd.s.c.ft.	
128)350(2 94	
256	
<hr/>	
94 cub.ft.	

9'

ale.

(4)		(5)		(6)
4 ft.		25 ft.=300 in.		
5½		20 " =240 "		
—		2 ft. 6 in.= 30 "		
20				
1				
—				
21 sq. ft.				
70				
<hr/>				
128)1470(11½ cords				
128				
—				
190				
128				
—				
62				
128)62(½				
64				
<hr/>				

## EXERCISE 40—Page 149.

(1)			
£93×400	= \$372.00	£276×400	= \$1104.00
14s.×20	= 2.80	19s.×20	= 3.80
7½d.=30 f.×5÷12	= .12½	10½d.=42 f.×5÷12	= .17½
<hr/>			
£93 14s. 7½d.	= \$374.92½	£276 19s. 10½d.	= \$1107.97½
£275×400	= \$1100.00	\$729.18	
4s.×20	= .80	710.50	
11½d.=47 f.×5÷12	= .19⅞	166.78	
<hr/>			
£275 4s. 11½d.	= \$1100.99⅞	374.92½	
		1107.97½	
		497.81	
		1100.99⅞	
		<hr/>	
		\$4688.16⅞	

(2)	
576=6+10×7+10×10×5	
m.fur.per.yds.ft.in.	m.fur.per.yds.ft.in.
47 6 17 4 2 7×6=	286 6 27 1 2 0
10	
478 0 18 4 1 10×7=	3346 3 11 4 2 4
10	
4780 4 28 2 0 4×5=	23902 7 21 4 3 2
	27536 1 21 0 1 6



(3)

$$243000 = 2^3 \times 3^5 \times 5^3$$

$$3+1=4$$

$$4+1=6$$

$$3+1=4$$

$$4 \times 6 \times 4 = 96$$

(4)

V	VIII
8)4234434	5)713427
8)241110..4	5)133721..2
8)13423..1	5)22303..2
8)1024..1	5)3532..1
8)32..3	5)570..2
2..1	5)113..1
	5)17..0
	3..0

VIII

713427

213114

500313

V

30012122

4234434

20222133

(5)

$$79 \cdot 342 \div .00006378 =$$

$$6378)7934200000(1243994 \cdot 98275$$

6378

15562

12756

28060

25512

25480

19134

63460

57402

60580

57402

31780

25512

6268'0

5740'2

527'80

510'24

17'560

12'756

4'8040

4'4646

•33940

•31890

•02050

[NAT. ARITH.

(7)

40	5.	7.	9.	11.	13.	15.	17.	19.	21.	23.	25.	27.	29.	31.	33.	35.	37.	39.	41.	43.	45.	47.	49.
21	7.	9.	11.	13.	15.	17.	19.	21.	23.	25.	27.	29.	31.	33.	35.	37.	39.	41.	43.	45.	47.	49.	51.
33	9.	11.	13.	15.	17.	19.	21.	23.	25.	27.	29.	31.	33.	35.	37.	39.	41.	43.	45.	47.	49.	51.	53.
10																							

$$40 \times 21 \times 33 \times 10 = 277200.$$

(9)

$$00006378 =$$

$$9999993000 = 10000000000 - 7000.$$

$$0(1243994 \cdot 98275$$

$$64276 \cdot 3427 \times 10000000000 = 642763427000000$$

$$64276 \cdot 3427 \times 7000 = 449934398 \cdot 9$$

$$642762977065601 \cdot 1$$

(10)

$$\begin{array}{r} \text{IX} \\ 5)78263 \\ \hline 5)15230 \cdot 3 \\ \hline 5)2760 \cdot 0 \\ \hline 5)511 \cdot 4 \\ \hline 5)102 \cdot 0 \\ \hline 5)17 \cdot 3 \\ \hline 3 \cdot 1 \end{array}$$

$$\begin{array}{r} \text{IX} \\ 11)78263 \\ \hline 11)6430 \cdot 3 \\ \hline 11)526 \cdot 6 \\ \hline 11)43 \cdot 0 \\ \hline 3 \cdot 6 \end{array}$$

IX	V	XI
7)78263 =	7)3130403 =	7)36063
7)11160 \cdot 3	7)214200 \cdot 3	7)5640 \cdot 3
7)1407 \cdot 5	7)13220 \cdot 5	7)884 \cdot 5
7)177 \cdot 3	7)1101 \cdot 3	7)128 \cdot 3
7)23 \cdot 4	7)41 \cdot 4	7)14 \cdot 4
3 \cdot 0	3 \cdot 0	3 \cdot 0

(12)

$$\begin{array}{rcl}
 £672 \times 400 & = & \$2688 \cdot 00 \\
 7s. \times 20 & = & 1 \cdot 40 \\
 7d. = 28f. \times 5 \div 12 & = & \cdot 11\frac{1}{2} \\
 \hline
 £672 \text{ } 7s. \text{ } 7d. & = & \$2689 \cdot 51\frac{1}{2}
 \end{array}$$

(13)

$$\begin{array}{r}
 891)243000(272 \\
 \underline{1782} \\
 6480 \\
 \underline{6237}
 \end{array}$$

(13 continued.)

$$81)37800(466$$

$$\underline{324}$$

$$540$$

$$\underline{486}$$

$$540$$

$$\underline{486}$$

$$54)81(1$$

$$\underline{54}$$

$$27)54(2$$

$$\underline{54}$$

Therefore G. C. M. = 27.

(17)

(18)

(19)

$$\begin{array}{rcl}
 £ \text{ } s. \text{ } d. & 2)276000 & \\
 178 \text{ } 16 \text{ } 4\frac{1}{2} & \underline{\hspace{1cm}} & 6 \text{ ft. } 2' \text{ } 7'' \text{ } 9''' \text{ } 10'''' \\
 97 \text{ } 15 \text{ } 11\frac{1}{2} & 2)138000 & 13 \text{ } 11 \text{ } 11 \text{ } 11 \text{ } 7 \\
 693 \text{ } 19 \text{ } 11\frac{1}{2} & \underline{\hspace{1cm}} & \\
 216 \text{ } 11 \text{ } 9\frac{1}{2} & 2)69000 & 3 \text{ } 7 \text{ } 6'''' \text{ } 6'''''' \text{ } 8'''''''' \text{ } 10'''''''' \\
 678 \text{ } 14 \text{ } 7\frac{1}{2} & \underline{\hspace{1cm}} & 5 \text{ } 8 \text{ } 5 \text{ } 2 \text{ } 0 \text{ } 2 \\
 197 \text{ } 13 \text{ } 11\frac{1}{2} & 2)34500 & 5 \text{ } 8 \text{ } 5 \text{ } 2 \text{ } 0 \text{ } 2 \\
 117 \text{ } 6 \text{ } 5 & \underline{\hspace{1cm}} & 5 \text{ } 8 \text{ } 5 \text{ } 2 \text{ } 0 \text{ } 2 \\
 91 \text{ } 1 \text{ } 1\frac{1}{2} & 2)17250 & 80 \text{ } 10 \text{ } 5 \text{ } 7 \text{ } 10 \\
 \hline
 2272 \text{ } 0 \text{ } 3\frac{1}{2} & 3)8625 & 87 \text{ } 1 \text{ } 1 \text{ } 3 \text{ } 0 \text{ } 10 \text{ } 8 \text{ } 10 \text{ } 10 \\
 \underline{\hspace{1cm}} & & \\
 & 5)2875 & \\
 \underline{\hspace{1cm}} & & \\
 & 5)575 & \\
 \underline{\hspace{1cm}} & & \\
 & 5)115 & \\
 \underline{\hspace{1cm}} & & \\
 & 23 & \\
 & 2^5 \times 3 \times 5^3 \times 23 &
 \end{array}$$

72

91(1

48

43)648(2

486

162)243(1

162

81)162(2

162

6'''' 8'''' 10''''

0 2

2

8 10 10

(20)

XII

713796)77e9·047(·011436

713796

977·277

713796

266·4110

245·3720

21·05e00

19·3e846

3·862760

3·67e490

·173290

(21)

IV

3333333

4

15

4

63

4

255

4

1023

4

4095

4

16383

(22)

VIII

10000

8

8

8

64

8

512

8

4096

(23)

74002702 ÷ 144 = 513907 ft. 94 in.

513907 ft. ÷ 9 = 57100 yards 7 ft.

57100 yds. ÷ 30½ = 1887 per. 18½ yds.

1887 per. 18 yds. 2 ft. 36 in.

Add 7 ft. 94 in.

40)1887 per. 19 yds. 0 ft. 130 in.

4)47 r. 7 per. 19 yds. 0 ft. 130 in.

11 a. 3 r. 7 per. 19 yds. 0 ft. 130 in.

(24)

1728 | 240..720..1620..1728

65 | 5..65..15

3 | 8

1728 × 65 × 3 = 336960.

(25)

6 children will have 6 children's shares

4 women will have  $4 \times 2 = 8$  " "3 men will have  $3 \times 5 \times 2 = 30$  " "

3 men 4 w'n &amp; 6 chl'n will have 44 children's sha.

4) \$7894.16

11) \$1973.54

\$179.41  $\frac{3}{4}$  = child's share.\$179.41  $\frac{3}{4} \times 2 = \$358.82 \frac{3}{4}$  = woman's share.\$358.82  $\frac{3}{4} \times 5 = \$1794.12 \frac{3}{4}$  = man's share.

(26)

(27)

II		II		yds. qrs. na. in.	yds. qrs. na. in.
1111111111	1000000000	7	1 1 1	729	3 2 i
2	2	4		4	
-	-	-		-	-
3	2	29		2919	
2	2	4		4	
-	-	-		-	-
7	4	117		11679	
2	2	24		24	
-	-	-		-	-
15	8	235		23359	
2	2	294		29194	
-	-	-		-	-
31	16	2644		262784	
2	2	4		4	
-	-	-		-	-
63	32	1057	)	105115(99 $\frac{472}{1051}$	
2	2			9513	
-	-	-		-	-
127	64			9985	
2	2			9513	
-	-	-		-	-
255	128			472	
2	2				
-	-	-		-	-
511	256				
2	2				
-	-	-		-	-
1023	512				

s shares  
"  
"  
children's sha.  
re.  
's share.  
share.

(28)	(29)	(30)
762'4978	723426	lbs. oz. drs. scr.
63'423	938'9126141	129 0 0 0
<hr/>	<hr/>	63 4 7 2
22874934	722487'0873859	<hr/>
15249956		65 7 0 1
30499912		
22874934		
45749868		
<hr/>		
48359'8979694		

(31)

$$1064 = 2^3 \times 7 \times 19.$$

1..2..4..8

1..7

1..2..4..8..7..14..28..56

1..19

1..2..4..8..7..14..28..56..19..38..76..152..133..266..532..1064=

1..2..4..7..8..14..19..28..38..56..76..133..152..266..532..1064

(32)

$$30 \text{ ft. } 6 \text{ in.} = 366 \text{ in.} \quad 366$$

$$20 \text{ ft. } 11 \text{ in.} = 251 \text{ in.} \quad 251$$

$$2 \text{ ft. } 7 \text{ in.} = 31 \text{ in.} \quad \text{—}$$

366

1830

732

in.

$$31)91866(2963\frac{1}{3}$$

62

298

279

$$2963\frac{1}{3} \div 36 = 82\frac{5}{6} \text{ yds.}$$

196

186

106

93

13

D

## EXERCISE 46—Page 158.

(1)

$$\begin{array}{r} 2 \times 7 \times 9 \times 5 \times 18 \quad 5 \times 5 \times 9 \times 5 \times 18 \quad 8 \times 5 \times 7 \times 5 \times 18 \\ 3, 4, 6, 2, 18 = \frac{\quad}{5 \times 7 \times 9 \times 5 \times 18} \quad \frac{\quad}{5 \times 7 \times 9 \times 5 \times 18} \quad \frac{\quad}{5 \times 7 \times 9 \times 5 \times 18} \\ 3 \times 5 \times 7 \times 9 \times 18 \quad 5 \times 5 \times 7 \times 9 \times 5 \quad 11340 \quad 20250 \quad 25200 \quad 17010 \quad 7875 \\ 5 \times 7 \times 9 \times 5 \times 18 \quad 5 \times 7 \times 9 \times 5 \times 18 \quad 28350 \quad 28350 \quad 28350 \quad 28350 \quad 28350 \end{array}$$

(2)

$$\begin{array}{r} 8 \times 13 \times 14 \quad 12 \times 11 \times 14 \quad 5 \times 11 \times 13 \quad 1456 \quad 1848 \quad 715 \\ 11, 13, 14 = \frac{\quad}{11 \times 13 \times 14} \quad \frac{\quad}{11 \times 13 \times 14} \quad \frac{\quad}{11 \times 13 \times 14} \quad \frac{\quad}{2002} \quad \frac{\quad}{2002} \quad \frac{\quad}{2002} \end{array}$$

(3)

$$\begin{array}{r} 6 \times 11 \times 13 \times 7 \times 2 \quad 4 \times 7 \times 13 \times 7 \times 2 \\ 9, 11, 13, 7, \frac{1}{2} = \frac{\quad}{7 \times 11 \times 13 \times 7 \times 2} \quad \frac{\quad}{7 \times 11 \times 13 \times 7 \times 2} \\ 5 \times 7 \times 11 \times 7 \times 2 \quad 4 \times 7 \times 11 \times 13 \times 2 \quad 1 \times 7 \times 11 \times 13 \times 7 \\ \frac{\quad}{7 \times 11 \times 13 \times 7 \times 2} \quad \frac{\quad}{7 \times 11 \times 13 \times 7 \times 2} \quad \frac{\quad}{7 \times 11 \times 13 \times 7 \times 2} = \\ 12012 \quad 5096 \quad 5390 \quad 8008 \quad 7007 \\ 14014 \quad 14014 \quad 14014 \quad 14014 \quad 14014 \end{array}$$

(4)

$$\begin{array}{r} 6 \times 7 \times 13 \quad 4 \times 11 \times 13 \quad 8 \times 11 \times 7 \quad 546 \quad 572 \quad 616 \\ 11, 7, 13 = \frac{\quad}{11 \times 7 \times 13} \quad \frac{\quad}{11 \times 7 \times 13} \quad \frac{\quad}{11 \times 7 \times 13} \quad \frac{\quad}{1001} \quad \frac{\quad}{1001} \quad \frac{\quad}{1001} \end{array}$$

(5)

$$\begin{array}{r} 5 \times 7 \times 5 \times 11 \quad 4 \times 6 \times 5 \times 11 \quad 4 \times 6 \times 7 \times 11 \\ 6, 7, 5, 11 = \frac{\quad}{6 \times 7 \times 5 \times 11} \quad \frac{\quad}{6 \times 7 \times 5 \times 11} \quad \frac{\quad}{6 \times 7 \times 5 \times 11} \\ 2 \times 6 \times 7 \times 5 \quad 1925 \quad 1320 \quad 1848 \quad 420 \\ \frac{\quad}{6 \times 7 \times 5 \times 11} = \frac{\quad}{2310} \quad \frac{\quad}{2310} \quad \frac{\quad}{2310} \quad \frac{\quad}{2310} \end{array}$$

(6)

$$\frac{1}{2}, \frac{3}{3}, \frac{3}{3}, \frac{4}{4} = \frac{1 \times 3 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{2 \times 2 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{3 \times 2 \times 3 \times 7}{2 \times 3 \times 5 \times 7},$$

$$\frac{2 \times 2 \times 3 \times 5}{2 \times 3 \times 5 \times 7} = \frac{105}{210}, \frac{140}{210}, \frac{126}{210}, \frac{60}{210}.$$

EXERCISE 47—Page 159.

(1)

$$\frac{1}{5}, \frac{3}{8}, \frac{1}{6}, \frac{3}{4}, \frac{7}{15}.$$

The least common multiple of 5, 8, 6, 4, 15 is 120.

The multiplier for both terms of the first fraction is  $\frac{120}{5} = 24$ ; for the second  $\frac{120}{8} = 15$ ; for the third  $\frac{120}{6} = 20$ ; for the fourth  $\frac{120}{4} = 30$ ; for the fifth  $\frac{120}{15} = 8$ .

Multiplying by these numbers, we obtain  $\frac{24}{120}, \frac{45}{120}, \frac{20}{120}, \frac{210}{120},$  and  $\frac{56}{120}.$

(2)

$$\frac{6}{11}, \frac{3}{3}, \frac{4}{4}, \frac{7}{7}, \frac{13}{33}.$$

The least common multiple of 11, 3, 7, 77 and 33 is 231.

The multiplier for both terms of the first fraction is  $\frac{231}{6} = 38.5$ ; for the second,  $\frac{231}{3} = 77$ ; for the third,  $\frac{231}{4} = 57.75$ ; for the fourth,  $\frac{231}{7} = 33$ ; and for the fifth,  $\frac{231}{33} = 7$ .

Multiplying by these numbers, we obtain  $\frac{229.5}{231}, \frac{231}{231}, \frac{132}{231}, \frac{154}{231},$  and  $\frac{91}{231}.$

(3)

$$\frac{1}{2}, \frac{3}{3}, \frac{5}{5}, \frac{7}{7}, \frac{9}{9}, \frac{13}{13}, \frac{7}{7}, \frac{37}{37}.$$

The least common multiple of 2, 3, 5, 6, 8, 10, 15, 16 and 80 is 240.

The multiplier for both terms of the first fraction is  $\frac{240}{2} = 120$ ; for the second,  $\frac{240}{3} = 80$ ; for the third,  $\frac{240}{5} = 48$ ; for the fourth,  $\frac{240}{6} = 40$ ; for the fifth,  $\frac{240}{8} = 30$ ; for the sixth,  $\frac{240}{10} = 24$ ; for the seventh,  $\frac{240}{15} = 16$ ; for the eighth,  $\frac{240}{16} = 15$ ; and for the ninth,  $\frac{240}{37} = 6.486$ .

Multiplying by these numbers, we obtain  $\frac{120}{240}, \frac{80}{240}, \frac{48}{240}, \frac{40}{240}, \frac{30}{240}, \frac{24}{240}, \frac{16}{240}, \frac{15}{240},$  and  $\frac{41.312}{240}.$



(4)

$$\frac{3}{8}, \frac{7}{16}, \frac{5}{24}, \frac{11}{30}, \frac{13}{48}, \frac{23}{60}.$$

The least common multiple of 5, 10, 25, 30, 45, and 60 is 900.

The multiplier for both terms of the first fraction is  $\frac{900}{10} = 180$ ; for the second,  $\frac{900}{20} = 45$ ; for the third,  $\frac{900}{25} = 36$ ; for the fourth,  $\frac{900}{30} = 30$ ; for the fifth,  $\frac{900}{45} = 20$ ; and for the sixth,  $\frac{900}{60} = 15$ .

Multiplying by these numbers, we obtain  $\frac{540}{900}, \frac{630}{900}, \frac{315}{900}, \frac{390}{900}, \frac{260}{900},$  and  $\frac{345}{900}.$

(5)

$$\frac{13}{20}, \frac{7}{30}, \frac{11}{40}, \frac{1}{50}.$$

The last common multiple of 20, 30, 40 and 50 is 600.

The multiplier for both terms of the first fraction is  $\frac{600}{20} = 30$ ; for the second,  $\frac{600}{30} = 20$ ; for the third,  $\frac{600}{40} = 15$ ; and for the fourth,  $\frac{600}{50} = 12$ .

Multiplying by these numbers, we obtain  $\frac{570}{600}, \frac{140}{600}, \frac{165}{600}$  and  $\frac{12}{600}.$

(6)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}.$$

The least common multiple of 2, 3, 4, 6, 8, 12, 16, and 24 is 48.

The multiplier for both terms of the first fraction is  $\frac{48}{2} = 24$ ; for the second,  $\frac{48}{3} = 16$ ; for the third,  $\frac{48}{4} = 12$ ; for the fourth,  $\frac{48}{6} = 8$ ; for the fifth,  $\frac{48}{8} = 6$ ; for the sixth,  $\frac{48}{12} = 4$ ; for the seventh,  $\frac{48}{16} = 3$ ; and for the eighth,  $\frac{48}{24} = 2$ .

Multiplying by these numbers, we obtain  $\frac{24}{48}, \frac{32}{48}, \frac{36}{48}, \frac{40}{48}, \frac{42}{48}, \frac{60}{48}, \frac{45}{48},$  and  $\frac{46}{48}.$

(7)

$$\frac{5}{7}, \frac{11}{12}, \frac{13}{16}, \frac{17}{27}, \frac{19}{36}, \frac{1}{40}.$$

The least common multiple of 7, 12, 15, 27, 35 and 40 is 7560.

The multiplier for both terms of the first fraction is  $\frac{7560}{7} = 1080$ ; for the second,  $\frac{7560}{12} = 630$ ; for the third,  $\frac{7560}{15} = 504$ ; for the fourth,  $\frac{7560}{27} = 280$ ; for the fifth,  $\frac{7560}{36} = 210$ ; for the sixth,  $\frac{7560}{40} = 189$ .

Multiplying by these numbers, we obtain  $\frac{5400}{7560}, \frac{6930}{7560}, \frac{5004}{7560}, \frac{2240}{7560}, \frac{1245}{7560},$  and  $\frac{3213}{7560}.$

(8)

$$\frac{14}{15}, \frac{7}{8}, \frac{1}{3}, \frac{11}{12}, \frac{5}{17}, \frac{13}{20}, \frac{6}{7}, \frac{32}{35}.$$

The least common multiple of 15, 8, 3, 12, 11, 20, 7, and 35 is 9240.

The multiplier for both terms of the first fraction is  $\frac{2240}{15} = 616$ ; for the second,  $\frac{2240}{8} = 1155$ ; for the third,  $\frac{2240}{3} = 3080$ ; for the fourth,  $\frac{2240}{12} = 770$ ; for the fifth,  $\frac{2240}{17} = 840$ ; for the sixth,  $\frac{2240}{20} = 462$ ; for the seventh,  $\frac{2240}{7} = 1320$ ; for the eighth,  $\frac{2240}{35} = 264$ .

Multiplying by these numbers, we obtain  $\frac{8624}{9240}, \frac{8085}{9240}, \frac{12320}{9240}, \frac{8470}{9240}, \frac{5040}{9240}, \frac{8778}{9240}, \frac{7920}{9240}$ , and  $\frac{7656}{9240}$ .

## EXERCISE 48—Page 160.

(1)

$$\frac{1}{7} \text{ of } \frac{3}{8} \text{ of } \frac{6}{17} \text{ of } \frac{35}{72} = \frac{4 \times 3 \times 6 \times 35}{7 \times 5 \times 11 \times 72} = \frac{2520}{27720} = \frac{1}{11}.$$

(2)

$$\frac{2}{3} \text{ of } \frac{1}{4} \text{ of } \frac{6}{7} \text{ of } \frac{81}{100} \text{ of } \frac{25}{24} = \frac{2 \times 4 \times 6 \times 81 \times 25}{3 \times 9 \times 7 \times 100 \times 24} = \frac{97200}{453600} = \frac{3}{14}.$$

(3)

$$\frac{31}{35} \text{ of } \frac{6}{17} \text{ of } \frac{77}{36} = \frac{21 \times 6 \times 77}{35 \times 11 \times 36} = \frac{7}{15}.$$

(4)

$$\frac{2}{5} \text{ of } \frac{1}{4} \text{ of } \frac{3}{17} \text{ of } \frac{13}{12} = \frac{2 \times 4 \times 3 \times 13}{5 \times 7 \times 11 \times 17} = \frac{312}{6545}.$$

## EXERCISE 49—Page 161.

(1)

$$\frac{5}{9} \text{ of } \frac{7}{7} \text{ of } \frac{2}{3} \text{ of } \frac{1}{6} = \frac{5 \times 6 \times 2 \times 3}{9 \times 7 \times 3 \times 16} = \frac{5 \times \overset{2}{\cancel{6}} \times \cancel{2} \times \cancel{3}}{\underset{3}{\cancel{9}} \times 7 \times \underset{8}{\cancel{3}} \times \underset{4}{\cancel{16}}} = \frac{5}{3 \times 7 \times 4} = \frac{5}{84}.$$

(2)

$$\frac{3}{2} \text{ of } \frac{5}{9} \text{ of } \frac{1}{13} \text{ of } \frac{6}{11} \text{ of } \frac{1}{13} \text{ of } \frac{1}{17} = \frac{2 \times 5 \times 18 \times 6 \times 11 \times 13}{3 \times 9 \times 132 \times 11 \times 13 \times 17} =$$

$$\frac{2 \times 5 \times \overset{2}{\cancel{18}} \times \overset{2}{\cancel{6}} \times 11 \times 13}{\underset{33}{\cancel{3} \times \cancel{9} \times 132} \times 11 \times 13 \times 17} = \frac{2 \times 5}{33 \times 17} = \frac{10}{561}.$$

(3)

$$\frac{2}{7} \text{ of } \frac{1}{11} \text{ of } 5\frac{1}{2} = \frac{2 \times 4 \times 11}{7 \times 11 \times 2} = \frac{2 \times 4 \times \cancel{11}}{7 \times \cancel{11} \times 2} = \frac{4}{7}.$$

(4)

$$\frac{1}{9} \text{ of } \frac{2}{13} \text{ of } \frac{1}{200} \text{ of } \frac{50}{169} \text{ of } \frac{1}{17} \text{ of } \frac{1}{6} = \frac{1 \times 8 \times 117 \times 50 \times 13 \times 13}{9 \times 13 \times 200 \times 169 \times 17 \times 6} =$$

$$\frac{1 \times \overset{2}{\cancel{8}} \times \overset{9}{\cancel{117}} \times \cancel{50} \times 13 \times 13}{\underset{4}{\cancel{9}} \times 13 \times \underset{13}{\cancel{200}} \times \underset{13}{\cancel{169}} \times 17 \times \underset{3}{\cancel{6}}} = \frac{1}{17 \times 3} = \frac{1}{51}.$$

(5)

$$\frac{3}{11} \text{ of } \frac{4}{7} \text{ of } \frac{9}{19} \text{ of } \frac{3}{11} \text{ of } \frac{3}{11} \text{ of } \frac{4}{7} = \frac{3 \times 4 \times 9 \times 33 \times 38 \times 47}{11 \times 7 \times 19 \times 47 \times 72 \times 7} =$$

$$\frac{3 \times 4 \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{33}} \times \cancel{38} \times 47}{11 \times 7 \times 19 \times 47 \times \underset{18}{\cancel{72}} \times \underset{2}{\cancel{7}}} = \frac{3 \times 3}{7 \times 7} = \frac{9}{49}.$$

(6)

$$\frac{5}{3 \times 7 \times 4} = \frac{5}{84}.$$

$$\frac{2}{7} \text{ of } \frac{3}{11} \text{ of } \frac{154}{1} = \frac{4 \times 3 \times 154}{7 \times 11 \times 1} = \frac{4 \times 3 \times \overset{2}{\cancel{14}}}{7 \times 11 \times 1} = \frac{2 \times 4 \times 3}{1} = 24.$$

## EXERCISE 50—Page 162.

(1)

$$\frac{6 \times 11 \times 13}{11 \times 13 \times 17} =$$

$$\frac{\frac{14}{45}}{1\frac{7}{25}} = \frac{\frac{14}{45}}{\frac{42}{25}} = \frac{14 \times 25}{45 \times 42} = \frac{14 \times \overset{5}{\cancel{25}}}{\underset{9}{45} \times \underset{3}{42}} = \frac{5}{9 \times 3} = \frac{5}{27}.$$

(2)

$$\frac{1\frac{1}{2}}{7\frac{18}{15}} = \frac{1\frac{1}{2}}{1\frac{42}{15}} = \frac{11 \times 18}{12 \times 143} = \frac{11 \times \overset{3}{\cancel{18}}}{\underset{2}{12} \times \underset{13}{143}} = \frac{3}{2 \times 13} = \frac{3}{26}.$$

(3)

$$\frac{50 \times 13 \times 13}{169 \times 17 \times 6} =$$

$$\frac{15\frac{3}{5}}{7\frac{1}{5}} = \frac{7\frac{3}{5}}{3\frac{2}{5}} = \frac{78 \times 5}{5 \times 39} = \frac{\overset{2}{78} \times 5}{5 \times 39} = 2.$$

(4)

$$\frac{11\frac{3}{5}}{12\frac{3}{5}}, \frac{3\frac{1}{4}}{9}, \frac{2}{\frac{3}{5}} = \frac{35}{\frac{60}{5}}, \frac{13}{\frac{9}{1}}, \frac{2}{\frac{3}{5}} = \frac{35 \times 5}{3 \times 68}, \frac{13 \times 1}{9 \times 4}, \frac{2 \times 5}{7 \times 3} = \frac{175}{204}, \frac{13}{36}, \frac{10}{21}.$$

(5)

$$\frac{33 \times 38 \times 47}{47 \times 72 \times 7} =$$

$$\frac{7\frac{7}{8}}{15\frac{1}{4}}, \frac{5\frac{7}{8}}{3\frac{1}{6}}, \frac{2\frac{3}{8}}{3\frac{3}{4}} = \frac{7\frac{7}{8}}{15\frac{1}{4}}, \frac{4\frac{7}{8}}{1\frac{3}{16}}, \frac{1\frac{2}{4}}{2\frac{1}{4}} = \frac{7 \times 4}{12 \times 63}, \frac{\overset{2}{47 \times 16}}{8 \times 3}, \frac{12 \times 7}{5 \times \underset{2}{24}} =$$

$$\frac{1}{3 \times 9}, \frac{47 \times 2}{3}, \frac{7}{5 \times 2} = \frac{1}{27}, 31\frac{1}{3}, 7\frac{1}{5}.$$

(6)

$$\frac{16\frac{1}{2}}{11\frac{1}{2}}, \frac{6\frac{1}{2}}{13}, \frac{17}{18\frac{1}{2}}, \frac{21\frac{1}{2}}{10\frac{1}{2}}, \frac{\frac{1}{2}}{4\frac{1}{2}} = \frac{\frac{4\frac{1}{2}}{2\frac{1}{2}}, \frac{3\frac{1}{2}}{1\frac{1}{2}}, \frac{17}{5\frac{1}{2}}, \frac{18\frac{1}{2}}{7\frac{1}{2}}, \frac{\frac{1}{2}}{2\frac{1}{2}}}{\frac{10}{8 \times 85 \times 7}} = \frac{10}{8 \times 85 \times 7}$$

$$\frac{31 \times 1}{5 \times 13}, \frac{17 \times 3}{55 \times 1}, \frac{108 \times 7}{72 \times 5}, \frac{1 \times 5}{2 \times 23} = \frac{10}{7}, \frac{31}{65}, \frac{51}{55}, \frac{21}{10}, \frac{5}{46} = 1\frac{13}{7}, 2\frac{11}{55}, 2\frac{1}{5}, 2\frac{1}{46}$$

## EXERCISE 51—Page 163.

(1)

$$\frac{4}{5} \text{ of } \frac{1}{18\frac{1}{4}} = \frac{1}{20} \text{ of a lb.}$$

(2)

$$\frac{2}{3} \text{ of } \frac{3}{7} \text{ of } \frac{1}{12\frac{1}{6}} \text{ of } \frac{1}{20} = \frac{1}{7 \times 6 \times 20} = \frac{1}{840}$$

(3)

$$\frac{2}{9} \text{ of } \frac{5}{35\frac{1}{2}} \text{ of } \frac{1}{7} = \frac{5}{9 \times 2} = \frac{5}{18} \text{ wk.}$$

(4)

$$\frac{5}{11} \text{ of } \frac{81}{5} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{81}{11 \times 4 \times 5} = \frac{81}{220} \text{ Eng. Ell.}$$

(5)

$$\frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{1}{5\frac{1}{2}} = \frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{2}{11} = \frac{24}{625} \text{ per.}$$

$$\frac{1}{2} = \frac{10}{50 \times 3} = \frac{3 \times 35}{3 \times 35 \times 7}$$

$$13, 31, 81, 21, 5, 45.$$

$$\frac{2}{3} \text{ of } \frac{4}{7} \text{ of } 21 \frac{1}{14} \text{ of } \frac{1}{8} = \frac{2 \times 4 \times 295 \times 1}{3 \times 7 \times 14 \times 8} = \frac{295}{294} = 1 \frac{1}{294} \text{ c.}$$

$$\frac{3}{19} \text{ of } \frac{4}{17} \text{ of } 9 \text{ of } \frac{1}{2} \text{ of } \frac{1}{40} \text{ of } \frac{1}{4} = \frac{3 \times 4 \times 19 \times 1 \times 1}{19 \times 17 \times 2 \times 40 \times 4} = \frac{3}{17 \times 2 \times 40} = \frac{3}{1360} \text{ a.}$$

## EXERCISE 52—Page 164.

$$\frac{14}{79} \text{ of } \frac{4}{1} \text{ of } \frac{2}{1} \text{ of } \frac{4}{1} = 4 \frac{4}{79} \text{ qt.}$$

$$2 \frac{1}{3} \frac{1}{10}.$$

$$\frac{2}{3} \text{ of } \frac{4}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{5}{1} \times \frac{3}{2} = \frac{2 \times 4 \times 4 \times 5}{3} = 10 \frac{2}{3}.$$

$$k.$$

$$\frac{7}{3} \times \frac{2}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{3}{2} = \frac{7 \times 2 \times 2 \times 4 \times 2}{3} = 23 \frac{1}{3}.$$

$$2 \frac{1}{10} \text{ Eng. Ell.}$$

$$\frac{17}{23} \times \frac{6}{12} \times \frac{8}{1} \times \frac{3}{1} = \frac{17 \times 6 \times 8 \times 3}{11} = 241 \frac{2}{11} \text{ scr.}$$

$$r. \text{ per.}$$

$$\frac{1}{5000} \times \frac{2}{3} \times \frac{3}{4} \times \frac{6}{11} \times \frac{22}{7} \times \frac{16}{1} \times \frac{4}{1} = \frac{2 \times 6 \times 2 \times 2 \times 4}{625 \times 7} = \frac{192}{4375} \text{ dr.}$$

## EXERCISE 53—Page 164.

(1)

bush.	pk.	gal.	qt.	pt.
11)3	0	0	0	0
<hr/>				
	1	0	0	1 $\frac{1}{11}$

lbs.	oz.	dr.
7)6	0	0
<hr/>		
	13	11 $\frac{1}{2}$

(2)			
yds.	qr.	na.	in.
13)7	(2	0	1 $\frac{5}{11}$
<hr/>			
	4		
<hr/>			
	28	qrs.	
<hr/>			
	26		
<hr/>			
	2		
<hr/>			
	4		
<hr/>			
	8	na.	
<hr/>			
	2 $\frac{1}{2}$		
<hr/>			
	18		
<hr/>			
	13		
<hr/>			
	5		

(4)			
fur.	per.	yds.	ft. in.
9)8	0	0	0 0
<hr/>			
	35	3	0 2

£	s.	d.
7)4	0	0
<hr/>		
	11	5 $\frac{1}{2}$

(3)

lbs.	oz.	dwt.	grs.
9)8	0	0	0
<hr/>			
	10	13	8

sq. m.	a.	r.	pr.	yds.	ft.	in.
113)11	(62	1	8	4	2	79 $\frac{11}{113}$
<hr/>						
	640					

7040 a.
678
<hr/>

260	484 yds.
226	452
<hr/>	

34	32
4	9
<hr/>	

136 r.	288 ft.
113	226
<hr/>	

23	62
40	144
<hr/>	

920 per.	248
904	248
<hr/>	
	62

16	
30 $\frac{1}{2}$	8928 in.
<hr/>	
	791

480	
4	1018
<hr/>	
	1017

484 yds.	
<hr/>	
	1

## EXERCISE 54—Page 165.

(1)

$$6 \text{ bus. } 1 \text{ pk. } 1 \text{ gal. } 1 \text{ qt. } 1 \text{ pt.} = 411 \text{ pts.}$$

$$50 \text{ bush.} = 3200 \text{ pts.}$$

And the required fraction is  $\frac{411}{3200}$ .

(2)

$$35 \text{ per. } 9 \text{ ft. } 2 \text{ in.} = 7040 \text{ in.}$$

$$1 \text{ fur.} = 7920 \text{ in.}$$

The required fraction is  $\frac{7040}{7920} = \frac{88}{99} = \frac{8}{9}$ .

(3)

$$7 \text{ hrs. } 12 \text{ min.} = 432 \text{ min.}$$

$$1 \text{ day} = 1440 \text{ min.}$$

Therefore the fraction is  $\frac{432}{1440} = \frac{3}{10}$ .

(4)

$$2 \text{ sq. yds. } 2 \text{ ft. } 120 \text{ in.} = 3000 \text{ in.}$$

$$3 \text{ sq. per. } 13\frac{1}{4} \text{ yds. } 1 \text{ ft. } 72 \text{ in.} = 135000 \text{ in.}$$

And the fraction is  $\frac{3000}{135000} = \frac{1}{45}$ .

(5)

$$7 \text{ oz. } 7 \text{ drs. } 2 \text{ scr. } 14 \text{ grs.} = 3834 \text{ grs.}$$

$$21 \text{ lbs.} = 120960 \text{ grs.}$$

The fraction is  $\frac{3834}{120960} = \frac{133}{4240} = \frac{71}{2240}$ .

(6)

$$9 \text{ min. } 48 \text{ sec.} = 588 \text{ sec.}$$

$$1 \text{ day} = 86400 \text{ sec.}$$

The required fraction is  $\frac{588}{86400} = \frac{7}{1080}$ .

(7)

$$16 \text{ bush. } 1 \text{ pk. } 1 \text{ pt.} = 1041 \text{ pts.}$$

$$69 \text{ bush.} = 4416 \text{ pts.}$$

Therefore the fraction is  $\frac{1041}{4416} = \frac{347}{1472}$ .

3)

wt. grs.

0 0

3 8

r. pr. yds. ft. in.

1 8 4 2 79  $\frac{1}{13}$ 

484 yds.

452

32

9

288 ft.

226

62

144

248

248

62

8928 in.

791

1018

1017

1



(8)

$$3 \text{ qrs. } 3\frac{1}{2} \text{ na.} = 15\frac{1}{2} = 15\frac{1}{2} \text{ na.}$$

$$1 \text{ Eng. ell} = 20 \text{ na.}$$

$$\text{And the fraction is } \frac{13\frac{1}{2}}{20} = \frac{136}{160} = \frac{34}{40}.$$

(9)

$$13 \text{ dwt. } 7 \text{ grs.} = 319 \text{ grs.}$$

$$1 \text{ lb. Troy} = 5760 \text{ grs.}$$

$$\text{The required fraction is } \frac{319}{5760}.$$

(10)

$$4800 \text{ cub. ft.}$$

$$54 \text{ cords} = 6912 \text{ cub. ft.}$$

$$\text{Therefore the fraction is } \frac{4800}{6912} = \frac{100}{144} = \frac{25}{36}.$$

## EXERCISE 55—Page 167.

(1)

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1.$$

(2)

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{2} = 1\frac{1}{2}.$$

(3)

$$4\frac{1}{2} + 11\frac{1}{2} + 16\frac{1}{2} + 21\frac{1}{2} + 19\frac{1}{2} = 4 + 11 + 16 + 21 + 19 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}) = 71 + \frac{5}{2} = 73\frac{1}{2}.$$

(4)

$$16\frac{1}{3} + 11\frac{1}{3} + 18\frac{1}{3} + 17\frac{1}{3} + 112\frac{1}{3} = 16 + 11 + 18 + 17 + 112 + (\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}) = 174 + \frac{5}{3} = 177\frac{1}{3}.$$

(5)

$$4\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 4 + 1 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2}) = 5 + \frac{3}{2} = 6\frac{1}{2}.$$

(6)

$$\frac{1}{2} + \frac{3}{4} + \frac{2}{3} + \frac{1}{6} + \frac{5}{8} + \frac{7}{12} + \frac{9}{16}.$$

These fractions reduced to their least common denominator become  $\frac{1280}{2560} + \frac{1536}{2560} + \frac{1600}{2560} + \frac{426\frac{2}{3}}{2560} + \frac{1600}{2560} + \frac{1536}{2560} + \frac{900}{2560} = \frac{15551}{2560} = 6\frac{131}{2560}.$

(7)

$\frac{1}{2} + \frac{5}{6} + \frac{1}{3}$  when reduced to their least common denominator become  $\frac{3}{6} + \frac{5}{6} + \frac{2}{6} = \frac{10}{6} = 1\frac{2}{3}$ .

(8)

$$\frac{1}{2} + \frac{5}{6} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8}$$

These fractions when reduced to their least common denominator become  $\frac{7323}{9240} + \frac{7700}{9240} + \frac{7920}{9240} + \frac{3465}{9240} + \frac{6720}{9240} = \frac{33197}{9240} = 3\frac{5477}{9240}$ .

(9)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

These fractions when reduced to their least common denominator become  $\frac{210}{420} + \frac{140}{420} + \frac{105}{420} + \frac{84}{420} + \frac{70}{420} + \frac{60}{420} = \frac{669}{420} = 1\frac{83}{140}$ .

(10)

$$16\frac{3}{4} + 47\frac{2}{3} + 21\frac{1}{3} + 17\frac{1}{8} + 19\frac{1}{2} = 16 + 47 + 21 + 19 + (\frac{3}{4} + \frac{2}{3} + \frac{1}{3} + \frac{1}{8} + \frac{1}{2})$$

$$16 + 47 + 21 + 19 = 103.$$

$$\frac{3}{4} + \frac{2}{3} + \frac{1}{3} + \frac{1}{8} + \frac{1}{2} = \frac{186}{1782} + \frac{396}{1782} + \frac{396}{1782} + \frac{693}{1782} + \frac{891}{1782} = \frac{2447}{1782}$$

$$103 + 1\frac{2447}{1782} = 104\frac{83}{140}$$

(11)

$$17\frac{1}{2} + 43\frac{3}{4} + 168\frac{1}{4} + 207\frac{2}{3} + 506\frac{1}{6} = 17 + 43 + 168 + 207 + 506 + (\frac{1}{2} + \frac{3}{4} + \frac{1}{4} + \frac{2}{3} + \frac{1}{6})$$

$$17 + 43 + 168 + 207 + 506 = 941.$$

$$\frac{1}{2} + \frac{3}{4} + \frac{1}{4} + \frac{2}{3} + \frac{1}{6} = \frac{63}{126} + \frac{91}{126} + \frac{31}{126} + \frac{84}{126} + \frac{21}{126} = \frac{247}{126}$$

$$941 + 2\frac{247}{126} = 943\frac{47}{63}$$

(12)

$$6\frac{1}{2} + 11\frac{1}{4} + 3\frac{3}{8} + 16\frac{7}{8} + \frac{1}{2} + \frac{1}{4} + 17\frac{1}{2} = 6 + 11 + 16 + 17 + (\frac{1}{2} + \frac{1}{4} + \frac{3}{8} + \frac{7}{8} + \frac{1}{2} + \frac{1}{4} + \frac{1}{2})$$

$$6 + 11 + 16 + 17 = 50.$$

$$\frac{1}{2} + \frac{1}{4} + \frac{3}{8} + \frac{7}{8} + \frac{1}{2} + \frac{1}{4} + \frac{1}{2} = \frac{168}{336} + \frac{84}{336} + \frac{105}{336} + \frac{280}{336} + \frac{168}{336} + \frac{84}{336} + \frac{168}{336} = \frac{1093}{336}$$

$$50 + 3\frac{1093}{336} = 53\frac{193}{112}$$

(13)

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + 68\frac{1}{8} = 68 + (\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{1}{8}).$$

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{1}{8} = \frac{4}{8} + \frac{6}{8} + \frac{7}{8} + \frac{1}{8} = \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}.$$

$$68 + 2\frac{1}{4} = 70\frac{1}{4}.$$

(14)

$$173\frac{1}{2} + 8\frac{5}{8} + 91\frac{1}{4} = 173 + 8 + 91 + (\frac{1}{2} + \frac{5}{8} + \frac{1}{4}).$$

$$173 + 8 + 91 = 272.$$

$$\frac{1}{2} + \frac{5}{8} + \frac{1}{4} = \frac{4}{8} + \frac{5}{8} + \frac{2}{8} = \frac{11}{8} = 1\frac{3}{8}.$$

$$272 + 1\frac{3}{8} = 273\frac{3}{8}.$$

(15)

$$1\frac{1}{8} + 2\frac{2}{4} + 3\frac{3}{4} + 4\frac{4}{8} = 1 + 2 + 3 + 4 + (\frac{1}{8} + \frac{2}{4} + \frac{3}{4} + \frac{4}{8}).$$

$$1 + 2 + 3 + 4 = 10.$$

$$\frac{1}{8} + \frac{2}{4} + \frac{3}{4} + \frac{4}{8} = \frac{1}{8} + \frac{4}{8} + \frac{6}{8} + \frac{4}{8} = \frac{15}{8} = 1\frac{7}{8}.$$

$$10 + 1\frac{7}{8} = 11\frac{7}{8}.$$

(16)

$$\frac{1}{8} + \frac{2}{4} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} + \frac{7}{8} + \frac{8}{8} = \frac{1}{8} + \frac{4}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} + \frac{7}{8} + \frac{8}{8} = \frac{32}{8} = 4.$$

(17)

$$7 + 11\frac{1}{2} + 18 + 26\frac{3}{4} + 79\frac{1}{4} = 7 + 11 + 18 + 26 + 79 + (\frac{1}{2} + \frac{3}{4} + \frac{1}{4}).$$

$$7 + 11 + 18 + 26 + 79 = 141.$$

$$\frac{1}{2} + \frac{3}{4} + \frac{1}{4} = \frac{2}{4} + \frac{3}{4} + \frac{1}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}.$$

$$141 + 1\frac{1}{2} = 142\frac{1}{2}.$$

(18)

$$\frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{21}{3} = \frac{1}{2} \text{ of } 3\frac{1}{2} = 1\frac{3}{4}.$$

$$\frac{3}{4} + \frac{21}{4} + \frac{3}{4} = \frac{3}{4} + \frac{21}{4} + \frac{3}{4} = \frac{27}{4} = 6\frac{3}{4}.$$

$$10 + 6\frac{3}{4} = 16\frac{3}{4}.$$

(19)

$$\frac{4\frac{1}{2}}{7\frac{1}{2}} = \frac{13}{7\frac{1}{2}} = \frac{13 \times 18}{3 \times 7} = 7\frac{1}{2} = 11\frac{1}{2}.$$

$$\frac{1}{2} \text{ of } 3\frac{1}{2} \text{ of } 1\frac{1}{2} \text{ of } 1\frac{1}{2} = \frac{1 \times 36 \times 4 \times 11}{2 \times 11 \times 15 \times 4} = \frac{13}{15} = 1\frac{3}{5}.$$

$$\frac{20\frac{3}{4}}{7\frac{1}{11}} = \frac{83}{11} = \frac{83 \times 11}{4 \times 83} = 11 = 2\frac{1}{2}.$$

$$11\frac{1}{2} + 1\frac{3}{5} + 2\frac{1}{2} = 11 + 1 + 2 + (\frac{1}{2} + \frac{3}{5} + \frac{1}{2}) = 14 + (\frac{1}{2} + \frac{3}{5} + \frac{1}{2}).$$

$$\frac{1}{2} + \frac{3}{5} + \frac{1}{2} = \frac{5}{10} + \frac{6}{10} + \frac{5}{10} = \frac{16}{10} = 1\frac{6}{10} = 1\frac{3}{5}.$$

$$14 + 1\frac{3}{5} = 15\frac{3}{5}.$$

(20)

$$3\frac{5}{8} + 11\frac{1}{8} + 14\frac{3}{8} = 3 + 11 + 14 + (\frac{5}{8} + \frac{1}{8} + \frac{3}{8}) = 28 + (\frac{5}{8} + \frac{1}{8} + \frac{3}{8}).$$

$$\frac{5}{8} + \frac{1}{8} + \frac{3}{8} = \frac{9}{8} + \frac{1}{8} + \frac{3}{8} = \frac{13}{8} = 1\frac{5}{8}. \quad 28 + 1\frac{5}{8} = 29\frac{5}{8}.$$

(21)

$$\frac{1}{2} \text{ of } 3 = \frac{3}{2}, \frac{2}{3} \text{ of } 6 = 4, \frac{3}{4} \text{ of } 7 = 5\frac{1}{4}, \frac{5}{6} \text{ of } 27 = 22\frac{1}{2}, \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2}$$

$$\text{of } \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{8}.$$

$$\frac{3}{2} + 4 + 5\frac{1}{4} + 22\frac{1}{2} + \frac{1}{8} = \frac{630}{80} + \frac{320}{80} + \frac{105}{40} + \frac{105}{40} + \frac{1}{80} = \frac{1630}{80} = 20\frac{3}{8} = 20\frac{3}{8}.$$

(22)

$$41\frac{1}{2} + 105\frac{2}{3} + 300\frac{3}{4} + 241\frac{1}{2} + 472\frac{1}{2} = 41 + 105 + 300 + 241 + 472 + (\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{1}{2} + \frac{1}{2}).$$

$$41 + 105 + 300 + 241 + 472 = 1159.$$

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{1}{2} + \frac{1}{2} = \frac{90}{180} + \frac{120}{180} + \frac{135}{180} + \frac{90}{180} + \frac{90}{180} = \frac{525}{180} = 2\frac{7}{12} = 2\frac{7}{12}.$$

$$1159 + 2\frac{7}{12} = 1161\frac{7}{12}.$$

(23)

$$92\frac{1}{4} + 37\frac{5}{9} + 7\frac{1}{2} = 92 + 37 + 7 + (\frac{1}{4} + \frac{5}{9} + \frac{1}{2}) = 136 + (\frac{1}{4} + \frac{5}{9} + \frac{1}{2}).$$

$$\frac{1}{4} + \frac{5}{9} + \frac{1}{2} = \frac{99}{36} + \frac{20}{36} + \frac{18}{36} = \frac{137}{36} = 3\frac{19}{36}.$$

$$136 + 3\frac{19}{36} = 139\frac{19}{36}.$$

(24)

$$\frac{10\frac{3}{4}}{\frac{2}{3}} = \frac{\frac{83}{4}}{\frac{1}{6}} = \frac{53 \times 5}{5 \times 12} = \frac{4\frac{3}{4}}{1} = 4\frac{3}{4} \quad \frac{1}{3} \text{ of } \frac{7}{4} = \frac{7}{12}$$

$$21\frac{1}{2} + 35\frac{1}{2} + 4\frac{1}{2} + \frac{7}{2} = 21 + 35 + 5 + (\frac{1}{2} + \frac{1}{2}) = 61\frac{1}{2}$$

(25)

$$\frac{1}{4} \text{ of } \frac{1}{3} = \frac{1}{12} = 10\frac{1}{12} \quad \frac{1}{6} \text{ of } \frac{3}{4} \text{ of } \frac{1}{2} = \frac{1}{16} = 15\frac{1}{16}$$

$$\frac{1}{16} = 6\frac{1}{16} \quad \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{4} \text{ of } \frac{1}{2} = \frac{1}{48} = 1\frac{1}{48}$$

$$10\frac{1}{12} + 6\frac{1}{16} + 15\frac{1}{16} + 1\frac{1}{48} = 10 + 6 + 15 + 1 + (\frac{1}{12} + \frac{1}{16} + \frac{1}{16} + \frac{1}{48})$$

$$\frac{1}{12} + \frac{1}{16} + \frac{1}{16} + \frac{1}{48} = \frac{4}{48} + \frac{3}{48} + \frac{3}{48} + \frac{1}{48} = \frac{11}{48} = 2\frac{1}{48}$$

$$32 + 2\frac{1}{48} = 34\frac{1}{24}$$

## Exercise 56—Page 169.

(1)

	oz.	dr.	scr.	grs.
$\frac{1}{16}$ of a lb. =	4	2	2	$14\frac{6}{16}$
$\frac{1}{8}$ of an oz. =	3	1		$5\frac{1}{8}$
$\frac{1}{4}$ of a dr. =		1		$1\frac{3}{4}$
$\frac{1}{8}$ of a scr. =				$16\frac{3}{8}$

$$4 \quad 6 \quad 2 \quad 18\frac{1}{2}$$

(3)

	in.
$\frac{1}{4}$ of a yd. =	$5\frac{1}{4}$
$\frac{1}{2}$ of a ft. =	$1\frac{1}{2}$
$\frac{1}{4}$ of an in. =	$\frac{1}{4}$
	<u>7</u>

(5)

	day	hrs.	min.
$\frac{1}{4}$ of a week =	1	18	0
$\frac{1}{3}$ of a day =		8	0
$\frac{1}{3}$ of an hour =			12
	<u>2</u>	<u>2</u>	<u>12</u>

(2)

	qr.	na.	in.
$\frac{3}{4}$ of a yard =	2	1	$1\frac{3}{4}$
$\frac{1}{2}$ of an Eng. ell. =		2	$1\frac{1}{2}$
$\frac{1}{4}$ of a qr. =		3	$0\frac{3}{4}$
	<u>3</u>	<u>3</u>	<u><math>1\frac{3}{4}</math></u>

(4)

	fur.	per.	ysds.	ft.	in.
$\frac{1}{16}$ of a mile =	5	3	3	1	6
$\frac{1}{12}$ of a fur. =		12	1	2	$0\frac{1}{3}$
$\frac{1}{2}$ of a yd. =				1	$2\frac{1}{4}$
	<u>5</u>	<u>16</u>	<u>0</u>	<u>0</u>	<u><math>3\frac{1}{4}</math></u>

(6)

	s.	d.
$\frac{1}{7}$ of a £ =	2	$10\frac{2}{7}$
$\frac{2}{3}$ of a s. =		$2\frac{2}{3}$
$\frac{1}{2}$ d. =		$\frac{1}{2}$
	<u>3</u>	<u><math>1\frac{3}{4}</math></u>

$$\frac{1}{3} \text{ of } \frac{7}{8} = \frac{7}{24}.$$

$$+ \frac{1}{4} = 61\frac{1}{4}.$$

$$\frac{2883}{10000} = 15\frac{1}{100}.$$

$$5 + 1 + (1\frac{1}{2} +$$

$$\frac{883}{10000} + \frac{8900}{10000} =$$

(7)

	£	s.	d.
$\frac{1}{4}$ of 21s.	=	13	1 $\frac{1}{2}$
$\frac{1}{4}$ of 5s.	=	3	1 $\frac{1}{2}$
$\frac{1}{4}$ of £3 12s. 6d.	=	2	5 3 $\frac{1}{2}$
$\frac{1}{4}$ of a £	=	10	9 $\frac{1}{2}$
$\frac{1}{4}$ d.	=		$\frac{1}{4}$
		3	12 4 $\frac{1}{4}$

## EXERCISE 57—Page 171.

(1)

$$\frac{1}{4} - \frac{1}{5} = \frac{10}{20} - \frac{4}{20} = \frac{6}{20} = \frac{3}{10}.$$

(2)

$$\begin{aligned} \frac{1}{7} \text{ of } \frac{1}{4} \text{ of } \frac{1}{4} &= \frac{3 \times 48}{17 \times 11} = \frac{144}{187}. \quad \frac{144}{187} + \frac{144}{187} = \frac{288}{187} + \frac{144}{187} = \\ &= \frac{432}{187} = 1\frac{245}{187} = 1\frac{1}{4}. \\ \frac{81}{17} &= \frac{36}{17} = \frac{35 \times 11}{4 \times 70} = \frac{11}{4 \times 2} = \frac{11}{8} = 1\frac{3}{8}. \\ \frac{64}{17} - \frac{17}{17} &= 0. \end{aligned}$$

(3)

$$\begin{aligned} 982\frac{1}{2} - 29\frac{1}{2} &= 982\frac{340}{400} - 29\frac{100}{400} = 981 + 1\frac{240}{400} = \\ 29\frac{100}{400} &= 981\frac{240}{400} - 29\frac{100}{400} = 952\frac{140}{400}. \end{aligned}$$

(4)

$$\begin{aligned} 69\frac{1}{4} - 18\frac{2}{4} &= 69\frac{146}{146} - 18\frac{200}{146} = 68 + 1\frac{146}{146} - 18\frac{200}{146} = \\ 68\frac{146}{146} - 18\frac{200}{146} &= 50\frac{246}{146} = 50\frac{123}{73}. \end{aligned}$$

(5)

$$100\frac{1}{2} - 9\frac{1}{2} = 100\frac{1}{2} - 9\frac{1}{2} = 99 + 1\frac{1}{2} - 9\frac{1}{2} = 99\frac{1}{2} - 9\frac{1}{2} = 90\frac{1}{2}$$

(6)

$$\begin{aligned} \frac{1}{4} \text{ of } \frac{3}{4} &= \frac{3}{4} = 4\frac{1}{2}. \quad 6\frac{1}{2} - 4\frac{1}{2} = 6\frac{1}{2} - 4\frac{1}{2} = 5 + 1\frac{1}{2} - 4\frac{1}{2} = \\ 5\frac{1}{2} - 4\frac{1}{2} &= 1\frac{1}{2}. \end{aligned}$$

(7)

$$\begin{aligned} 611\frac{1}{2} - 610\frac{1}{2} &= 611\frac{2557}{2557} - 610\frac{2557}{2557} = 610 + 1\frac{2557}{2557} - \\ 610\frac{2557}{2557} &= 610\frac{2557}{2557} - 610\frac{2557}{2557} = \frac{2557}{2557}. \end{aligned}$$

E

(8)

$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{4}, \quad \frac{1}{2} + \frac{1}{2} = \frac{2}{2} + \frac{2}{2} = \frac{4}{2}, \quad \frac{1}{2} \text{ of } \frac{1}{4} = \frac{1}{8},$$

(9)

	oz.	dr.
$\frac{3}{4}$ of a lb. =	10	10 $\frac{3}{4}$
$\frac{3}{4}$ of a dr. =		$\frac{9}{8}$
	10	9 $\frac{3}{4}$

$\frac{1}{2}$  of a dr. =  $\frac{1}{2}$   $\frac{1}{2}$

$\frac{1}{2}$  10 98

(10)

$$\begin{aligned} 24\frac{1}{2} - 21\frac{1}{8} &= 24\frac{7}{8} - 21\frac{1}{8} = \\ 23 + 1\frac{7}{8} - 21\frac{1}{8} &= 23\frac{7}{8} - \\ 21\frac{1}{8} &= 2\frac{6}{8}. \end{aligned}$$

$$23 + 1\frac{7}{168} - 21\frac{8}{168} = 23\frac{7}{168} -$$

$$21_{\overline{188}} = 21_{\overline{88}}.$$

(11)

	fur.	per.	yds.	ft.	in.
$\frac{3}{4}$ of a mile =	1	31	0	1	10
$\frac{1}{11}$ of a fur. =		25	2	1	6
	1	5	3	1	10

$\frac{1}{11}$ of a fur. =	25	2	1	6
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(12)

$$71 - 13\frac{1}{2} = 71\frac{1}{2} - 13\frac{1}{2} = 6 + 13\frac{1}{2} - 13\frac{1}{2} = 6\frac{1}{2} - 13\frac{1}{2} = 53\frac{1}{2}.$$

$$71 - 136 = 712 - 132 = 6 + 116 - 136 = 642 - 132 = 510.$$

(13)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{5}{6} \text{ of } \frac{7}{8} = \frac{1 \times 3 \times 2 \times 3 \times 6 \times 5}{2 \times 3 \times 4 \times 6 \times 8} = \frac{135}{512}$$

[illegible]

$$\frac{111}{133} = \frac{11}{44} = \frac{196 \times 83}{11 \times 56} = \frac{21}{2} = 10\frac{1}{2}$$

$$12\frac{3}{8} - 10\frac{1}{2} = 12\frac{3}{8} - 10\frac{4}{8} = 2\frac{3}{8}.$$

(14)

$$3\frac{1}{8} + 8\frac{1}{9} + 5\frac{1}{7} + 6\frac{1}{2} = 3 + 8 + 5 + 6 + (\frac{1}{8} + \frac{1}{9} + \frac{1}{7} + \frac{1}{2}) = 22 + (\frac{1}{8} + \frac{1}{9} + \frac{1}{7} + \frac{1}{2}).$$

$$\frac{1}{18} + \frac{1}{9} + \frac{1}{6} + \frac{1}{3} = \frac{16}{180} + \frac{20}{180} + \frac{36}{180} + \frac{60}{180} = \frac{132}{180}.$$

$$22 + \cancel{190} = 22\cancel{180}.$$

$$3^3_0 + 2^2_6 + 16_4 = 3 + 2 + 16 + (1^3_0 + \frac{4}{6} + \frac{1}{4}) = 21 + (1^3_0 + \frac{4}{6} + \frac{1}{4}).$$

$$1^3 + 8 + 1 = 1^3 + 8 + 1 = 10 = 10. \quad 21 + 10 = 31.$$

$$22188 - 2288 = 22188 - 22188 = 0 = 0.$$

$$= \frac{1}{18}.$$

$$\frac{7}{68} - \frac{21}{136} =$$

$$= 23\frac{7}{136} -$$

$$\begin{array}{r} \frac{1}{11} \text{ of an acre} = 1 \text{ r. } 18 \text{ yds. } 5 \text{ ft. } 4 \text{ in.} \\ \frac{1}{5} \text{ of a per.} = \quad \quad \quad 13 \text{ } 4 \\ \hline 1 \text{ } 17 \text{ } 22 \text{ } 2 \text{ } 108 \end{array}$$

(16)

$$\begin{aligned} 16\frac{1}{13} - 9\frac{1}{13} &= 16\frac{1}{13} - 9\frac{22}{13} = 15 + 1\frac{1}{13} - 9\frac{22}{13} = 15\frac{1}{13} - 9\frac{22}{13} = 6\frac{11}{13}. \\ 169\frac{17}{100} - 83\frac{17}{100} &= 169\frac{221}{1000} - 83\frac{221}{1000} = 168 + 1\frac{221}{1000} - 83\frac{221}{1000} = 168\frac{221}{1000} - 83\frac{221}{1000} = 85\frac{671}{1000}. \end{aligned}$$

## EXERCISE 58—Page 173.

(1)

$$\frac{7}{12} \text{ of } \frac{5}{6} = \frac{7 \times 5}{12 \times 6} = \frac{35}{72}.$$

(2)

$$\frac{5}{8} \times \frac{4}{5} = \frac{1}{2}.$$

(3)

$$\frac{4}{15} \times \frac{5}{24} = \frac{1}{9}.$$

(4)

$$\frac{1}{2} \times \frac{4}{5} \times \frac{7}{15} = \frac{28}{75}.$$

(5)

$$\frac{14}{1} \times \frac{241}{18} \times \frac{82}{9} = \frac{14 \times 241 \times 82}{9} = \frac{27942}{9} = 749\frac{2}{3}$$

(6)

$$\frac{3}{10} \times \frac{7}{4} \times \frac{9}{11} \times \frac{11}{12} = \frac{3 \times 7 \times 9}{2 \times 4 \times 4} = \frac{189}{32} = 5\frac{29}{32}.$$

(7)

$$\frac{4}{5} \times \frac{3}{11} \times \frac{9}{17} \times \frac{182}{200} \times \frac{5}{9} = \frac{3 \times 182}{11 \times 17 \times 25} = \frac{546}{4675}.$$



(8)

$$\frac{\overset{3}{8}}{7} \times \frac{11}{\underset{\frac{1}{2}}{8}} \times \frac{\overset{3}{8}}{\underset{\frac{1}{2}}{88}} \times \frac{\overset{3}{21}}{1} \times \frac{8}{5} \times \frac{5}{1} = \frac{3 \times 3 \times 3}{2} = \frac{27}{2} = 13\frac{1}{2}.$$

(9)

$$\frac{\overset{2}{8}}{\underset{\frac{1}{2}}{8}} \times \frac{8}{5} \times \frac{6}{11} \times \frac{4}{19} \times \frac{11}{\underset{1}{200}} = \frac{2 \times 6 \times 4}{5} = \frac{48}{5} = 9\frac{3}{5}.$$

(10)

$$\frac{18}{2} \times \frac{80}{7} \times \frac{\overset{2}{180}}{11} \times \frac{2}{18} \times \frac{7}{80} \times \frac{1}{50} = 1\frac{1}{2}.$$

(11)

$$\frac{4}{7} \times \frac{3}{11} \times \frac{9}{\underset{4}{18}} \times \frac{7}{1} \times \frac{8}{7} \times \frac{8}{18} \times \frac{\overset{18}{91}}{1} \times \frac{167}{\underset{\frac{1}{2}}{24}} = \frac{3 \times 9 \times 167}{4} = \frac{4509}{4} = 1127\frac{1}{4}.$$

(12)

$$\frac{\frac{1}{7}}{\frac{8}{1}} \times \frac{\frac{8}{19}}{\frac{1}{2}} \times \frac{\frac{64}{9}}{\frac{8}{5}} \times \frac{\frac{19}{4}}{\frac{101}{14}} \times \frac{3}{27} \times \frac{2}{32} =$$

$$\frac{1}{7 \times 8} \times \frac{8 \times 2}{7 \times 19} \times \frac{8 \times 5}{9 \times 8} \times \frac{19 \times 14}{101 \times \frac{1}{2}} \times \frac{1}{9} \times \frac{2}{8} = \frac{1}{7 \times 101} = \frac{1}{707}.$$

(13)

$$\frac{1}{4} \times \frac{\overset{2}{8}}{1} \times \frac{2}{7} \times \frac{19}{1} = \frac{2 \times 2 \times 19}{7} = \frac{76}{7} = 10\frac{6}{7}.$$

## EXERCISE 58.]

KEY.

69

$$3 = \frac{27}{7} = 13\frac{1}{7}.$$

(14)

$$\frac{9}{10} \times \frac{7}{5} \times \frac{11}{15} \times \frac{32}{11} = \frac{9 \times 7 \times 32}{5} = 403\frac{1}{5}.$$

(15)

$$4\frac{2}{3} = 9\frac{1}{3}.$$

$$\frac{27}{4} \times \frac{7}{8} \times \frac{4}{5} \times \frac{4}{7} = 1\frac{1}{5} = 2\frac{1}{5}.$$

(16)

$$\frac{11}{8} \times \frac{13}{8} \times \frac{15}{1} = \frac{11 \times 13 \times 15}{8} = 268\frac{1}{8}.$$

(17)

$$= 4502 = 1127\frac{1}{2}.$$

$$\frac{1}{8} \times \frac{7}{4} \times \frac{8}{19} \times \frac{19}{2} \times \frac{94}{11} \times \frac{16}{17} \times \frac{49}{8} \times \frac{4}{5} \times \frac{27}{81} \times \frac{31}{2} \times \frac{191}{188} = \frac{7 \times 49 \times 27 \times 191}{2 \times 11 \times 17} = 4729\frac{3}{4}.$$

(18)

$$\frac{1}{101} = 7\frac{1}{7}.$$

$$\frac{27}{37\frac{1}{2}} \times \frac{87\frac{1}{2}}{98\frac{1}{2}} \times \frac{7}{2\frac{1}{2}} \times \frac{81\frac{1}{2}}{128} = \frac{27}{128} \times \frac{78\frac{1}{2}}{28\frac{1}{2}} \times \frac{7}{7} \times \frac{89\frac{1}{2}}{12\frac{1}{2}} = \frac{27 \times 5}{189} \times \frac{78\frac{1}{2} \times 8}{8 \times 78\frac{1}{2}} \times \frac{7 \times 8}{8 \times 7} \times \frac{89\frac{1}{2} \times 1}{11 \times 128} = \frac{5}{3 \times 11} = \frac{5}{33}.$$

(19)

$$\frac{5}{11} \times \frac{1}{7} \times \frac{3}{5} \times \frac{17}{19} = \frac{3 \times 17}{11 \times 7} = \$\frac{1}{7}.$$

(20)

$$\begin{aligned}
 & \frac{75\frac{3}{4}}{6\frac{1}{11}} \times \frac{\frac{3}{4} \text{ of } 8\frac{1}{2} \times \frac{1}{15} \text{ of } 28}{\frac{2}{11} \text{ of } 6\frac{3}{4} \times \frac{1}{17} \text{ of } 24} \times \frac{7\frac{1}{2}}{15} \times \frac{2}{4} \times 14\frac{3}{4} \times \frac{100}{121} \times \\
 & \quad \frac{4}{5\frac{1}{2}} \times \frac{5}{9} = \\
 & \frac{69\frac{3}{4}}{41} \times \frac{\frac{3}{4} \times \frac{3}{4} \times \frac{2}{15} \times \frac{2}{17}}{\frac{2}{11} \times \frac{6\frac{1}{8}}{8} \times \frac{1}{17} \times \frac{2}{11}} \times \frac{3\frac{3}{4}}{1\frac{1}{2}} \times \frac{2}{4} \times \frac{101}{7} \times \frac{100}{121} \times \\
 & \quad \frac{1}{1\frac{1}{2}} \times \frac{5}{9} = \\
 & \frac{608 \times 11}{8 \times 87} \times \frac{8 \times 88 \times 28 \times 11 \times 8 \times 17}{7 \times 4 \times 15 \times 2 \times 5 \times 1 \times 24} \times \frac{88}{5 \times 15} \times \frac{8 \times 7}{4 \times 5} \times \\
 & \quad \frac{101}{7} \times \frac{100}{121} \times \frac{4 \times 8}{16} \times \frac{5}{7 \times 9} = \\
 & \quad \frac{11 \times 9 \times 101}{5 \times 7 \times 16} = \frac{9999}{2660} = 17\frac{179}{2660}.
 \end{aligned}$$

EXERCISE 59—Page 174.

(1)

$$\frac{4\frac{1}{2}}{36} \text{ of 4 days, 5 hours, } = \frac{180 \text{ d. } 23 \text{ h.}}{36} = 5 \text{ d. } 0 \text{ h. } 38 \text{ min. } 20 \text{ sec.}$$

(2)

$$\frac{1\frac{1}{2} \text{ of } £29}{42} = \frac{£29 \times 13}{42} = \frac{£377}{42} = £8 \text{ } 19\text{s. } 6\frac{1}{2}\text{d.}$$

$$\frac{100}{121} \times \frac{100}{121} \times$$

$$\frac{101}{7} \times \frac{100}{121} \times$$

$$\frac{8 \times 7}{4 \times 5} \times$$

min. 20 sec.

(3)

$$\frac{7}{9} \text{ of } 186 \text{ a. } 3 \text{ r.} = \frac{186 \text{ a. } 3 \text{ r.} \times 7}{9} = \frac{1307 \text{ a. } 1 \text{ r.}}{9} = 145 \text{ a. } 1 \text{ r.}$$

(4)

$$\frac{1}{4} \text{ of } \frac{7}{8} \text{ of } \frac{1}{10} \text{ of } \frac{4}{5} \text{ of } 24 \text{ h. } 30 \text{ m.} = \frac{1}{16} \text{ of } 24 \text{ h. } 30 \text{ m.} = 1 \text{ h. } 38 \text{ m.}$$

(5)

$$\frac{7}{9} \text{ of } \frac{4}{5} \text{ of } \frac{1}{10} \text{ of } \frac{7}{8} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} = \frac{7}{90} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} =$$

$$\frac{33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} \times 7}{90} = \frac{235 \text{ b. } 1 \text{ p. } 1 \text{ g.}}{90} = 2 \text{ b. } 2 \text{ p. } 0 \text{ g. } 3 \text{ q } 1 \frac{1}{2} \text{ p.}$$

## EXERCISE 60—Page 175.

(1)

$$\frac{1}{2} \text{ of } \frac{3}{4} \div \frac{1}{2} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{4} \times \frac{4}{3} \times \frac{4}{35} = \frac{2 \times 4}{5 \times 35} = \frac{8}{175}.$$

(2)

$$\frac{15}{12} \div \frac{2}{3} \div \frac{1}{5} = \frac{15}{24} \times \frac{3}{2} \times \frac{11}{5} = \frac{5}{2 \times 3} = \frac{5}{6}$$

(3)

$$82 \frac{1}{7} \div 26 \frac{5}{11} = \frac{155}{17} \times \frac{41}{1071} = \frac{155 \times 41}{17 \times 119} = \frac{6355}{2023} = 3 \frac{225}{2023}.$$

(4)

$$2 \frac{1}{2} \div \frac{1}{4} + \frac{1}{5} = \frac{5}{2} \div \frac{1}{4} = \frac{5}{2} \times \frac{4}{1} = \frac{5 \times 4}{11} = \frac{20}{11} = 1 \frac{9}{11}$$

(5)

$$1\frac{1}{2} \div \frac{1}{4} \text{ of } 2\frac{1}{2} \text{ of } 16 \text{ of } \frac{3}{4} \text{ of } \frac{1}{10} = \frac{7}{4} \times \frac{7}{1} \times \frac{4}{11} \times \frac{1}{18} \times \frac{4}{85} \times$$

$$\frac{2}{10} = \frac{7 \times 7}{2 \times 11} = \frac{49}{22} = 2\frac{5}{11}.$$

(6)

$$2\frac{1}{2} \div (\frac{1}{2} \div \frac{1}{15} \text{ of } 9) = \frac{7}{2} \div (\frac{1}{2} \text{ of } \frac{3}{2} \text{ of } \frac{1}{3}) = \frac{7}{2} \times \frac{9}{5} \times \frac{3}{16} \times$$

$$\frac{3}{2} = \frac{7 \times 9 \times 3 \times 3}{5 \times 16} = \frac{567}{80} = 7\frac{7}{80}.$$

(7)

$$48\frac{1}{2} \div \frac{2}{3} + \frac{1}{2} \text{ of } 6 = \frac{97}{2} \div \frac{2}{3} + \frac{1}{2} = \frac{97}{2} \div \frac{2}{3} = \frac{97}{2} \times \frac{3}{2} = \frac{97 \times 18}{4} = \frac{1746}{4} = 193\frac{3}{4}.$$

(8)

$$6\frac{1}{2} \div \frac{3}{5} \text{ of } \frac{2}{10} + \frac{1}{17} = \frac{13}{2} \div \frac{3}{10} + \frac{1}{17} = \frac{13}{2} \div \frac{3}{10} = \frac{13}{2} \times \frac{10}{3} = \frac{13 \times 5}{3} = \frac{65}{3} = 21\frac{2}{3}.$$

(9)

$$\frac{1}{2} \times \frac{1}{3} \div \frac{1}{4} \times \frac{2}{5} = \frac{1}{2} \times \frac{10}{3} \times \frac{4}{2} \times \frac{4}{5} = \frac{4 \times 4}{3 \times 5} = \frac{16}{15} = 1\frac{1}{15}.$$

(10)

$$\frac{27}{15} \div \frac{7}{25} = \frac{67 \times 3}{9 \times 35} \div \frac{3 \times 8}{7 \times 33} = \frac{67 \times 3}{9 \times 35} \times \frac{7 \times 33}{3 \times 8} = \frac{67 \times 11}{3 \times 5 \times 8} = \frac{737}{120} = 6\frac{17}{120}.$$

(11)

$$\frac{1}{2} \text{ of } \frac{10}{11} \div \frac{1}{11} \text{ of } \frac{13}{7} = \frac{10}{11} \times \frac{11}{4} \times \frac{7}{13} = \frac{5 \times 10 \times 7}{9 \times 61} = \frac{350}{61}.$$

(12)

$$\frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5} \div \frac{1}{6} \text{ of } \frac{1}{7} \text{ of } \frac{1}{8} \text{ of } \frac{1}{9} = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \div \frac{1}{6} \times \frac{1}{7} \times \frac{1}{8} \times \frac{1}{9} = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \frac{6}{1} \times \frac{7}{1} \times \frac{8}{1} \times \frac{9}{1} = \frac{252}{65} = 3\frac{7}{65}.$$

(13)

$$\frac{7}{8} \div \frac{3}{4} = \frac{7 \times 2}{4 \times 9} \div \frac{7 \times 4}{3 \times 9} = \frac{7 \times 2}{4 \times 9} \times \frac{3 \times 9}{7 \times 4} = \frac{3}{2 \times 4} = \frac{3}{8}.$$

(14)

$$\frac{3}{25} \div \frac{21}{35} = \frac{3}{25} \div \frac{21 \times 2}{5 \times 35} = \frac{3}{25} \times \frac{5 \times 35}{21 \times 2} = 1.$$

(15)

$$\frac{113}{13} \times \frac{1}{9} \div \frac{1}{3} \times \frac{107}{13} \times \frac{17}{13} = \frac{113}{13} \times \frac{1}{9} \times \frac{7}{3} \times \frac{17}{107} \times \frac{17}{13} = \frac{113 \times 2 \times 17}{9 \times 3 \times 107} = \frac{3842}{2889} = 1\frac{253}{2889}.$$

(16)

$$\begin{aligned}
 & \frac{31}{2} \times \frac{7}{3} \times \frac{1}{7} \times \frac{16}{7} \div \frac{41}{9} \times \frac{7}{19} \times \frac{7}{8} \times \frac{11}{4} = \\
 & \frac{31}{2} \times \frac{2 \times 9}{7 \times 7} \times \frac{7 \times 3}{2} \times \frac{16}{10 \times 3} \div \frac{41}{9 \times 7} \times \frac{3 \times 4}{19} \times \frac{7 \times 2}{8 \times 7} \times \frac{11 \times 7}{4 \times 4} = \\
 & \frac{31}{2} \times \frac{2 \times 9}{7 \times 7} \times \frac{7 \times 3}{2} \times \frac{16}{10 \times 3} \times \frac{9 \times 7}{41} \times \frac{3 \times 4}{19} \times \frac{7 \times 2}{8 \times 7} \times \frac{11 \times 7}{4 \times 4} = \\
 & \frac{31 \times 3 \times 9 \times 19 \times 4}{5 \times 41 \times 11} = \frac{63612}{2255} = 28 \frac{472}{2255}.
 \end{aligned}$$

## EXERCISE 61—Page 176.

(1)

$$\begin{aligned}
 \frac{19}{4} &= \frac{19 \times 3}{11 \times 5} = \frac{57}{55}. \quad £8 \text{ 14s. } 6\frac{1}{2}\text{d.} \div \frac{57}{55} = £8 \text{ 14s. } 6\frac{1}{2}\text{d.} \times \frac{55}{57} = \\
 & \frac{£8 \text{ 14s. } 6\frac{1}{2}\text{d.} \times 55}{57} = £8 \text{ 8s. } 5\frac{1}{2}\text{d.}
 \end{aligned}$$

(2)

$$\begin{aligned}
 \frac{22}{5} \times \frac{10}{11} &= \frac{11}{5}. \quad 1 \text{ m. } 5 \text{ fur. } 91 \text{ yds. } 2 \text{ ft.} \div \frac{11}{5} = 1 \text{ m. } \\
 5 \text{ fur. } 91 \text{ yds. } 2 \text{ ft.} \times \frac{5}{11} &= \\
 1 \text{ m. } 5 \text{ fur. } 91 \text{ yds. } 2 \text{ ft.} \times 22 &= 2 \text{ fur. } 124 \text{ yds. } 2 \text{ ft.}
 \end{aligned}$$

(3)

$$\begin{aligned}
 3 \text{ a. } 3 \text{ r. } 3 \text{ per.} \div \frac{3}{8} &= 3 \text{ a. } 3 \text{ r. } 3 \text{ p.} \times \frac{8}{3} = \frac{3 \text{ a. } 3 \text{ r. } 3 \text{ p.} \times 8}{3} = \\
 6 \text{ a. } 1 \text{ r. } 5 \text{ per.}
 \end{aligned}$$

(4)

$$\begin{aligned}
 £7 \text{ 16s. } 2\text{d.} \div \frac{4}{9} &= £7 \text{ 16s. } 2\text{d.} \times \frac{9}{4} = \frac{£7 \text{ 16s. } 2\text{d.} \times 9}{4} = \\
 £17 \text{ 11s. } 4\frac{1}{2}\text{d.}
 \end{aligned}$$

## EXERCISE 62—Page 178.

(1)

$$\begin{array}{r}
 12\frac{1}{2} \\
 \underline{7} \\
 3\frac{1}{2} \\
 \underline{9} \\
 \frac{2}{3} \\
 \underline{7} \\
 5
 \end{array}
 =
 \begin{array}{r}
 4\frac{2}{3} \\
 \underline{7} \\
 1\frac{2}{3} \\
 \underline{9} \\
 \frac{2}{3} \\
 \underline{7} \\
 4
 \end{array}
 =
 \begin{array}{r}
 7 \\
 \underline{1\frac{1}{2}} \\
 2\frac{1}{2} \\
 \underline{7} \\
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 7\frac{1}{3} \\
 \underline{2\frac{1}{2}} \\
 4\frac{2}{3} \\
 \underline{7} \\
 \frac{2}{3}
 \end{array}
 =
 \begin{array}{r}
 7\frac{1}{3} \\
 \underline{2\frac{1}{2}} \\
 4\frac{2}{3} \\
 \underline{7} \\
 \frac{2}{3}
 \end{array}
 =
 \begin{array}{r}
 7\frac{1}{3} \\
 \underline{2\frac{1}{2}} \\
 4\frac{2}{3} \\
 \underline{7} \\
 \frac{2}{3}
 \end{array}
 =
 \begin{array}{r}
 7 \times 35 \\
 \underline{13 \times 27}
 \end{array}$$

$$\begin{array}{r}
 4\frac{1}{2} \\
 \underline{\frac{2}{3} \text{ of } 32} \\
 7 \\
 \underline{9\frac{1}{2}} \\
 3\frac{1}{2} \\
 \underline{7\frac{1}{2}} \\
 7 \times 35 \\
 \underline{13 \times 27}
 \end{array}
 =
 \begin{array}{r}
 4\frac{1}{2} \\
 \underline{7} \\
 2\frac{1}{2} \\
 \underline{7} \\
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 4\frac{1}{2} \\
 \underline{7} \\
 2\frac{1}{2} \\
 \underline{7} \\
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 4\frac{1}{2} \\
 \underline{7} \\
 2\frac{1}{2} \\
 \underline{7} \\
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 4\frac{1}{2} \\
 \underline{7} \\
 2\frac{1}{2} \\
 \underline{7} \\
 \frac{1}{2}
 \end{array}
 = 3.$$

$$\frac{7 \times 35}{13 \times 27} \times \frac{3}{1} = \frac{735}{351} = 2\frac{1}{3} = 2\frac{1}{3}.$$

(2)

$$\begin{array}{r}
 \frac{1}{3} \\
 \underline{7} \\
 6\frac{1}{3} \\
 \underline{9\frac{1}{3}} \\
 3 \\
 \underline{1} \\
 \frac{5}{6}
 \end{array}
 =
 \begin{array}{r}
 \frac{1}{3} \\
 \underline{7} \\
 1\frac{2}{3} \\
 \underline{1\frac{2}{3}} \\
 \frac{1}{3} \\
 \underline{1} \\
 \frac{5}{6}
 \end{array}
 =
 \begin{array}{r}
 \frac{1}{3} \\
 \underline{1\frac{2}{3}} \\
 1\frac{2}{3} \\
 \underline{1\frac{2}{3}} \\
 \frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 2 \\
 \underline{21 \times 13} \\
 1\frac{2}{3}
 \end{array}
 =
 \begin{array}{r}
 8 \\
 \underline{7 \times 13 \times 19}
 \end{array}$$

$$\frac{5}{6} = \frac{5}{42}, \quad \frac{8}{7 \times 13 \times 19} \div \frac{5}{42} = \frac{8}{7 \times 13 \times 19} \times \frac{42}{5} = 2\frac{1}{3}.$$



(3)

$$\begin{array}{r} 12\frac{1}{2} \\ \hline 5\frac{1}{4} \\ \hline 3\frac{3}{4} \\ \hline 5\frac{1}{2} \\ \hline 2\frac{1}{4} \\ \hline 5 \\ \hline 4\frac{1}{2} \\ \hline 3\frac{3}{4} \\ \hline 16\frac{3}{4} \\ \hline \frac{1}{2} \end{array} = \begin{array}{r} 3\frac{1}{2} \\ \hline 3\frac{1}{4} \\ \hline 1\frac{1}{4} \\ \hline 1\frac{1}{2} \\ \hline \frac{9}{4} \\ \hline \frac{5}{1} \\ \hline \frac{9}{2} \\ \hline 1\frac{1}{4} \\ \hline 5\frac{3}{4} \\ \hline \frac{1}{2} \end{array} = \begin{array}{r} 50 \\ \hline 1\frac{1}{2} \\ \hline 20 \\ \hline \frac{9}{2} \\ \hline 1\frac{1}{4} \\ \hline 100 \\ \hline \frac{1}{3} \end{array} = \frac{220}{63} \div \frac{8}{9} = \frac{220}{63} \times \frac{9}{8} = \frac{55}{14} = 3\frac{1}{2}.$$

**EXERCISE 63—Page 180.**

(1)

$$\frac{800}{2000} = \frac{2}{5}, \quad \frac{420}{2000} = \frac{21}{100}, \quad \frac{100}{2000} = \frac{1}{20}, \quad \frac{160}{2000} = \frac{2}{25},$$

$$\frac{35}{2000} = \frac{7}{400}.$$

(2)

$$\frac{2}{3} \text{ of } \frac{4}{5} \text{ of } \frac{3}{8} \text{ of } \frac{1}{2} \text{ of } \frac{5}{6} \text{ of } \frac{4}{5} = \frac{2}{5} \times \frac{4}{5} \times \frac{3}{8} \times \frac{1}{2} \times \frac{5}{6} \times \frac{4}{5} = \frac{2}{15}$$

(3)

$$6\frac{7}{8} \times 65\frac{3}{4} \text{ cts.} = \frac{55}{8} \times \frac{263}{4} \text{ cts.} = \frac{14165}{32} \text{ cts} = \$4.52\frac{1}{2}.$$

(4)

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}.$$

(5)

$$\frac{1}{2} + \frac{1}{10} + \frac{1}{5} + \frac{1}{5} = \frac{5}{10} + \frac{1}{10} + \frac{2}{5} + \frac{2}{5} = \frac{10}{10} = 1.$$

1 or  $\frac{10}{10} - \frac{9}{10} = \frac{1}{10}.$

(6)

$$\frac{5\frac{1}{2} - 2\frac{1}{2}}{3\frac{1}{2} + \frac{1}{2}} \text{ of } \frac{4\frac{1}{2} + 5\frac{1}{2}}{4\frac{1}{2}} \text{ of } \frac{2\frac{1}{2} + 1\frac{1}{2}}{7\frac{1}{2} - 2\frac{1}{2}} = \frac{5\frac{1}{2} - 2\frac{1}{2}}{3\frac{1}{2} + \frac{1}{2}} \text{ of } \frac{4\frac{1}{2} + 5\frac{1}{2}}{\frac{9}{2}} \text{ of } \frac{2\frac{1}{2} + 1\frac{1}{2}}{7\frac{1}{2} - 2\frac{1}{2}} = \frac{3\frac{1}{2} + \frac{1}{2}}{\frac{9}{2}} \text{ of } \frac{9\frac{1}{2}}{5\frac{1}{2}} \text{ of } \frac{5\frac{1}{2}}{1\frac{1}{2}} =$$

$$\frac{7}{8} \times \frac{2 \times 47}{5 \times 3} \times \frac{8 \times 64}{5 \times 133} = \frac{2 \times 64}{5 \times 3 \times 5} = \frac{128}{75} = 1\frac{53}{75}$$

(7)

$$1670\frac{1}{2} \times 12\frac{1}{2} \text{ cts.} = 2117\frac{1}{2} \times 5\frac{1}{2} \text{ cts.} = 11074\frac{1}{2} \text{ cts.} = \$212.99\frac{1}{2}.$$

(8)

$\frac{2}{3}$  of the longer =  $\frac{1}{2}$  of the shorter; therefore  $\frac{1}{2}$  of the longer =  $\frac{1}{3}$  of  $\frac{2}{3}$  =  $\frac{1}{3}$  of the shorter.

Hence the longer =  $\frac{2}{3} \times 3 = 2$  of the shorter.

The whole tree = longer + shorter =  $2 + 1$  of shorter =  $3$  of the shorter.

If 136 ft. =  $\frac{1}{3}$  of the shorter,  $\frac{1}{3}$  of 136 =  $45\frac{1}{3}$  =  $\frac{1}{3}$  of the shorter.

Hence shorter =  $45\frac{1}{3} \times 3 = 136$  ft.; and longer =  $136 - 45\frac{1}{3} = 90\frac{2}{3}$  ft.

(9)

$$97\frac{1}{2} + 127\frac{1}{2} + 500\frac{1}{2} + 333\frac{1}{2} = 97\frac{1}{2} + 127\frac{1}{2} + 500\frac{1}{2} + 333\frac{1}{2} = 1058\frac{1}{2}.$$

$$\$1000 + \$1375\frac{1}{2} + \$6831 + \$4013\frac{1}{2} = \$1000 + \$1375\frac{1}{2} + \$6831 + \$4013\frac{1}{2} = \$13219\frac{1}{2} = \$13219.68\frac{1}{2}.$$

(10)

$$12\frac{1}{2} + \frac{1}{5} = 13\frac{1}{10}. \quad 8\frac{1}{2} + 1\frac{1}{10} = 9\frac{6}{10}. \quad 13\frac{1}{10} - 9\frac{6}{10} = 3\frac{1}{10} = \frac{31}{100}.$$

$$7\frac{1}{2} - 6\frac{1}{2} = \frac{1}{2}. \quad \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = \frac{125}{1000} = 14\frac{61}{100}.$$

$$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3}. \quad \frac{4}{5} \div \frac{1}{10} = \frac{4}{5} \times \frac{10}{1} = \frac{40}{5} = 8. \quad \frac{1}{5} - \frac{1}{10} = \frac{1}{10} = \frac{27}{270}.$$

(11)

$$19\frac{1}{2} \times \$6\frac{1}{2} = 125 \times \$1\frac{1}{2} = \$187\frac{5}{2} = \$134.156.$$

(12)

$$376\frac{1}{2} \times \$75\frac{1}{2} = 2775 \times \$29\frac{1}{2} = 81412\frac{1}{2} = \$28387.061.$$

(13)

$$147\frac{1}{2} + 320\frac{1}{2} = 147\frac{1}{2} + 320\frac{1}{2} = 467\frac{1}{2}. \quad 467\frac{1}{2} - 156\frac{1}{2} = 311\frac{1}{2}.$$

(14)

$$\frac{7(1\frac{1}{2} \text{ of } \frac{1}{2})}{\frac{1}{2} \left( \frac{3}{3\frac{1}{2}} \text{ of } 7 \right)} \div 7\frac{1}{2} = \frac{1 \times 2 \times 1}{\frac{1}{2} \times \frac{1}{2} \times 1} \div 4\frac{1}{2} = \frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{1}{2\frac{1}{2}} =$$

$$\frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{2}{8} = 1. \quad \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = \frac{1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2}}{2} =$$

$$\frac{1\frac{1}{2}}{2} = \frac{1\frac{1}{2}}{2} = \frac{1\frac{1}{2}}{2} = \frac{2535}{2176} = 1\frac{2535}{2176}.$$

(15)

$$17\frac{1}{2} \div 7\frac{1}{2} = 19\frac{1}{2} \div 4\frac{1}{2} = 19\frac{1}{2} \times \frac{2}{9} = 19\frac{1}{2} = 2\frac{1}{2}.$$

(16)

$$3\frac{3}{4} + 4\frac{1}{2} + 4\frac{1}{2} = 3\frac{3}{4} + 4\frac{1}{2} + 4\frac{1}{2} = 13\frac{3}{4} = 7\frac{3}{4}.$$

$$7\frac{3}{4} - 5\frac{1}{2} = 7\frac{3}{4} - 5\frac{1}{2} = 2\frac{1}{4} = \frac{5}{10}.$$

$$94\frac{1}{2} + 93\frac{1}{2} = 94\frac{1}{2} + 93\frac{1}{2} = 187\frac{1}{2} = 187\frac{1}{2}.$$

$$7\frac{3}{4} \times \frac{2}{5} \div \frac{13\frac{1}{2}}{7\frac{1}{2}} = \frac{7\frac{3}{4}}{\frac{5}{1}} \times \frac{17}{85} \times \frac{8}{72} = \frac{1}{1}.$$

(17)

$$2\frac{3}{4} + \frac{1}{2} + 4 = 2\frac{3}{4} + 1\frac{1}{2} + 4 = 7\frac{1}{2} = 7\frac{1}{2}.$$

$$2 \div \frac{1}{10} = 2 \times \frac{10}{1} = 20. \quad 1\frac{1}{2} - \frac{1}{2} = 1\frac{1}{2} - \frac{1}{2} = 1.$$

$$\frac{1}{2} + \frac{1}{2} = 1. \quad 5\frac{1}{2} - 4\frac{1}{2} = 1. \quad 4\frac{1}{2} - 4\frac{1}{2} = 0.$$

$$\frac{583}{72} \times \frac{7}{10} = 4\frac{1}{2}.$$

(18)

$$\frac{1}{2} + \frac{1}{2} = 1. \quad 1\frac{1}{2} + 2\frac{1}{2} = 4\frac{1}{2} = 4\frac{1}{2}. \quad 2\frac{1}{4} - 1\frac{1}{4} = 1\frac{1}{4} = 1\frac{1}{4}.$$

$$3\frac{1}{10} - \frac{7}{10} = 2\frac{4}{10} = 1\frac{2}{5}. \quad \frac{5}{8} \times \frac{49}{12} \times \frac{187}{7} = \frac{187}{3 \times 12} = \frac{187}{36} = 5\frac{1}{6}.$$

$$1\frac{1}{2} \div 2\frac{1}{2} = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}. \quad 5\frac{1}{2} \div 3\frac{1}{2} = \frac{11}{2} \times \frac{2}{7} = \frac{11}{7} = 1\frac{4}{7}. \quad 1\frac{1}{2} + 1\frac{1}{2} = 2\frac{1}{2}.$$

(19)

$$1 - (\frac{1}{2} + \frac{1}{2}) = 0. \quad \frac{2}{3} \text{ of } \frac{1}{2} = \frac{1}{3}. \quad \frac{1}{2} - \frac{1}{2} = 0. \quad \frac{1}{2} + \frac{1}{2} = 1. \quad \frac{1}{2} - \frac{1}{2} = 0. \quad \frac{1}{2} \text{ of } \$40000 = \$1333\frac{1}{3}.$$

EXERCISE 66—Page 183.

(1)

$$\frac{1}{2} = 2)1 \quad \frac{3}{8} = 8)3$$

.5

.375

(2)

$$\frac{9}{25} = 25)9$$

$$\frac{1}{4} = 4)1$$

.36 =  $\frac{36}{100}$ .25 =  $\frac{25}{100}$

(3)

75)73 (.9733+

67.5

5.50

5.25

.250

.225

250

225

25

123)574(4.666+

492

82.0

73.8

8.20

7.38

.820

.738

82

34)15 (.44117+

13.6

1.40

1.36

40

34

60

34

260

238

22

(4)

7)6

.857142+

12)5

.4166+

9)4

.44444+

(5)

112)17 (.15178571428+

11.2

5.80

5.60

.200

.112

880

784

960

896

640

560

800

800

784

160

112

480

448

320

224

960

896

64

1296)718 (.554012+

648.0

70.00

64.80

5.200

5.184

1600

1296

3040

2592

448

$\frac{1}{2}$  of  $\frac{1}{2}$  of  
 $\frac{2}{3}$  of  $\frac{1}{2}$  of  
 27.

## EXERCISE 67—Page 184.

(1)

$$\begin{array}{r}
 12) 1.0 \text{ in.} \\
 \hline
 3) 2.083333 \text{ ft.} \\
 \hline
 5\frac{1}{2}) 3.694444 \text{ yd.} \\
 \begin{array}{r}
 2 \phantom{00} \\
 \hline
 2 \phantom{00} \\
 \hline
 \end{array} \\
 11) 7.388888 \\
 \hline
 40) .671717 \text{ per.} \\
 \hline
 .01679+ \text{ fur.}
 \end{array}$$

(2)

$$\begin{array}{r}
 12) 17.0 \text{ grs.} \\
 \hline
 2) 1.41666666 \\
 \hline
 20) 3.70833333 \text{ dwt.} \\
 \hline
 12) .18541666 \text{ oz.} \\
 \hline
 .01545138+ \text{ lb.}
 \end{array}$$

(3)

$$\begin{array}{r}
 20) 7.0 \text{ grs.} \\
 \hline
 3) 2.35 \text{ scr.} \\
 \hline
 8) .7833333 \text{ dr.} \\
 \hline
 12) .0979166 \text{ oz.} \\
 \hline
 .0081597+ \text{ lb.}
 \end{array}$$

(4)

$$\begin{array}{r}
 12) 9.0 \text{ in} \\
 \hline
 3) 2.75 \text{ ft.} \\
 \hline
 5\frac{1}{2}) 2.91666 \\
 \begin{array}{r}
 2 \phantom{00} \\
 \hline
 2 \phantom{00} \\
 \hline
 \end{array} \\
 11) 5.83333 \\
 \hline
 40) 35.53030 \text{ per.} \\
 \hline
 8) 5.88825 \text{ fur.} \\
 \hline
 .73603+ \text{ mile.}
 \end{array}$$

(5)

$$\begin{array}{r}
 4) 2.0 \text{ na.} \\
 \hline
 4) 3.5 \text{ qr.} \\
 \hline
 .875 \text{ yd.}
 \end{array}$$

(7)

$$\begin{array}{r}
 60) 21.0 \text{ sec.} \\
 \hline
 60) 55.35 \text{ min.} \\
 \hline
 12) 12.9225 \text{ hr.} \\
 \hline
 2) 1.076875 \\
 \hline
 .5384375 \text{ day.}
 \end{array}$$

(6)

$$\begin{array}{r}
 13\text{s. } 4\text{d.} = 160\text{d.} \\
 5\text{s.} = 60\text{d.} \\
 \frac{60}{160} = \frac{3}{8} = .375
 \end{array}$$

(8)

$$\begin{array}{l}
 \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 6\frac{1}{2}\text{d.} = 3\frac{1}{4}\text{d. and } £\frac{1}{4} = 80\text{d.} \\
 \frac{1}{2}\text{d.} = \frac{1}{20} \text{ of } \frac{1}{2} \text{ of } £\frac{1}{4} = \frac{1}{2240} \text{ of } £\frac{1}{4}. \\
 27 \div 2240 = 0.012053.
 \end{array}$$

(9)

$$\begin{array}{r}
 \frac{3}{4} \text{ of } \frac{1}{2} \text{ of } 1 \text{ mil.} = 12672 \text{ in.} \\
 3\frac{1}{2}) 12672 \\
 \begin{array}{r}
 2 \phantom{00} \\
 \hline
 2 \phantom{00} \\
 \hline
 7) 25344 \\
 \hline
 \end{array}
 \end{array}$$

$$3620.571428+$$

(10)

$$\frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 4^{13} \text{ lbs.} = \frac{1}{4} \text{ lb.} = 110\frac{1}{4} \text{ drs.} = 1664 \text{ drs.}$$

$$\frac{1}{2} \text{ of an oz.} = 12 \text{ drs. } 1664 \div 12 = 138\frac{4}{3}$$

$$180)1664(9\cdot2444+$$

(11)

1620

2)1·0 pts.

440

4)1·5 qt.

360

2)1·375 gal.

800

4)3·6875 pk.

720

·921875 bush.

800

720

800

740

80

## EXERCISE 68—Page 186.

(1)

(2)

(3)

·3945

·3965

·309153

24

8

20

15780

3·1720 fur.

6·183060 dwt.

7890

40

24

9·4680 hrs.

6·8800 per.

732240

60

5½

366120

28·0800 min.

44000

4·393440 grs.

60

4400

4·8000 sec.

4·8400 yds.

3

2·5200 ft.

12

6·2400 in.

drs.

11)

0 pts.

5 qt.

375 gal.

6875 pk.

921875 bush.

(4)

$$22.75 = 22\frac{75}{100} = 22\frac{3}{4}$$

$$£2 \text{ 2s. 6d.} \times 22\frac{3}{4} = £48 \text{ 6s. 10}\frac{1}{2}\text{d.}$$

$$7 \text{ b. 1 p. 1 g. 1 qt.} = 237 \text{ qts.}$$

$$11 \cdot 17825 \times 237 = 2649 \cdot 24525 \text{ qt.} =$$

$$82 \text{ b. 3 p. 0 g. 1 q. 0.4905 pts}$$

(5)

(6)

$$\begin{array}{r} .2057 \\ 12 \end{array}$$

$$\begin{array}{r} 2.4684 \text{ oz.} \\ 20 \end{array}$$

$$\begin{array}{r} 9.3680 \text{ dwt.} \\ 24 \end{array}$$

$$\begin{array}{r} 14720 \\ 7360 \end{array}$$

$$8.8320 \text{ grs.}$$

(7)

$$1 \text{ f. 36 p. 2 y. 5 in.} = 15125 \text{ in.}$$

$$15125 \times .176 = 2662 \text{ in.} =$$

$$13 \text{ per. 2 yds. 1 ft. 4 in.}$$

(8)

$$\begin{array}{r} .625 \\ 3 \end{array}$$

$$\begin{array}{r} 1.875 \text{ mil.} \\ 8 \end{array}$$

$$7.000 \text{ fur.}$$

(3)

$$\begin{array}{r} .309153 \\ 20 \end{array}$$

$$\begin{array}{r} 6.183060 \text{ dwt.} \\ 24 \end{array}$$

$$\begin{array}{r} 732240 \\ 366120 \end{array}$$

$$4.393440 \text{ grs.}$$

(9)

$$\begin{array}{r} .015625 \\ 4 \end{array}$$

$$\begin{array}{r} .062500 \text{ pk.} \\ 2 \end{array}$$

$$\begin{array}{r} .125000 \text{ gal.} \\ 4 \end{array}$$

$$\begin{array}{r} .500000 \text{ qt.} \\ 2 \end{array}$$

$$1.000000 \text{ pt.}$$

(10)

$$\begin{array}{r} .9378 \\ 4 \end{array}$$

$$\begin{array}{r} 3.7512 \text{ r.} \\ 40 \end{array}$$

$$\begin{array}{r} 30.0480 \text{ per.} \\ 30\frac{1}{4} \end{array}$$

$$\begin{array}{r} 14400 \\ 120 \end{array}$$

$$\begin{array}{r} 1.4520 \text{ yd.} \\ 9 \end{array}$$

$$\begin{array}{r} 4.0680 \text{ ft.} \\ 144 \end{array}$$

$$2720$$

$$2720$$

$$680$$

$$9.7920 \text{ in.} = 9\frac{99}{128} \text{ in.}$$

(11)

$$1 \text{ sq. yd. 3 ft. 72 in.} = 1800 \text{ in.}$$

$$.2775 \times 1800 = 499.5 \text{ in.} =$$

$$3 \text{ ft. } 67\frac{1}{2} \text{ in.}$$



## EXERCISE 71—Page 191.

(1)

$$\begin{array}{r} \cdot 8 \\ \hline \end{array} = \frac{8}{10}.$$

$$\begin{array}{r} \cdot 05 \\ \hline \end{array} = \frac{5}{100}.$$

$$\begin{array}{r} \cdot 342 \\ \hline \end{array} = \frac{342}{1000} = \frac{171}{500}.$$

$$\begin{array}{r} \cdot 7004 \\ \hline \end{array} = \frac{7004}{10000}.$$

$$\begin{array}{r} \cdot 002003 \\ \hline \end{array} = \frac{2003}{100000}.$$

(2)

$$\begin{array}{r} \cdot 19 \\ \hline \end{array} = \frac{19}{100}.$$

$$\begin{array}{r} \cdot 1067 \\ \hline \end{array} = \frac{1067}{10000} = \frac{1067}{10000}.$$

$$\begin{array}{r} \cdot 11115 \\ \hline \end{array} = \frac{11115}{100000} = \frac{2223}{20000}.$$

$$\begin{array}{r} \cdot 704103 \\ \hline \end{array} = \frac{704103}{1000000} = \frac{704103}{1000000}.$$

(3)

$$\begin{array}{r} \cdot 102 \\ \hline \end{array} = \frac{102}{1000} = \frac{51}{500}.$$

$$\begin{array}{r} \cdot 0013 \\ \hline \end{array} = \frac{13}{10000}.$$

$$\begin{array}{r} \cdot 00007103 \\ \hline \end{array} = \frac{7103}{100000000}.$$

$$\begin{array}{r} \cdot 01020304 \\ \hline \end{array} = \frac{1020304}{100000000}.$$

$$\begin{array}{r} \cdot 987654321 \\ \hline \end{array} = \frac{987654321}{1000000000} = \frac{987654321}{1000000000}.$$

## EXERCISE 72—Page 192.

(1)

$$\begin{array}{r} \cdot 8325 \\ \hline 83 \\ \hline \end{array}$$

$$\frac{8325}{83} = 100.$$

$$\begin{array}{r} \cdot 147658 \\ \hline 147 \\ \hline \end{array}$$

$$\frac{147658}{147} = 1004.$$

$$\begin{array}{r} \cdot 4320075. \\ \hline 432 \\ \hline \end{array}$$

$$\frac{4320075}{432} = 10000.$$

(2)

$$\begin{array}{r} 875 \cdot 4965 \\ \hline 49 \\ \hline \end{array}$$

$$\frac{8754965}{49} = 178672.$$

$$\begin{array}{r} 301 \cdot 82756 \\ \hline 82 \\ \hline \end{array}$$

$$\frac{30182756}{82} = 368081.$$

(3)

$$\begin{array}{r} \cdot 083 \\ 8 \\ \hline 800 = 1\frac{1}{2} \end{array}$$

$$\begin{array}{r} \cdot 123456 \\ 123 \\ \hline 123456 = 111111 \end{array}$$

$$\begin{array}{r} \cdot 714285 \\ 999999 = 714285 = 107142 = 142857 = 142857 \\ 8734 = 14. \end{array}$$

(4)

$$\begin{array}{r} \cdot 7034 \\ 703 \\ \hline 9880 \\ \cdot 00207 \\ 2 \\ \hline 99000 = 19900 \end{array}$$

$$\begin{array}{r} \cdot 96432 \\ 96 \\ \hline 99932 = 10704 = 3488 = 832 \\ \cdot 143271 \\ 1432 \\ \hline 141838 \\ 990000 \end{array}$$

EXERCISE 73—Page 194.

(1)

Dissimilar.      Similar.      Similar and Coterminous.

$$\begin{array}{lcl} \cdot 9 & = & \cdot 99999 = \cdot 999999999 \\ 6 \cdot 327 & = & 6 \cdot 327272 = 6 \cdot 327272727 \\ 19 \cdot 43 & = & 19 \cdot 43000 = 19 \cdot 430000000 \\ 27 \cdot 0278 & = & 27 \cdot 027878 = 27 \cdot 027878787 \\ \cdot 0347123 & = & \cdot 0347123 = \cdot 0347123123 \end{array}$$

2 carried.

$$\text{Sum,} = 53 \cdot 8198638274$$

(2)

Dissimilar.		Similar.		Similar and Coterminous.
7.427	=	7.42727	=	7.427272727272727
9.1234	=	9.123423	=	9.123423423423423
17.2987643	=	17.2987643	=	17.298764376437643
18.67	=	18.67676	=	18.676767676767676
				2 carried

$$\text{Sum,} = 52.526228203901471$$

(3)

Dissimilar.		Similar.		Similar and Coterminous.
4.95	=	4.959595	=	4.9595959595
7.164	=	7.1641641	=	7.1641641641
4.7123	=	4.7123123	=	4.7123123123
.97317	=	.97317	=	.9731777777
				2 carried.

$$\text{Sum,} = 17.9092502138$$

(4)

Dissimilar.		Similar.		Similar and Coterminous
1.5	=	1.5000	=	1.500000000
99.083	=	99.0830	=	99.083000000
.162	=	.162162	=	.162162162
.814	=	.814814	=	.814814814
2.93	=	2.93939	=	2.939393939
3.769230	=	3.769230769	=	3.769230769
97.26	=	97.2666	=	97.266666666
134.09	=	134.09090	=	134.090909090
				3 carried.

$$\text{Sum,} = 339.626177443$$

Coterminous.

7272727

423423

437643

767676

2 carried

901471

Coterminous.

95

41

23

77

2 carried.

38

Coterminous

00000

00000

62162

14814

93939

30769

66666

09090

3 carried.

77443

## EXERCISE 74—Page 195.

(1)

Dissimilar.		Similar.		Similar and Coterminous.
729·3427	=	729·342742	=	729·342742
93·126	=	93·1260	=	93·126000
				<u>636·216742</u>

(2)

Dissimilar.		Similar.		Similar and Coterminous.
1·437291	=	1·43729137	=	1·4372913729137
·00713	=	·00713	=	·0071313131313
				<u>1·4301300597824</u>

(3)

Dissimilar.		Similar.		Similar and Coterminous
1·12754	=	1·12754	=	1·12754754754754
·47384	=	·473847	=	·47384738473847
				<u>·65370016280907</u>

(4)

Dissimilar.		Similar.		Similar and Coterminous.
42·18763	=	42·1876333	=	42·1876333333
17·0000008432	=	17·0000008432	=	17·0000008432
				<u>25·1876324900</u>



(4)

$$\begin{aligned} \cdot\ddot{45} &= \frac{45}{100} = \frac{9}{20} \text{ and } \cdot\ddot{118881} = \frac{118881}{1000000} = \frac{17}{145} \\ \frac{9}{20} \div \frac{17}{145} &= \frac{9}{20} \times \frac{145}{17} = \frac{9}{4} = 2\frac{1}{4} = 2.25 = 3.8235294117647058. \end{aligned}$$

EXERCISE 77.

(1)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{4}{5} \text{ of } 14 = \frac{1}{2} \times \frac{2}{3} \times \frac{4}{5} \times 14 = \frac{56}{15} = 3\frac{16}{15} = 3.7333333333333333.$$

(2)

$$\begin{aligned} \cdot\dot{67} &= \frac{67}{100} \text{ and } 2\cdot\ddot{13} = \frac{213}{100} = \frac{213}{100} \\ \frac{67}{100} \times \frac{213}{100} &= \frac{14271}{10000} = 1.4271 = 1.4445566778 + \end{aligned}$$

(3)

wk.

$$\cdot 678125 = 4 \text{ days } 17 \text{ hours } 55 \text{ minutes } 30 \text{ seconds.}$$

7

$$4.746875 \text{ days.}$$

24

$$2987500$$

$$1493750$$

$$17.925000 \text{ hours.}$$

60

$$55.500000 \text{ minutes.}$$

60

$$30.000000 \text{ seconds.}$$

(4)

$$.92437$$

92

$$\cdot 92437 = \frac{92437}{100000} = \frac{184874}{200000} = 0.92437$$

(5)

Dissimilar.		Similar.		Similar and Coterminous.
-------------	--	----------	--	--------------------------

$$67\cdot23\dot{4} = 67\cdot234343\dot{4} = 67\cdot234343434\dot{4}$$

$$98\cdot71\dot{3} = 98\cdot7137137\dot{1} = 98\cdot7137137137\dot{1}$$

$$91\cdot0347123\dot{4} = 91\cdot0347123\dot{4} = \underline{91\cdot0347123423\dot{4}}$$

$$\text{Sum,} = 256\cdot9827694903\dot{9}$$

Dissimilar.		Similar and Coterminous.
-------------	--	--------------------------

$$256\cdot9827694903\dot{9} = 256\cdot9827694903\dot{9}$$

$$100\cdot12345678\dot{9} = \underline{100\cdot1234567894\dot{5}}$$

$$\text{Difference} = 156\cdot8593127009\dot{4}$$

(6)

$$12) 9 \text{ in.}$$

$$3) 2\cdot75 \text{ ft.}$$

$$5\frac{1}{2}) 2\cdot916 \text{ yds.}$$

$$\underline{2} \quad \underline{2}$$

$$11) 5\cdot833$$

$$40) 36\cdot5303 \text{ rds.}$$

$$8) 5\cdot913257 \text{ fur.}$$

$$\underline{\cdot739157196 \text{ miles.}}$$

(7)

$$17\cdot428571 \text{ sq. ft.} = 17\frac{3}{7} \text{ sq. ft.} = 17\frac{3}{7} \text{ sq. ft.} = 17 \text{ sq. ft. } 61\frac{1}{7} \text{ in}$$

$$100\cdot8 \text{ sq. in.} = \underline{100\frac{4}{5}}$$

$$\text{Difference,} = 16 \text{ sq. ft. } 104\frac{4}{5} \text{ in.}$$

terminous.

434

371

234

039

inous.

(8)

$$\begin{array}{r} .91789772 \\ 917897 \\ \hline \end{array}$$

$$.91789772 \text{ of } 2 \text{ a.} = \frac{91789772}{100000000} \times \frac{1}{2} \text{ a.} = \frac{91789772}{200000000} = \frac{114737215}{100000000} = 1 \text{ a. } 3 \text{ r. } 13 \text{ per. } 22 \text{ yds.}$$

$$11.287$$

(9)

$$11.287 = \frac{11287}{1000} = 11\frac{287}{1000}. \quad 1.0428571 = \frac{10428571}{10000000} = 1\frac{428571}{1000000} = 1\frac{3}{7}.$$

(10)

$$47.345 = 47\frac{345}{1000} \text{ and } 1.76 = 1\frac{76}{100} = 1\frac{19}{25}. \\ \frac{47345}{1000} \div \frac{176}{100} = \frac{47345}{1000} \times \frac{100}{176} = \frac{47345}{17600} = 26.7837428571.$$

(11)

Dissimilar.                      Similar.      Similar and Coterminous.

$$85.62 = 85.626 = 85.62626$$

$$13.76432 = 13.76432 = 13.76432$$

$$\text{Difference,} = 71.86193$$

(12)

(13)

$$.734 \text{ of a lb.} = 11.744 \text{ oz.} \quad 2 \text{ ft. } 5\frac{1}{2} \text{ in.} = 29\frac{1}{2} \text{ in.} = \frac{59}{2} \text{ in.}$$

$$.198 \text{ of an oz.} = .198 \text{ oz.}$$

$$27.3 \text{ ft.} = 27\frac{3}{10} \text{ ft.} = 328 \text{ in}$$

$$\text{Difference,} = 11.546 \text{ oz.}$$

$$20.16 \text{ ft.} = 20\frac{16}{100} \text{ ft.} = 242 \text{ in.}$$

$$328 \times 242 \div \frac{59}{2} = 378 \times \frac{242}{59} \times \frac{2}{1} = 2706 \text{ in.} = 75\frac{1}{2} \text{ yds.}$$

(14)

$$3.145 = 3\frac{145}{1000} = 3\frac{29}{200} = \frac{173}{100} \text{ and } 4.297 = 4\frac{297}{1000} = 4\frac{1}{3} = \frac{14}{3}.$$

$$\frac{173}{100} \times \frac{14}{3} = \frac{2422}{300} = 13.5169533.$$

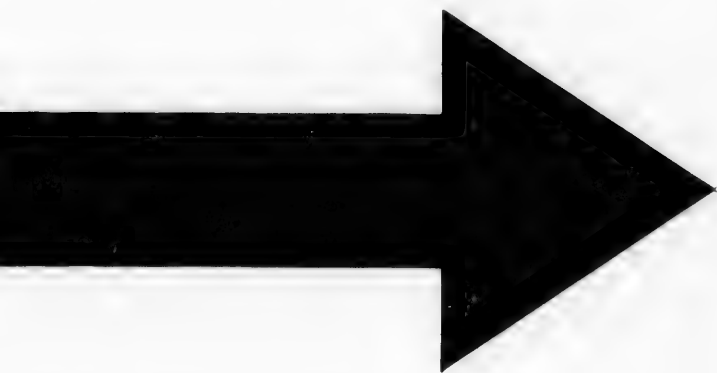
ft. 61 in

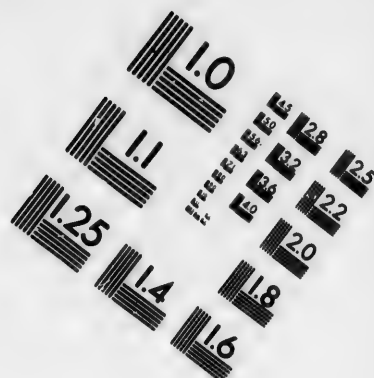
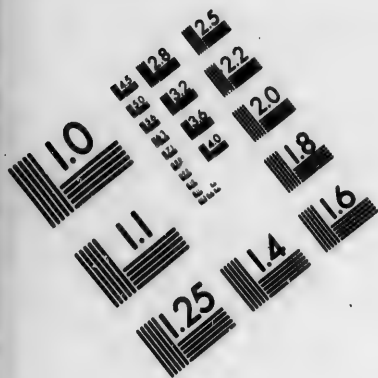
1008

. 1048 in.

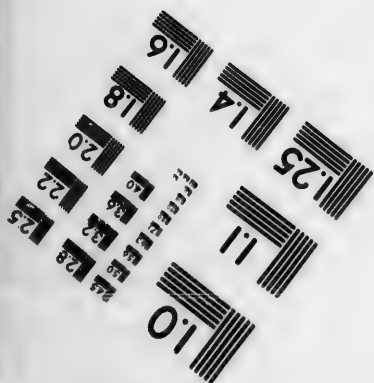
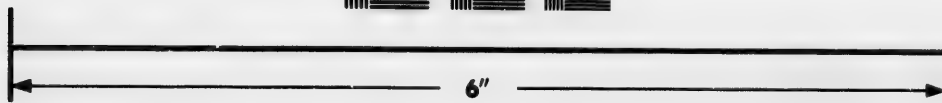
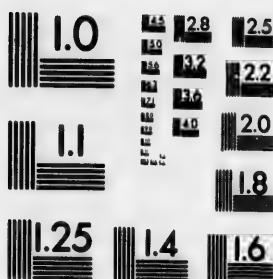








# IMAGE EVALUATION TEST TARGET (MT-3)



# Photographic Sciences Corporation

**23 WEST MAIN STREET  
WEBSTER, N.Y. 14580  
(716) 872-4503**

28 25 22 20 18

10 01

(15)

$\frac{3}{40}$ . Here  $40 = 2^3 \times 5$ . Therefore the equivalent decimal will contain 3 places.

$\frac{3}{40}$	"	$24 = 2^3 \times 3$	"	"	"	"	3	"
$\frac{3}{80}$	"	$15 = 5 \times 3$	"	"	"	"	1	"
$\frac{3}{160}$	"	$144 = 2^4 \times 3^2$	"	"	"	"	4	"
$\frac{3}{320} = \frac{1}{106}$	"	$15 = 5 \times 3$	"	"	"	"	1	"
$\frac{3}{3584}$	"	$3584 = 2^9 \times 7$	"	"	"	"	9	"

(16)

Dissimilar.		Similar		Similar and Coterminous
$81\dot{6}$	=	$81\cdot666$	=	$81\cdot66666666$
$61\cdot12\dot{6}$	=	$61\cdot126$	=	$61\cdot12666666$
$328\cdot2\dot{3}$	=	$328\cdot232\dot{3}$	=	$328\cdot2323232\dot{3}$
$5\cdot624$	=	$5\cdot6246\dot{2}$	=	$5\cdot6246246\dot{2}$
				2 carried

$$\text{Sum,} = 476\cdot65028119$$

(17)

$$\begin{aligned} & \left( \frac{4\cdot4 - 2\cdot83}{1\cdot6 + 2\cdot629} \times \frac{6\cdot8 \times 3}{2\cdot25} \right) + \frac{2\cdot8 \times 2\cdot27}{1\cdot136} \\ &= \left( \frac{1\cdot61}{4\cdot296} \times \frac{20\cdot4}{2\cdot25} \right) + \frac{2\frac{1}{2} \times 2\frac{27}{100}}{1\frac{136}{100}} \\ &= \left( \frac{1\frac{61}{100}}{4\frac{296}{100}} \times \frac{20\frac{4}{10}}{2\frac{25}{10}} \right) + \frac{2\frac{1}{2} \times 2\frac{27}{100}}{1\frac{136}{100}} \\ &= \left( \frac{1\frac{11}{100}}{4\frac{296}{100}} \times \frac{1\frac{22}{10}}{2} \right) + \frac{1\frac{1}{2} \times 1\frac{27}{100}}{1\frac{136}{100}} \\ &= \left( \frac{1\frac{11}{100}}{4\frac{296}{100}} \times \frac{3\frac{1}{2}}{2} \right) + \frac{1\frac{1}{2} \times 1\frac{1}{100}}{1\frac{136}{100}} \\ &= \left( \frac{1}{111} \times \frac{3\frac{1}{2}}{2} \right) + \frac{1\frac{1}{2}}{1\frac{136}{100}} = \left( \frac{111}{111} \times \frac{1\frac{1}{2}}{1\frac{136}{100}} \right) + \frac{2\frac{1}{2}}{1\frac{136}{100}} \\ &= \left( \frac{1}{1} \times \frac{1\frac{1}{2}}{1\frac{136}{100}} \right) + \frac{2\frac{1}{2}}{1\frac{136}{100}} = \frac{1}{1} + \frac{2\frac{1}{2}}{1\frac{136}{100}} = \frac{4\frac{1}{2}}{1\frac{136}{100}} = 9. \end{aligned}$$

ent decimal  
ain 3 places.

3 "  
1 "  
4 "  
1 "  
9 "

eterminous

6668

6668

323

462

2 carried

119

EXERCISE 78—Page 198.

(1)

V	V	V
9)4312131	3)4312131	8)4312131
9)224322..8	3)1234023..2	8)242343..7
9)12043..5	3)224322..2	8)14022..2
9)344..7	3)41240..2	8)1032..1
9)21..0	3)12043..1	8)32..6
1..2	3)2144..1	2..1
	3)344..2	
	3)113..0	
	3)21..0	
	3)8..2	
	1..0	

V	IX	III	VIII
4312131 =	120758 =	10200211222 =	216127
5	9	3	8
23	11	3	17
5	9	3	8
116	99	11	898
5	9	3	3
582	898	33	2695
5	9	3	3
2911	8087	99	8087
5	9	3	3
14558	72791 dec.	299	24263
5			3
72791 decimal.		72791 dec.	

28

= 9.

(3)

$$976.432 \div .00000096 = 97643200000 \div 96 \text{ and } 96 = 12 \times 8.$$

$$12)97643200000$$

$$8)8136933333.3$$

$$1017116666.6$$

(4)

$$(2\frac{1}{2} + .5625 - 1.5 + \frac{1}{16}) \div \frac{1}{11}$$

$$(1\frac{1}{11} \times \frac{1}{2} \times 296 \times \frac{1}{101} \div \frac{1}{8}) \div .9472947 =$$

$$\frac{(2\frac{1}{2} + \frac{9}{16} - 1\frac{1}{2} + \frac{1}{16}) \times \frac{1}{11}}{\frac{1}{8} \times \frac{1}{11}} =$$

$$\frac{(\frac{11}{2} \times \frac{1}{2} \times \frac{296}{101} \times \frac{1}{8}) \div \frac{1}{8}}{\frac{11}{2} \times \frac{1}{2} \times \frac{296}{101} \times \frac{1}{8} \times \frac{1}{8}} =$$

$$\frac{\frac{11}{2}}{\frac{11}{2}} = \frac{\frac{11}{2}}{\frac{1}{11}} = \frac{11}{2} = 2\frac{1}{2}.$$

(5)

lbs. oz. dr. scr. lbs. oz. dr. scr. grs.

9 7 7 2 ) 97 3 4 1 17

12 12

116 1167

8 8

927 9340

3 3

2783 28021

20 20

55660 ) 560437(10,3837.

55660

3837

122

d 96 = 12 × 8.

(6)

15 yds. = 540 in. and 7 ft. = 84 in.

6 ft. = 72 in. and 4 ft. = 48 in.

$(540 \times 84 \times 13) - (72 \times 48 \times 13) = 589680 - 44928 = 544752.$

$544752 \div 108 = 5044.$

(7)

9 ft. 6' 4" 7"									
11	7	9	11						
				8	8	10"	2"	5"	
				7	1	9	5	3	
				5	6	8	8	1	
104	10	2	5						
<hr/>									
111	0	9	7	4	5	5			

(8)

$\frac{47}{13} + \frac{8}{13} - \frac{7}{13} = \frac{474}{13} - \frac{7}{13} = \frac{1157}{13} = \frac{1157}{13} = \frac{42123}{13} = 81555.$   
 $\frac{2}{3} \text{ of } \frac{1}{2} + \frac{1}{6} \text{ of } \frac{1}{3} = \frac{2}{13} + \frac{1}{13} = \frac{389}{13} = \frac{389}{13} = \frac{42123}{13} = 81555.$

(9)

(10)

pts. 77 77 42 27 21 33 14 7 11 63 30  
 2) 782436  
 4) 391218 0 pt.  
 2) 97804 2 qt.  
 4) 48902 0 gal.  
 12225 2 pks.  
 12225 bush. 2 pks. 0 gal. 2 qts.

$77 \times 27 \times 10 = 20790 = 1 \text{ c. m.}$



(11)

XII	IX
28e4)36t87942(1375t·12	3762814
28e4	9
9e47	34
82t0	9
18679	312
17274	9
14054	2810
11888	9
23882	25298
23554	9
32t·0	227683
28e·4	9
5t·80	2049151
55·t8	
4·94	

(12)

$$150528 = 2^{10} \times 3 \times 7^2.$$

$$10+1=11$$

$$1+1=2$$

$$2+1=3$$

$$11 \times 3 \times 2 = 66.$$

(13)

2 wks. 2 dys. = 16 dys.	728½ = 8½ + 2 × 10 + 7 × 10 × 10.
1234625	lbs. oz. dr.
16	27 4 3 × 8½ = 231 11 9½
7407750	10
1234625	272 9 14 × 2 = 545 3 12
1·9754000 dys.	10
24	2726 2 12 × 7 = 19023 3 4
39016000	19860 2 9½
19508000	
23·4096000 hrs.	
60	
24·5760000 min.	
60	
34·5600000 = 34 ½ sec.	

(12—

(15)

$$\begin{aligned} £16 \text{ 3s. 8\frac{1}{2}\text{d.}} &= \$64.74\frac{1}{2} \text{ and } £67 \text{ 17s. 7\frac{1}{2}\text{d.}} = \$271.52\frac{1}{2}. \\ \$98.17 + \$42.29 + \$64.74\frac{1}{2} + \$97.19 + \$127.87\frac{1}{2} &= \$430.27\frac{1}{2} \\ \$430.27\frac{1}{2} - \$271.52\frac{1}{2} &= \$158.75. \end{aligned}$$

(16)

$$\begin{aligned} .8 &= \frac{8}{10} \quad .76 = \frac{76}{100} \quad .9123 = \frac{9123-91}{9900} = \frac{9032}{9900} = \frac{2258}{2475} \\ .003327 &= \frac{3327-3}{999000} = \frac{3324}{999000} = \frac{277}{83250} \end{aligned}$$

(18)

$$\begin{aligned} &[(2\frac{1}{2} \times .5 \text{ of } 1\frac{1}{2}) + 9\frac{1}{2} + .09 + \frac{23}{11}] - 11\frac{1}{7} \div (\frac{1}{2} \text{ of } .16) \\ &[(.7632763 \times 11) \times \frac{1}{2} \text{ of } \frac{10}{11}] \times (\frac{1}{2} \text{ of } .2 \text{ of } .3 \text{ of } .25 \text{ of } 96) \div .2 \end{aligned}$$

$$\frac{1}{2} \text{ of } .6732467 \div \frac{1}{2}$$

$$\begin{aligned} &[(\frac{1}{3} \times \frac{1}{2} \times 1\frac{1}{2}) + 9\frac{1}{2} + 1\frac{1}{7} + \frac{23}{11}] - 11\frac{1}{7} \div (\frac{1}{2} \text{ of } \frac{1}{6}) \\ &(\frac{7632}{11} \times \frac{1}{2} \times \frac{1}{2} \times \frac{10}{11} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{25}{1}) \div \frac{1}{2} \\ &\frac{1}{2} \times \frac{7632}{11} \div \frac{1}{2} \end{aligned}$$

$$\begin{aligned} &[(2 + 9\frac{1}{2} + 1\frac{1}{7} + \frac{23}{11}) - 11\frac{1}{7}] \div (\frac{1}{2} \times \frac{1}{6}) \\ &\frac{7632}{11} \times \frac{1}{2} \times \frac{1}{2} \times \frac{10}{11} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{25}{1} \times \frac{1}{2} \\ &\frac{1}{2} \times \frac{7632}{11} \times \frac{1}{2} \end{aligned}$$

$$\begin{aligned} &\frac{(12 - 11\frac{1}{7}) \div (\frac{1}{2} \times \frac{1}{6})}{\frac{1}{6}} = \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{\frac{1}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}} = \frac{1}{1} = 1 \\ &\frac{11111}{11111} = 3\frac{5062}{100000} \end{aligned}$$

(19)

8 children will have 8 children's shares.

One woman will have 3 children's shares  $\therefore$  6 women will have

$$6 \times 3 = 18 \text{ children's shares.}$$

One man will have 6 children's shares  $\therefore$  4 men will have

$$4 \times 6 = 24 \text{ children's shares.}$$

4 men, 6 women, and 8 child. will therefore have 50 child. shares.

$$£550 \text{ 3s. 1½d. } \div 50 = £11 \text{ 0s. 0½d.} = \text{child's share.}$$

$$£11 \text{ 0s. 0½d. } \times 3 = £33 \text{ 0s. 2½d.} = \text{woman's share.}$$

$$£33 \text{ 0s. 2½d. } \times 2 = £66 \text{ 0s. 4½d.} = \text{man's share.}$$

(20)

$$16\frac{7}{11} + 19\frac{1}{2} + 23\frac{7}{11} + 129\frac{4}{11} = 16 + 19 + 23 + 129 + \\ (\frac{7}{11} + \frac{1}{2} + \frac{7}{11} + \frac{4}{11}) = 187 + 3\frac{13}{11} = 190\frac{3}{11}.$$

(21)

$$8100 = 2^3 \times 3^4 \times 5^2.$$

1..3..9..27..81

1..2..4

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324

1..5..25

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324..

5..15..45..135..405..10..30..90..270..810..20..60..180..

540..1620..25..75..225..675..2025..50..150..450..1350..

4050..100..300..900..2700..8100.

Therefore the divisors of 8100 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2025, 2700, 4050, 8100.

(22)

$$\begin{array}{r} 2691)11817(4 \\ 10764 \\ \hline \end{array}$$

$$\begin{array}{r} 1053)2691(2 \\ 2106 \\ \hline \end{array}$$

$$\begin{array}{r} 585)1053(1 \\ 585 \\ \hline \end{array}$$

$$\begin{array}{r} 468)585(1 \\ 468 \\ \hline \end{array}$$

$$\begin{array}{r} 117)468(4 \\ 468 \\ \hline \end{array}$$

9828 is divisible by 117  $\therefore$  117 is the G. C. M.

365 d., 5 h., 48 m., 48 sec.

(24)

$$\begin{array}{rcl} 14 \text{ ft. } 11 \text{ in.} & = & 179 \text{ in.} \\ 38 \text{ miles} & = & 2407680 \text{ in.} \\ 2407680 \div 179 & = & 13450\frac{1}{2} \end{array}$$

(25)

$$\begin{array}{l} 11 \text{ ft.} \times 13 \text{ ft.} \times 15 \text{ ft.} = 2145 \text{ cub. ft.} \\ \text{One cubic foot weighs } 62\frac{1}{2} \text{ lbs. } 2145 \times 62\frac{1}{2} = 134062\frac{1}{2} = \text{weight} \\ \text{of } 2145 \text{ cub. ft.} \\ \text{One gallon weighs } 10 \text{ lbs. } 134062\frac{1}{2} \div 10 = 13406\frac{1}{2} = \text{gals. in} \\ 134062\frac{1}{2} \text{ lbs.} \end{array}$$

(26)

$$\begin{array}{rcl} £73 \times 400 & = & \$292.00 \\ 17s. \times 20 & = & 3.40 \\ 11\frac{1}{2}d. = 47 \text{ far.} \times 5 \div 12 & = & .19\frac{1}{2} \\ \hline £73 \quad 17s. \quad 11\frac{1}{2}d. & = & \$295.59\frac{1}{2} \end{array}$$

(27)

$$\begin{array}{rcl} 93\frac{1}{2} - 76\frac{1}{2} = 17 \\ 4206 \div 258 = 16\frac{1}{3} = 16\frac{2}{3} \\ 258 \div 17 = 15\frac{1}{3} = 15\frac{2}{3} \\ 16\frac{2}{3} \times 15\frac{2}{3} = 250\frac{2}{3} = 250\frac{4}{6} = 250\frac{2}{3} \\ 250\frac{2}{3} \div 17 = 14\frac{2}{3} = 14\frac{4}{6} = 14\frac{2}{3} \end{array}$$

arcs.

omen will have

men will have

50 child. shares.

ld's share.

man's share.

n's share.

$$3 + 129 +$$

$$90\frac{1}{2} -$$

$$36..108..324$$

$$36..108..324..$$

$$20..60..180..$$

$$450..1350..$$

$$4, 5, 6, 9, 10, 12,$$

$$90, 100, 108, 135,$$

$$675, 810, 900,$$

(28)

$$\frac{5\frac{1}{2} \div \frac{2}{3}}{1\frac{1}{2} \text{ of } \frac{2}{3} + 10\frac{1}{2}} \times \frac{\frac{2}{3} \text{ of } 1\frac{1}{2} \text{ of } 4\frac{1}{2}}{13\frac{1}{2} \text{ of } 5\frac{1}{2}} = \frac{4\frac{1}{2} \times \frac{2}{3}}{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} \times \frac{\frac{2}{3} \times \frac{2}{3}}{1\frac{1}{2} \times 1\frac{1}{2}} =$$

$$\frac{\frac{2}{3} \times \frac{2}{3}}{1\frac{1}{2} \times 1\frac{1}{2}} \times \frac{2}{3} \times \frac{2}{3} =$$

$$\frac{4\frac{1}{2} \times 3 \times 31}{16 \times 2} \times \frac{2}{5} \times \frac{27}{2 \times 2 \times 2 \times 2} = \frac{3 \times 9 \times 31}{16 \times 2 \times 2 \times 2} =$$

$$\frac{41}{16} = 6\frac{5}{16}$$

(29)

$$\begin{array}{r} \text{XI} \\ 5)91342 \\ \hline 5)19074..4 \\ \hline 5)4015..1 \\ \hline 5)891..0 \\ \hline 5)184..3 \\ \hline 5)39..3 \\ \hline 5)8..2 \\ \hline 1..8 \end{array}$$

$$\begin{array}{r} \text{XI} \\ 12)91342 \\ \hline 12)8344..9 \\ \hline 12)773..1 \\ \hline 12)70..3 \\ \hline 6..5 \end{array}$$

$$\begin{array}{r} \text{XI} \\ 2)91342 \\ \hline 2)46176..1 \\ \hline 2)23093..0 \\ \hline 2)11541..1 \\ \hline 2)6246..0 \\ \hline 2)3153..0 \\ \hline 2)1627..0 \\ \hline 2)869..0 \\ \hline 2)434..1 \\ \hline 2)217..1 \\ \hline 2)109..0 \\ \hline 2)54..0 \\ \hline 2)24..1 \\ \hline 2)15..0 \\ \hline 2)8..0 \\ \hline 2)4..0 \\ \hline 2)2..0 \\ \hline 1..0 \end{array}$$

(29 continued.)

$$\frac{\frac{3}{4} \times \frac{3}{4}}{\frac{1}{4} \times \frac{1}{4}} =$$

$$\frac{\times 9 \times 31}{\times 2 \times 2 \times 2} =$$

XI  
 2)91342  
 2)46176..1  
 2)23093..0  
 2)11541..1  
 2)6248..0  
 2)3153..0  
 2)1627..0  
 2)869..0  
 2)434..1  
 2)217..1  
 2)109..0  
 2)54..0  
 2)24..1  
 2)15..0  
 2)8..0  
 2)4..0  
 2)2..0  
 1..0

XI	V	XII	II
91342	13233014	65319	100000100110000101
11	5	12	2
100	8	77	2
11	5	12	2
1103	42	927	4
11	5	12	2
12137	213	11125	8
11	5	12	2
133509 dec.	1068	133509 dec.	16
	5		2
	5340		32
	5		2
	26701		65
	5		2
	133509 dec.		130
			2
			300
			39377
			2
			66754
			3
			133509 dec.

260  
2

521  
2

1043  
2

2086  
2

4172  
2

8344  
2

16688  
2

39377  
2

66754  
3

$$2)7680 = 2^5 \times 3 \times 5$$

$$2)3840$$

$$2)1920$$

$$2)960$$

$$2)480$$

$$2)240$$

$$2)120$$

$$2)60$$

$$2)30$$

$$3)15$$

$$5$$

$$(31) \quad \begin{array}{ccccccc} \text{m.} & \text{f.} & \text{p.} & \text{y.} & \text{ft.} & \text{in.} \\ 72 & 3 & 7 & 2 & 1 & 7 \end{array}$$

$$8$$

$$579 \text{ fur.}$$

$$40$$

$$23167 \text{ per.}$$

$$5\frac{1}{2}$$

$$115837$$

$$11583\frac{1}{2}$$

$$127420\frac{1}{2} \text{ yds.}$$

$$3$$

$$382262\frac{1}{2} \text{ ft.}$$

$$12$$

$$4587157 \text{ in.}$$

$$12$$

$$55045884 \text{ lines}$$

(34)

$$93 \cdot 723 = 93\frac{111}{111} = 93\frac{111}{111} \text{ and } 29 \cdot 4173 = 29\frac{111}{111} = 29\frac{111}{111}$$

$$29\frac{111}{111} \div 29\frac{111}{111} = \frac{92786}{293879} \times \frac{111}{111} = \frac{92786 \times 111}{11 \times 293879} =$$

$$\frac{10399246}{1111111} = 3 \cdot 185988 +$$

(35)

One bushel of oats weighs 34 lbs.  $\therefore$  in 73429 lbs. there are  $73429 \div 34 = 2159\frac{1}{2}$  bushels.

(36)

In 719630 lbs. of wheat there are  $719630 \div 60 = 11993\frac{1}{2}$  bus.

$$\$1 \cdot 80 \times 11993\frac{1}{2} = \$21588 \cdot 90,$$

Or  $\$1 \cdot 80$  per bushel = 3 cents per lb.

$$719630 \times 3 = 2158890 \text{ cents.} = \$21588 \cdot 90.$$

The

The

210; for

for the

sixth, 2

Multi

580

2370, 2

\$11 \times

\$31 \cdot 4

\$1 \cdot 87 +

\$20 \cdot 6

(32)

$$97 = \$45.59.$$

(33)

$$) \div 128 = 25\frac{3}{4}.$$

$$3\frac{1}{2} = \$90.96\frac{1}{4}.$$

(38)

$$\begin{array}{r} 21389)180781(8 \\ 171112 \\ \hline \end{array}$$

(37)

$$\begin{array}{r} 9669)21389(2 \\ 19338 \\ \hline \end{array}$$

$$\$72.14 + \$93.76 = \$165.90$$

$$19338$$

$$\$165.90 \times 9.47 = \$1571.0730$$

$$2051)9669(4$$

$$8204$$

$$\$1571.0730 \div 11 = \$142.8248+$$

$$1465)2051(1$$

$$1465$$

$$586)1465(2$$

$$1172$$

$$293)586(2$$

Last divisor 293 = G. C. M.

(39)

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.$$

$$= 2310.$$

$$\begin{array}{r} \times 111 \\ \hline 93879 \end{array}$$

ps. there are

$$11993\frac{1}{2} \text{ bus.}$$

90.

The least common multiple of 11, 5, 7, 33, 14, 10 and 2 is 2310.

The multiplier for both terms of the first fraction is  $2310 = 210$ ; for the second,  $2310 = 462$ ; for the third,  $2310 = 330$ ; for the fourth,  $2310 = 70$ ; for the fifth,  $2310 = 165$ ; for the sixth,  $2310 = 231$ ; for the seventh,  $2310 = 1155$ .

Multiplying by these numbers, we obtain  $\frac{1}{11}, \frac{1}{5}, \frac{1}{7}, \frac{1}{33}, \frac{1}{14}, \frac{1}{10}, \frac{1}{2}$  for the required fractions.

(40)

$$\begin{array}{l} \$11 \times 17 = \$1.87. \quad \$37\frac{1}{2} \times 19 = \$7.12\frac{1}{2}. \quad \$2.17 \times 14\frac{1}{2} = \\ \$31.46\frac{1}{2}. \quad \$27 \times 67 = \$18.09. \quad \$1.37\frac{1}{2} \times 15 = \$20.62\frac{1}{2}. \\ \$1.87 + \$7.12\frac{1}{2} + \$31.46\frac{1}{2} + \$4.75 + \$11.50 + \$18.09 + \\ \$20.62\frac{1}{2} + \$7.93 = \$103.35\frac{1}{2}. \end{array}$$



## EXERCISE 84—Page 210.

(1)

$$\begin{array}{r} \text{Baskets.} \qquad \qquad \qquad 1 \cdot 22 \\ 11 : 87 :: \$13 \cdot 42 : \frac{\$13 \cdot 42 \times 87}{11} = \$106 \cdot 14. \end{array}$$

(2)

$$\begin{array}{r} \text{Cords.} \qquad \qquad \qquad 19 \\ 28 : 25 :: \$266 : \frac{\$266 \times 25}{28} = \$237 \cdot 50. \end{array}$$

(3)

$$\begin{array}{r} \text{days} \qquad \qquad \qquad 4 \\ \$29 \cdot 20 : \$83 \cdot 60 :: 16 : \frac{16 \times 83 \cdot 60}{7 \cdot 3} = 456\frac{2}{3} \text{ days.} \end{array}$$

(4)

$$\begin{array}{r} \text{Bags.} \qquad \qquad \qquad \cdot 8 \\ 16 : 156 :: \$12 \cdot 80 : \frac{\$12 \cdot 80 \times 156}{16} = \$124 \cdot 80. \end{array}$$

(5)

$$\begin{array}{r} \text{Feet.} \qquad \text{ft.} \qquad 7 \times 112 \\ 5 : 112 :: 7 : \frac{7 \times 112}{5} = 156\frac{4}{5} \text{ ft.} \end{array}$$

(6)

$$\begin{array}{r} \text{Cows.} \quad \text{days} \qquad 9 \\ 35 : 27 :: 99 : \frac{99 \times 27}{5} = 48\frac{3}{5} \text{ days.} \end{array}$$

(7)

$$\begin{array}{r} \text{Acres.} \quad \text{bus.} \quad 9 \times 48 \\ 5 : 48 :: 9 : \frac{9 \times 48}{5} = 86\frac{4}{5} \text{ bush.} \end{array}$$

(8)

$$\begin{array}{r} 11 \\ \text{Perches. days. } 2 \times 803 \\ 73 : 803 :: 2 : \frac{11}{73} = 22 \text{ days.} \end{array}$$

(9)

$$\begin{array}{r} 141 \\ \text{Pails. lbs. } 100 \times 1128 \\ 170 : 1128 :: 100 : \frac{141}{1128} = 640\frac{1}{2} \text{ lbs.} \end{array}$$

(10)

$$\begin{array}{r} .58 \quad 155 \\ 108 : 465 :: \$20.88 : \frac{.58 \times 155}{108} = \$89.90. \end{array}$$

(11)

$$\begin{array}{r} 9 \quad 639 \\ \$ \quad \$ \quad \text{brls. } 72 \times 1278 \\ 16 : 1278 :: 72 : \frac{9 \times 639}{16} = 5751 \text{ barrels.} \end{array}$$

(12)

$$\begin{array}{r} 15 \\ \text{Men. Acres } 165 \times 3 \\ 11 : 3 :: 165 : \frac{15}{11} = 45 \text{ acres.} \end{array}$$

(13)

$$\begin{array}{r} 125 \\ \text{Barrels. loaves } 250 \times 67 \\ 4 : 67 :: 250 : \frac{125}{4} = 416\frac{1}{4} \text{ loaves.} \end{array}$$

(14)

$$\begin{array}{r} 16 \times 88 \\ \text{Bushels. brls. } 16 \times 88 \\ 190 : 38 :: 16 : \frac{16 \times 88}{38} = 3\frac{1}{2} \text{ barrels,} \end{array}$$

(15)

$$\begin{array}{r} \text{Days.} \quad \text{men} \quad 30 \times 12 \\ 15 : 12 :: 90 : \frac{30 \times 12}{15} = 72 \text{ men} \end{array}$$

(16)

$$\begin{array}{r} \text{D'. work. brls.} \quad 2 \times 279 \\ 17 : 279 :: 2 : \frac{2 \times 279}{17} = 32\frac{1}{4} \text{ barrels.} \end{array}$$

(17)

$$\begin{array}{r} \text{Hours.} \quad \text{miles.} \\ 1 : 24 :: 27 : 27 \times 24 = 648 \text{ miles.} \end{array}$$

(18)

$$\begin{array}{r} \text{Cows.} \quad \text{lbs.} \quad 30 \times 23 \\ 7 : 23 :: 30 : \frac{30 \times 23}{7} = 98\frac{1}{7} \text{ lbs.} \end{array}$$

## EXERCISE 85—Page 211.

(1)

$$\frac{1}{16} : \frac{1}{16} :: \$9750 : \frac{375}{1} \times \frac{7}{28} \times \frac{16}{3} = \$42000.$$

(2)

$$\begin{array}{r} \text{Yard.} \quad \text{s.} \quad 5 \quad 1 \quad 8 \quad \text{s.} \\ \frac{1}{2} : \frac{1}{4} :: \frac{5}{8} \times \frac{1}{4} \times \frac{8}{7} = \frac{5}{7} = 2\frac{1}{2} \text{ d.} \end{array}$$

(3)

$$\begin{array}{r} \text{Tons.} \\ \frac{1}{3} : 8\frac{1}{2} :: \$7.49 : \frac{\$7.49 \times 8\frac{1}{2}}{\frac{1}{3}} = \frac{1.07}{1} \times \frac{25}{8} \times \frac{3}{7} = \$80.25. \end{array}$$

(4)

$$\begin{array}{r} \text{Yards.} \\ 5\frac{1}{2} : \frac{1}{4} :: \$28.42 : \frac{.14}{1} \times \frac{4}{7} \times \frac{5}{28} = \$2.80. \end{array}$$

(5)

$$\text{Dollar. bag } \frac{4}{12} : \frac{7}{20} :: \frac{4}{5} : \frac{7}{20} \times \frac{5}{12} = \frac{7}{12} \text{ of a bag.}$$

(6)

$$\begin{array}{r} \$ \\ 100 : 472\frac{1}{2} :: 98\frac{1}{2} : \frac{98\frac{1}{2} \times 472\frac{1}{2}}{100} = \frac{98.875 \times 472.44}{100} = \$467.12\frac{1}{2}. \end{array}$$

(7)

$$\begin{array}{r} \text{Tons. days. } 107\frac{3}{4} \times 11\frac{1}{4} : 107\frac{3}{4} :: \frac{107\frac{3}{4} \times 11\frac{1}{4}}{17\frac{1}{2}} = \frac{1180}{11} \times \frac{198}{17} \times \frac{5}{88} = 70\frac{1}{4} \text{ dys.} \end{array}$$

(8)

$$\begin{array}{r} \text{Tons. cords. } 22\frac{1}{2} \times 11\frac{2}{3} : 22\frac{1}{2} :: \frac{22\frac{1}{2} \times 11\frac{2}{3}}{15\frac{1}{2}} = \frac{202}{9} \times \frac{295}{28} \times \frac{12}{202} = 16\frac{1}{2} \text{ cords.} \end{array}$$

(9)

$$\begin{array}{r} \text{Yds. yds. } \frac{1}{2} \text{ of } \frac{3}{4} \text{ of } 3\frac{1}{2} : \frac{3}{4} \text{ of } \frac{1}{2} \text{ of } 4\frac{1}{2} :: \frac{3}{4} \text{ of } \frac{1}{2} \text{ of } 4\frac{1}{2} : \frac{1}{2} \text{ of } \frac{3}{4} \text{ of } 3\frac{1}{2} = \end{array}$$

$$\frac{4}{11} \times \frac{15}{224} = \$1\frac{1}{4}.$$

EXERCISE 86—Page 212.

(1)

$$37 \text{ sq. yds. 4 ft. 120 in.} = 48648 \text{ in., and } 9 \text{ sq. yds. 2 ft.} = 11952 \text{ in.}$$

$$\begin{array}{r} \text{Inches. } 11952 : 48648 :: \$3.50 : \frac{3.50 \times 48648}{11952} = \$14.245 + \end{array}$$

(2)

12 lbs. 10 oz. = 154 oz.

Ounces.

$$1 : 154 :: \$1.25 : 1.25 \times 154 = \$192.50.$$

(3)

10 yds. = 40 qrs., and 3 yds. 2 qrs. = 14 qrs.

.17 7

Quarters.

\$ .40 \times 14

$$40 : 14 :: \$3.40 : \frac{\$3.40 \times 14}{40} = \$1.19.$$

40

40

(4)

15 oz. 12 dwt. 16 grs. = 7504 grs., and 13 oz. 14 grs. = 6254 grs.

.95 3127

Grains.

\$ .80 \times 7504

$$7504 : 6254 :: \$3.80 : \frac{\$3.80 \times 7504}{6254} = \$3.67 +$$

7504

1876

938

(5)

3 lbs. 1 oz. 11 dwt. = 751 dwt. and 12 lbs. 6 oz. 4 dwt. = 3004 dwt.

150

Dwt.

\$ .80 \times 751

$$3004 : 751 :: 600 : \frac{600 \times 751}{3004} = \$150.$$

8004

4

(6)

Barrels. h. m. s. 2 h. 46 m. 39 s. \times 24

$$54 : 24 :: 2\ 46\ 30 : \frac{2\ 46\ 30 \times 24}{54} = 1\ \text{hr.}\ 14\ \text{min.}$$

54

9

(7)

73 yds. 3 qrs. 2 na. 1 in = 2660½ in. 3 Fl. e. 2 qrs. 1 na. = 101½ in.  
And £4 17s. 8½d. = 1172½d.

$$\begin{array}{l} \text{Inches.} \quad d. \quad 1172\frac{1}{2} \times 2660\frac{1}{2} \quad \begin{array}{c} 521 \\ 4889 \end{array} \quad 5321 \quad \begin{array}{c} 4 \\ 405 \\ 45 \end{array} \\ 101\frac{1}{2} : 2660\frac{1}{2} :: 1172\frac{1}{2} : \frac{1172\frac{1}{2}}{101\frac{1}{2}} = \frac{4889}{4} \times \frac{5321}{2} \times \frac{4}{405} = \end{array}$$

$$277241 \text{ d.} = £128 \text{ 6s. } 10\frac{1}{2}\text{d.}$$

(8)

$$\begin{array}{l} 8\frac{1}{2} \text{ lbs.} = 136\frac{1}{2} \text{ oz.} \\ \text{Ounces.} \quad s. \quad \begin{array}{c} 7 \quad 205 \quad 3 \\ 287 \quad 410 \quad 9 \end{array} \\ 4\frac{1}{2} : 136\frac{1}{2} :: 8\frac{1}{2} : \frac{287}{82} \times \frac{410}{8} \times \frac{9}{41} = \frac{4784}{15} = £13 \text{ 9s. } 0\frac{1}{2}\text{d.} \end{array}$$

6254 grs.

(9)

$$\begin{array}{l} \text{Pages.} \quad \begin{array}{c} 52 \\ 158 \times 400 \end{array} \\ 327 : 400 :: 156 : \frac{827}{109} = 190\frac{20}{109}, \text{ i. e. on the 191st p.} \end{array}$$

(10)

46 a., 3 r., 14 p. = 7494 p., and 35 a., 2 r., 10 p. = 5690 p.

$$\begin{array}{l} \text{Perches.} \quad £ \quad \begin{array}{c} 50 \\ 100 \times 5690 \end{array} \\ 7494 : 5690 :: 100 : \frac{5494}{3747} = £75 \text{ 18s. } 6\frac{1}{2}\text{d.} \end{array}$$

(11)

$$\begin{array}{l} \text{Days.} \quad \text{miles.} \quad \begin{array}{c} 17 \\ 12 \times 88 \end{array} \\ 48 : 68 :: 12 : \frac{48}{4} = 17 \text{ miles per day.} \end{array}$$

min.

(12)

$$\begin{array}{r} \text{Shillings. lbs.} \quad 113 \quad 113 \\ 21\frac{1}{2} : 32\frac{1}{2} :: 16\frac{1}{2} : \frac{7}{7} \times \frac{238}{7} \times \frac{3}{84} = \frac{11307}{1000} = 24\frac{875}{1000} \text{ lbs} \end{array}$$

(13)

$$\begin{array}{l} 17493 \times 1000 \times 5 \text{ cub. ft.} = 87465000 \text{ cub. ft.} \\ 192724 \times 1000 \times 4 \text{ cub. ft.} = 770896000 \text{ cub. ft.} \\ 87465000 + 770896000 = 858361000 \text{ cub. ft.} \\ \text{Cubic feet.} \quad \text{ton. } 858361000 \\ 9000 : 858361000 :: 1 : \frac{9000}{858361000} = 95373\frac{1}{2} \text{ tons.} \end{array}$$

(14)

$$\begin{array}{l} 50000 \times 9000 = 450000000 = \text{cub. ft. of gas in 50000 tons of coal} \\ \text{Cubic feet.} \quad \text{hour.} \\ 4 : 450000000 :: 1 : \frac{450000000}{4} = 112500000 \text{ h.} = 12842 \text{ y. } 170 \text{ d.} \end{array}$$

(15)

$$\begin{array}{l} \text{lbs. lbs. lbs. lb. lb.} \\ 4 + 3 + 2 + 1 + \frac{1}{2} = 10\frac{1}{2} \text{ lbs.} \\ \text{lbs.} \quad 11270 \\ 10\frac{1}{2} : 11270 :: 1 : \frac{11270}{10\frac{1}{2}} = 1073, \text{ and } 3\frac{1}{2} \text{ lbs. remaining.} \end{array}$$

(16)

$$\begin{array}{l} 180 \text{ miles} = 180 \times 1760 = 316800 \text{ yards.} \\ \text{Yards.} \quad \text{day.} \\ 100 : 316800 :: 1 : \frac{316800 \times 1}{100} = 3168 \text{ dys. or about } 8\frac{2}{3} \text{ yrs.} \end{array}$$

EXERCISE 87—Page 216.

(1)

22½ lbs

$$\left. \begin{array}{l} 120 : 90 \text{ bush.} \\ 6 : 14 \text{ horses.} \end{array} \right\} :: 56 \text{ days} : \frac{7 \times 15}{56 \times 90 \times 14} = 7 \times 14 = 98 \text{ days.}$$

(2)

$$\left. \begin{array}{l} 28 : 32 \text{ ft. high.} \\ 8 : 15 \text{ days.} \end{array} \right\} :: 63 \text{ men} : \frac{9 \times 4}{8 \times 32 \times 15} = 9 \times 15 = 135 \text{ men.}$$

(3)

s of coal

$$\left. \begin{array}{l} 3 : 45 \text{ length.} \\ 1\frac{1}{4} : 1 \text{ width.} \end{array} \right\} :: 1 \text{ lb.} : \frac{45}{3 \times 1\frac{1}{4}} = \frac{45}{\frac{15}{4}} = \frac{3}{15} \times 4 = 3 \times 4 = 12 \text{ lbs.}$$

y. 170 d.

(4)

$$\left. \begin{array}{l} 10 : 100 \text{ length.} \\ 1\frac{1}{4} : 1\frac{1}{4} \text{ width.} \end{array} \right\} :: 3 \text{ lbs.} : \frac{3 \times 1\frac{1}{4} \times 100}{1\frac{1}{4} \times 10} = 2 \times 1\frac{1}{4} \times 10 = 25 \text{ lbs.}$$

(5)

$$\left. \begin{array}{l} 44 : 132 \text{ tons.} \\ 18 : 5 \text{ days.} \end{array} \right\} :: 12 \text{ horses} : \frac{2 \times 8}{12 \times 5 \times 18} = 2 \times 5 = 10 \text{ horses.}$$

(6)

3 yrs.

$$\left. \begin{array}{l} 4 : 14 \text{ men.} \\ 7 : 10 \text{ days.} \end{array} \right\} :: 27\text{s.} : \frac{2 \times 5}{27 \times 14 \times 10} = 27 \times 5 = 135\text{s.} = £6 \text{ } 15\text{s.}$$



(7)

$$\left. \begin{array}{l} 3:5 \text{ masters.} \\ 8:10 \text{ apprentices.} \\ 5:8 \text{ weeks} \\ 6:5\frac{1}{2} \text{ days per wk.} \end{array} \right\} :: \$144 : \frac{\overset{8}{144} \times \overset{24}{5\frac{1}{2}} \times 8 \times 10 \times 5}{8 \times 8 \times 5 \times 8} = \$440.$$

(8)

$$\left. \begin{array}{l} 6:18 \text{ s.mak.} \\ 4:5 \text{ weeks.} \end{array} \right\} :: 36 \text{ pairs of men's shoes : } \frac{\overset{9}{36} \times \overset{3}{18} \times 5}{8 \times 4} =$$

135 pairs men's and the women's =  $\frac{2}{3} = \frac{1}{3}$  of 135 = 90 pairs.

(9)

$$\left. \begin{array}{l} 9:18 \text{ feet high.} \\ 4:6 \text{ days.} \end{array} \right\} :: 12 \text{ men : } \frac{\overset{3}{12} \times \overset{2}{18} \times 6}{2 \times 4} = 3 \times 2 \times 6 = 36 \text{ men.}$$

(10)

$$\left. \begin{array}{l} 130:390 \text{ miles.} \\ 7:14 \text{ hours.} \end{array} \right\} :: 3 \text{ days : } \frac{\overset{2}{3} \times \overset{3}{14} \times \overset{3}{390}}{180 \times 7} = 3 \times 2 \times 3 = 18 \text{ days.}$$

(11)

$$\left. \begin{array}{l} 10:60 \text{ oz.} \\ 22\frac{1}{2}:30 \text{ d.} \end{array} \right\} :: 1 \text{ d. : } \frac{60 \times 30}{10 \times 22\frac{1}{2}} = \frac{\overset{4}{60} \times \overset{8}{30}}{1 \times 1} \times \frac{1}{10} \times \frac{2}{45} = 4 \times 2 = 8 \text{d.}$$

(12)

$$\left. \begin{array}{l} 10:5 \text{ composers} \\ 7:14 \text{ hours.} \\ 20:40 \text{ sheets.} \\ 24:16 \text{ pages.} \\ 50:60 \text{ lines.} \\ 40:50 \text{ letters.} \end{array} \right\} :: 16 \text{ days : } \frac{\overset{2}{16} \times \overset{2}{5} \times \overset{2}{14} \times \overset{8}{40} \times 16 \times \overset{8}{60} \times \overset{8}{50}}{10 \times 7 \times 20 \times 24 \times 50 \times 40} =$$

$2 \times 16 = 32 \text{ days.}$

386 : 240 men.

5 : 9 days.

10 : 12 hours.

6 : 5 degrees.

5 : 3 yards wide

3 : 2 yards deep

(13)

:: 70 yards :

$$\frac{7 \quad 5 \quad 2}{70 \times 240 \times 9 \times 12 \times 5 \times 3 \times 2} = \frac{336 \times 5 \times 10 \times 4 \times 5 \times 3}{48}$$

$$9 \times 2 \times 2 = 36 \text{ yards.}$$

(14)

6 : 12 horses.

4 : 9 months.

:: 16 acres :

$$\frac{4 \quad 2}{16 \times 12 \times 9} = 4 \times 2 \times 9 = 72 \text{ acres.}$$

(15)

25 : 139 persons

1 : 7 years.

:: 300 bush. :

$$\frac{12}{2500 \times 139 \times 7} = 11676 \text{ bushels.}$$

(16)

48 : 32 men.

36 : 864 feet long.

8 : 5 feet high.

4 : 3 feet wide.

:: 4 days :

$$\frac{3 \quad 2 \quad 108}{4 \times 864 \times 864 \times 5 \times 3} = 30 \text{ days.}$$

(17)

679 : 22407 sold's.

336 : 112 days.

:: 702 bushels :

$$\frac{234 \quad 33}{702 \times 22407 \times 112} = \frac{679 \times 336}{8}$$

$$234 \times 33 = 7722 \text{ bushels.}$$

(18)

13 : 494 suits.

19 : 27 days.

:: 12 tailors :

$$\frac{2 \quad 25}{12 \times 494 \times 27} = 648 \text{ tailors.}$$

H

(19)

$$\begin{array}{l} 17:40 \text{ head of cattle} \\ 30:51 \text{ days.} \end{array} \left. \vphantom{\begin{array}{l} 17:40 \\ 30:51 \end{array}} \right\} :: 5 \text{ a. } 2 \text{ r. } 10 \text{ p.} : \frac{5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 40 \times 51}{17 \times 50} =$$

$$5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 4 = 23 \text{ a. } 1 \text{ r.}$$

(20)

$$\begin{array}{l} 20 : 100 \text{ ft. long} \\ 6 : 4 \text{ feet wide.} \end{array} \left. \vphantom{\begin{array}{l} 20 \\ 6 \end{array}} \right\} :: 180 \text{ bricks} : \frac{30 \times 5 \times 180 \times 4}{20 \times 6} =$$

$$30 \times 5 \times 4 = 600 \text{ bricks.}$$

## EXERCISE 88.—Page 21

(1)

$$\begin{array}{ll} .7 \text{ cords} & = 116 \text{ lbs.} \\ 87 \text{ lbs.} & = 23 \text{ barrels} \\ 19 \text{ barrels} & = 34 \text{ days' work} \\ 92 \text{ days' work} & = 57 \text{ baskets peaches} \\ 31 \text{ baskets peaches} & = 24 \text{ dollars} \\ 12 \text{ dollars} & = 2 \text{ tons} \\ 35 \text{ tons} & = x \text{ cords} \end{array} \left. \vphantom{\begin{array}{l} .7 \\ 87 \\ 19 \\ 92 \\ 31 \\ 12 \\ 35 \end{array}} \right\} =$$

$$\frac{17 \times 87 \times 19 \times 92 \times 31 \times 12 \times 35}{116 \times 23 \times 34 \times 57 \times 24 \times 2} = \frac{31 \times 35}{2 \times 2 \times 2} = \frac{1085}{8} = 135\frac{5}{8}$$

(2)

$$\begin{array}{ll} 6 \text{ lbs. tea} & = 29 \text{ lbs. sugar} \\ 17 \text{ lbs. sugar} & = 1 \text{ bushel} \\ 27 \text{ bushels} & = 4 \text{ tons} \\ 34 \text{ tons} & = 15 \text{ cows} \\ 29 \text{ cows} & = 1160 \text{ dollars} \\ 20 \text{ dollars} & = x \text{ lbs. tea.} \end{array} \left. \vphantom{\begin{array}{l} 6 \\ 17 \\ 27 \\ 34 \\ 29 \\ 20 \end{array}} \right\} =$$

$$\frac{6 \times 17 \times 27 \times 34 \times 29 \times 20}{29 \times 1 \times 4 \times 15 \times 1160} = \frac{17 \times 17 \times 27}{5 \times 58} = \frac{7803}{290} = 26\frac{263}{290}$$

11x1

21

7

29x7x

16x17

4

7 A =

5 B =

15 C =

11 D =

42 E =

$$\begin{array}{r} 4 \quad 8 \\ p. \times 40 \times 51 \\ \hline \end{array}$$

$$\begin{array}{r} 89 \\ 19 \\ \hline \end{array}$$

(3)

11 bush. barley	=	21 bush. potatoes	}	=
19 " potatoes	=	29 " oats		
115 " oats	=	44 " wheat		
14½ " wheat	=	38 " peas		
60 " peas	=	55 " rye		
75 " rye	=	11½ " clover sd.		
36 " clover sd.	=	x " barley		

$$\begin{array}{r} 2 \quad 5 \\ 28 \quad 20 \quad 18 \\ 11 \times 19 \times 115 \times 14 \frac{1}{2} \times 60 \times 75 \times 36 \\ \hline 21 \times 29 \times 44 \times 38 \times 55 \times 11 \frac{1}{2} \\ 7 \quad 2 \quad 11 \quad 2 \quad 5 \end{array} = \frac{5 \times 75 \times 18}{7 \times 11} = \frac{2142}{77} = 879 \frac{1}{2}$$

(4)

16 baskets pears	=	29 turkeys	}	=
17 turkeys	=	7 days' work		
7½ days' work	=	187 loaves		
3½ loaves	=	4 lbs. veal		
1 lb. veal	=	11 cents		
792 cents	=	63 lbs. sugar		
x lbs. sugar	=	21 baskets pears		

$$\begin{array}{r} 4 \quad 11 \quad 2 \\ 28 \times 7 \times 187 \times 4 \times 11 \times 63 \times 21 \\ \hline 16 \times 17 \times 7 \frac{1}{2} \times 3 \frac{1}{2} \times 1 \times 792 \\ 4 \quad 2 \quad 2 \quad 2 \end{array} = \frac{11 \times 7 \times 21}{4} = \frac{1617}{4} = 404 \frac{1}{4}$$

(5)

$$\left. \begin{array}{l} 7 A = 11 B \\ 5 B = 8 C \\ 15 C = 21 D \\ 11 D = 5 E \\ 42 E = x A \end{array} \right\} = \frac{7 \times 5 \times 15 \times 11 \times 42}{11 \times 8 \times 21 \times 5} = \frac{7 \times 15}{4} = \frac{105}{4} = 26 \frac{1}{4}$$

$$= 26 \frac{1}{4}$$

(6)

7 barrels flour = 23 cords  
 6 cords = 11 cwt.  
 46 cwt. = £28  
 £77 = 9 sheep  
 8 sheep = 8 tons  
 9 tons = x barrels flour

$$\begin{array}{r}
 \begin{array}{r} 3 \quad 2 \quad 7 \\ 7 \times 8 \times 16 \times 77 \times 5 \times 9 \end{array} \\
 \hline
 \begin{array}{r} 28 \times 11 \times 23 \times 8 \times 8 \end{array}
 \end{array}
 = \frac{3 \times 7 \times 5}{8} = \frac{105}{8} = 13\frac{1}{8}$$

(7)

15 N. England = 20 New York  
 24 New York = 22½ N. Jersey  
 30 New Jersey = 20 Canada  
 4807½ Canada = x N. England

$$\begin{array}{r}
 \begin{array}{r} 2 \quad 6 \quad 8 \quad 961\frac{1}{2} \\ 15 \times 24 \times 30 \times 4807\frac{1}{2} \end{array} \\
 \hline
 \begin{array}{r} 20 \times 22\frac{1}{2} \times 20 \end{array}
 \end{array}
 = 961\frac{1}{2} \times 6 = 5769 \text{ s.} = £288 \text{ 9s.}$$

## EXERCISE 89.—Page 222.

(1)

$$\frac{7}{8} \times \frac{17}{11} \times \frac{23}{29} \times \frac{\frac{11}{319}}{\frac{119}{17}} \times \frac{\frac{2}{18}}{\frac{69}{3}} = \frac{2}{3} = 2 : 3.$$

(2)

$$\begin{array}{rcl}
 £119 \times 400 & = & \$476.00 \\
 16s. \times 20 & = & 3.20 \\
 6\frac{1}{2}d. = 26 \text{ far.} \times 5 \div 12 & = & .10\frac{1}{2} \\
 \hline
 £119 \text{ 16s. 6}\frac{1}{2}d. & = & \$479.30\frac{1}{2}
 \end{array}$$

\* To reduce separately.

(4)

$$\left. \begin{array}{l} 9 : 13 = 9 \div 13 = .692 \\ 21 : 27 = 21 \div 27 = .777 \\ 7 : 10 = 7 \div 10 = .7 \\ 11 : 15 = 11 \div 15 = .733 \end{array} \right\}$$

Hence 21 : 27 is the greatest,  
and 9 : 13 the least.

(5)

Dissimilar.

Similar.

Similar and Coterminous.

$$76 \cdot 23478 = 76 \cdot 234784 = 76 \cdot 234784784784784$$

$$19 \cdot 1342291 = 19 \cdot 1342291 = \underline{19 \cdot 134229122912291}$$

$$\text{Difference,} = 57 \cdot 100555661872493$$

(6)

71324t undenary = 1146287 denary, 23421 quinary = 1736 denary, and 4e7 duodenary = 17995 denary.

$1146287 \times 1736 = 1989954232 \div 17995 = 110583\frac{13447}{17995}$   
 110583 $\frac{13447}{17995}$  denary\* = 53ee3 $\frac{7437}{17995}$  duodenary, 12014313 $\frac{410043}{17995}$  quinary, and 76070 $\frac{2272}{17995}$  undenary.

(7)

$$\left. \begin{array}{l} 5 \cdot 63 : 7 \cdot 9 \text{ cubic inches.} \\ 1 : 1 \cdot 220 \text{ spec. grav.} \end{array} \right\} \begin{array}{l} \text{oz.} \\ \therefore 3 \cdot 254 : \end{array} \frac{3 \cdot 254 \times 7 \cdot 9 \times 1 \cdot 220}{5 \cdot 63} =$$

$$\frac{31 \cdot 362052}{5 \cdot 63} = 5 \cdot 57052 \text{ oz. Ans.}$$

\* To reduce the fractional part, reduce both numerator and denominator separately.

$$\begin{array}{r} \text{yds. qrs. na. in.} \quad \text{yds. qrs. na. in.} \\ 17)63 \quad 3 \quad 2 \quad 1 \quad (3 \quad 3 \quad 0 \quad 0\frac{1}{2} \end{array} \quad (9)$$

51

—

12

4

—

51

51

—

0

4

—

2

2\frac{1}{2}

—

$$5\frac{1}{2} = \frac{1}{2} \div 17 = \frac{1}{34}$$

$$\cdot 913625 \text{ of an acre} = \cdot 913625 \times 4840 =$$

$$4421 \cdot 945 \text{ sq. yds.}$$

$$4421 \cdot 945 \times \$ .67 = \$ 2962 \cdot 70 +$$

(10)

$$\frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{7}{8} \text{ of } 20 \text{ bush. } \times \cdot 5 \times \cdot 6 \times \frac{7}{8} =$$

$$\frac{1}{2} \times \frac{3}{4} \times \frac{7}{8} \times 20 \times \frac{1}{2} \times \frac{3}{4} \times \frac{7}{8} =$$

$$\frac{3}{32} \text{ bush.} = 1 \text{ bush. } 2 \text{ pks. } 0 \text{ gal. } 1 \text{ qt.}$$

(12)

$$\text{Whole amount of increase} = 2571437 - 1842265 = 729172.$$

$$1842265 : 100 :: 729172 : \frac{729172 \times 100}{1842265} = 39 \text{ per cent.}$$

(13)

$$\frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{1}{2} - \frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{1}{4} = \frac{6}{16} - \frac{3}{16} = \frac{3}{16}$$

(14)

$$100 : 7 :: 11 : \frac{11 \times 7}{100} = \frac{77}{100} \quad 11 - \frac{77}{100} = 10\frac{23}{100}$$

(15)

$$79 \times 16 \times £ \cdot 00163 = £ 2 \cdot 06032 = £ 2 \text{ ls. } 2\frac{3}{4} \text{ d.}$$

(16)

$$\begin{array}{l} 4:3 \text{ men} \\ 10:12 \text{ hours} \\ 20:35 \text{ acres} \end{array} \left. \vphantom{\begin{array}{l} 4:3 \text{ men} \\ 10:12 \text{ hours} \\ 20:35 \text{ acres} \end{array}} \right\} :: 2\frac{1}{2} \text{ days} : \frac{2\frac{1}{2} \times 3 \times 12 \times 35}{4 \times 10 \times 20} = \frac{48}{4} = 12 \text{ days.}$$

2 roods 7

(17)

$$(\frac{1}{2} \text{ of } \frac{1}{2} \times .02 \times .456) \div (\frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 51) =$$

$$\begin{array}{r} 2 \quad 8 \quad 38 \\ 4 \quad 9 \quad 1 \quad 152 \quad 17 \quad 8 \quad 8 \quad 1 \\ 5 \quad 11 \quad 50 \quad 888 \quad 18 \quad 2 \quad 1 \quad 51 \\ \quad \quad 25 \quad 111 \quad 8 \quad \quad \quad 8 \\ \quad \quad \quad 37 \end{array} \quad \begin{array}{l} 2 \times 38 \\ 5 \times 11 \times 25 \times 37 \end{array} = \frac{76}{50875}$$

(18)

$$\frac{2}{1} \times \frac{4}{7} \times \frac{13}{5} \times \frac{7}{2} \times \frac{5}{2} = 4 \times 13 = 52.$$

(19)

$$\begin{array}{l} 50 \text{ barrels} = 125 \text{ yards} \\ 80 \text{ yards} = 6 \text{ bales,} \\ 13 \text{ bales} = 3\frac{1}{2} \text{ hogsheads} \\ x \text{ hogsheads} = 1000 \text{ barrels} \end{array} =$$

$$\begin{array}{r} 5 \quad 3 \quad 125 \\ 125 \times 6 \times 3\frac{1}{2} \times 1000 \\ \hline 50 \times 40 \times 13 \\ \quad 2 \quad 18 \\ \quad \quad 2 \end{array} = \frac{125 \times 3 \times 3\frac{1}{2}}{2 \times 13} = 50\frac{1}{2}.$$

(20)

$$\begin{array}{r} 73 \cdot 47 \times .0063 \div 17 \cdot 2345 = \frac{7347}{100} \times \frac{63}{10000} \div \frac{57391}{3330} = \\ \frac{7347}{100} \times \frac{63}{10000} \times \frac{3330}{57391} = \frac{161133713}{8739100000} = .026856599989+ \end{array}$$

(21)

$$2 \text{ roods } 7 \text{ per. } 4 \text{ yds. } 3 \text{ ft. } 117 \text{ in.} = 3416481 \text{ in. and } 7 \text{ acres} = 43908480 \text{ inches.}$$

$$3416481 \div 43908480 = .0778+$$



(22)

 $\frac{7}{8}$  of  $\frac{1}{2}$  of  $\frac{1}{3}$  of 70 miles =  $\frac{1}{3}$  miles = 5.33333+ miles.

 $\cdot 73$  of 11 fur. = 8.03 fur. = 1.00375 mile.

 $5.33333 - 1.00375 = 4.32958$  miles.

(23)

274312 nonary = 167195 denary, 1101011010 = 858 denary, and  
 .5555 septenary = 2000 denary.

 $167195 - 858 = 166337 \times 2000 = 332674000.$ 
 $332674000$  denary = 764876837 nonary.

 $= 10011110101000011001111010000$  binary,

 $= 11146453021$  septenary.

(24)

$$\begin{array}{r|rrrrrr} 275 & 44..275..18..190..200..225 \\ 38 & 4 & 18..38..19..9 \\ 18 & 2 & 9 & 9 \\ \hline 275 \times 38 \times 18 & = 188100 = 1. \text{ c. m.} \end{array}$$

(25)

10:6 weeks	}	
6:5 days		
11:10 hours		
2400:8742 feet long		
18:20 feet wide		
11:8 feet high		

$$\begin{array}{l} \text{men } 5 \times 5 \times 5 \times 10 \times 2914 \times 2 \\ \therefore 60: \frac{10 \times 5 \times 11 \times 2400 \times 18 \times 11}{240 \times 8} = \\ \frac{5 \times 2914 \times 2}{11 \times 3 \times 11} = \frac{29140}{33} = 883.33 \end{array}$$

(26)

$172000 = 2^5 \times 5^3 \times 43$ . Increasing each exponent by 1 and multiplying them together we obtain  $6 \times 4 \times 2 = 48$ .

33+miles.  
mile.  
es.

358 denary, and

74000.

1010000 binary,

$$\begin{array}{r} 2914 \quad 2 \\ 8742 \times 20 \times 8 \\ \hline 0 \times 18 \times 11 \\ 0 \quad 8 \\ 2 \\ 3 \end{array} =$$

onent by 1 and  
 $\times 2 = 48$ .

$$42 \cdot 7 = 42\frac{7}{10} = 42\frac{35}{100} \text{ and } 9 \cdot 7123 = 9\frac{7123}{10000} = 9\frac{1188}{10000} = 9\frac{16171}{100000}.$$

$$42\frac{35}{100} \times 9\frac{16171}{100000} = 692583\frac{35}{100000} = 415 \cdot 471137804.$$

(27)

$$100 : 27 :: \$73 \cdot 42 : \frac{73 \cdot 42 \times 27}{100} = \$19 \cdot 8234.$$

$$\$73 \cdot 42 - \$19 \cdot 8234 = \$53 \cdot 5966.$$

(28)

$$6300 = 2^2 \times 3^2 \times 5^2 \times 7.$$

$$\begin{array}{l} 1..5..25 \\ 1..2..4 \\ 1..5..25..2..10..50..4..20..100 \\ 1..3..9 \\ 1..5..25..2..10..50..4..20..100..3..15..75..6..30..150.. \\ 12..60..300..9..45..225..18..90..450..36..180..900 \\ 1..7 \\ 1..5..25..2..10..50..4..20..100..3..15..75..6..30..150.. \\ 12..60..300..9..45..225..18..90..450..36..180..900..7.. \\ 35..175..14..70..350..28..140..700..21..105..525..42.. \\ 210..1050..84..420..2100..63..315..1575..126..630..3150 \\ ..252..1260..6300. \end{array}$$

Therefore the divisors of 6300 are 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 25, 28, 30, 35, 36, 42, 45, 50, 60, 63, 70, 75, 84, 90, 100, 105, 126, 140, 150, 175, 180, 210, 225, 252, 300, 315, 350, 420, 450, 525, 630, 700, 900, 1050, 1260, 1575, 2100, 3150, 6300

(29)

$$\frac{7}{8} \text{ of } \frac{3}{4} \text{ of } 3\frac{1}{2} \text{ lbs.} = \frac{3}{8} \text{ lbs.,} \quad \frac{7}{8} \text{ of } \frac{3}{4} \text{ of } \frac{3}{4} \text{ of } \frac{1}{2} \text{ of } \$1 = \$\frac{7}{8},$$

$$\text{and } \frac{3}{8} \text{ of } \frac{7}{8} \text{ of } \frac{1}{10} \text{ of } \frac{3}{4} \text{ of } 90 \text{ lbs.} = \frac{1323}{1000} \text{ lbs.}$$

63

$$\begin{array}{r} \text{lbs.} \\ 3 : \frac{1323}{1000} :: \frac{7}{8} : \frac{\$ \frac{7}{8} \times \frac{1323}{1000}}{\frac{3}{8}} = \frac{2}{7} \times \frac{1323}{200} \times \frac{8}{7} = \frac{132}{10} = \$5 \cdot 04. \end{array}$$

(31)

7 men will have 7 men's shares.

One woman has  $\frac{1}{11}$  of a man's share;  $\therefore$  2 women will have 2  $\times \frac{1}{11} = \frac{2}{11}$  of a man's share.

One child has  $\frac{1}{11}$  of  $\frac{1}{11} = \frac{1}{121}$  of a man's share;  $\therefore$  11 children will have  $11 \times \frac{1}{121} = \frac{1}{11}$  of a man's share.

7 men, 2 women, and 11 children will have  $7 + \frac{2}{11} + \frac{1}{11} = 8\frac{3}{11}$  men's shares.

$\$2739.18 \div 8\frac{3}{11} = \$325.99\frac{1}{11}$  = a man's share.

$\frac{1}{11}$  of  $\$325.99\frac{1}{11} = \$88.90\frac{1}{11}$  = a woman's share.

$\frac{1}{11}$  of  $\$88.90\frac{1}{11} = \$25.40\frac{1}{11}$  = a child's share.

(33)

(34)

	yds.	ft.	in.	
$\frac{1}{2}$ of $6\frac{1}{2}$ yds. = $2\frac{1}{2}$ yds.	2	2	8	$\left. \begin{array}{l} 2 \ 28 : 7 \ 2 \\ 4 : 11 \\ 2 \ 8 : 5 \\ 13 : 11\frac{1}{2} \end{array} \right\} = 104 : 5.$
$\frac{1}{2}$ of $\frac{1}{2}$ of $8\frac{1}{2}$ ft.	= 1	0	0	
$\frac{1}{2}$ of $\frac{1}{11}$ of $7\frac{1}{10}$ in.	=		$\frac{1}{2}$	
Sum =	3	2	$8\frac{1}{2}$	

(35)

23 bush. 2 pks. 1 gal. 1 qt. 1 pt. = 1515 pts.

$1515 \times 9000 \times \frac{1}{2} = 4545000$  in. = 71 miles 5 fur. 34 per. 3 yds.

(36)

$$\frac{4158}{10395} = \frac{462}{1155} = \frac{66}{165} = \frac{22}{55} = \frac{2}{5}.$$

(37)

VIII.

$\frac{1}{2}, \frac{3}{4}, \frac{1}{5}, \frac{1}{7}$ . Here the common denominator is  $2 \times 3 \times 5 \times 7 = 322$ . The numerators of the fractions are, for the first,  $1 \times 3 \times 5 \times 7 = 151$ ; for the second,  $2 \times 2 \times 5 \times 7 = 214$ ; for the third,  $4 \times 2 \times 3 \times 7 = 250$ ; for the fourth,  $2 \times 2 \times 3 \times 5 = 74$ ; and the equivalent fractions are,  $\frac{151}{322}, \frac{214}{322}, \frac{250}{322}$ , and  $\frac{74}{322}$ , which when added together =  $\frac{731}{322} = 2\frac{65}{322}$ , the numbers all through being in the octenary scale.

n will have 2  
11 children  
 $\frac{1}{2} + \frac{1}{2} = 8\frac{1}{2}$

(38)

$$\left. \begin{array}{l} 17 \text{ sheep} = 6 \text{ cows} \\ 26 \text{ cows} = 27\frac{1}{2} \text{ acres} \\ 12 \text{ acres} = 13 \text{ horses} \\ 11 \text{ horses} = 28 \text{ goats} \\ x \text{ goats} = 68 \text{ sheep} \end{array} \right\} = \frac{3}{6} \times \frac{27\frac{1}{2}}{26} \times \frac{13}{12} \times \frac{28}{11} \times \frac{17}{68} = 2\frac{1}{2} \times 28 = 70 \text{ goats.}$$

re.  
share.  
are.

(39)

$$\left. \begin{array}{l} 27 : 54 \text{ days} \\ 24 : 18 \text{ cel.} \\ 36 : 48 \text{ ft. l.} \\ 21 : 28 \text{ ft. w.} \\ 10 : 9 \text{ ft. d.} \\ 3 : 5 \text{ hrs.} \end{array} \right\} :: 50 \text{ men} : \frac{50 \times 54 \times 18 \times 48 \times 28 \times 9 \times 5}{27 \times 24 \times 36 \times 21 \times 10 \times 3} = 200 \text{ men.}$$

$\} = 104 : 5.$

EXERCISE 90.—Page 226.

(1)

$$\$35 \times 92647 = \$32426.45.$$

(2)

	£	s	d.	
4d.   $\frac{1}{4}$	4746	17	0	= cost of 94937 pails at 1s.
1d.   $\frac{1}{4}$	1582	5	8	= " " " at 4d.
	395	11	5	= " " " at 1d.
<hr/>				
	£6724	14	1	= cost of 94937 pails at 1s. 5d.

(3)

$$\$0.71 \times 95972 = \$7197.90$$

(4)

$$\$28.80 \times 62 = \$1785.60.$$

(5)

$$\$3.21 \times 2310 = \$750.75.$$

(6)

$$\$3.71 \times 2117 = \$793.87\frac{1}{2}.$$

(7)

$3 \times 5 \times 7$   
ne first,  $1 \times$   
 $= 214$ ; for  
 $\times 2 \times 3 \times$   
 $\frac{1}{2}, \frac{359}{222}$ , and  
numbers all

	£	s	d.	
6d.   $\frac{1}{4}$	375	6	0	= price of 7506 pairs at 1s.
3d.   $\frac{1}{4}$	187	13	0	= " " at 6d.
1d.   $\frac{1}{4}$	93	16	6	= " " at 3d.
	23	9	11	= " " at 1d.
<hr/>				
	£680	4	71	= price of 7506 pairs at 1s. 91d.

$$(8) \quad \$ \cdot 17\frac{1}{2} \times 1217 = \$212.97\frac{1}{2}. \quad (9) \quad \$3 \cdot 07\frac{1}{2} \times 2103 = \$6466.72\frac{1}{2}.$$

10s.	$\frac{1}{2}$	2096							
		3							
		<hr/>							
		£6288	0	0	= cost of 2096 oz. at £3.				
5s.	$\frac{1}{2}$	1048	0	0	= " " at 0 10s.				
2s. 6d.	$\frac{1}{2}$	524	0	0	= " " at 0 5s.				
1s. 3d.	$\frac{1}{2}$	262	0	0	= " " at 0 2s. 6d.				
1½d.	$\frac{1}{10}$	131	0	0	= " " at 0 1s. 3d.				
		13	2	0	= " " at 0 0 1½d.				
		<hr/>							
		£8266	2	0	= " " at £3 18s. 10½d.				

10 dwt.	$\frac{1}{2}$	\$1.55							
		6							
		<hr/>							
		\$9.30	= cost of 6 oz.						
5 dwt.	$\frac{1}{2}$	.77½	= " 10 dwt.						
2 dwt. 12 grs.	$\frac{1}{2}$	.38½	= " 5 dwt.						
1 dwt. 6 grs.	$\frac{1}{2}$	.19½	= " 2 dwt. 12 grs.						
2 grs.	$\frac{1}{18}$	.09½	= " 1 dwt. 6 grs.						
		.00½	= " 2 grs.						

$$\$10.75\frac{1}{2} = \text{cost of 6 oz. 18 dwt. 20 grs.}$$

10s.	$\frac{1}{2}$	£98	0	0	= cost of 98 yards at £1.				
5s.	$\frac{1}{2}$	49	0	0	= " " 0 10s.				
		24	10	0	= " " 0 5s.				
		<hr/>							
		£171	10	0	= cost of 98 yards at £1 15s.				
2 qrs.	$\frac{1}{2}$	£1	15						
		<hr/>							
1 qr.	$\frac{1}{2}$	17	6	= cost of 2 qrs.					
1 na.	$\frac{1}{4}$	8	9	= " 1 qr.					
		2	2½	= " 1 na.					
		<hr/>							
		£1	8	5½	= cost of 3 qrs. 1 na.				

$$\text{Then } £171 \ 10 \ 0 = \text{cost of 98 yards at } £1 \ 15s.$$

$$1 \ 8 \ 5\frac{1}{2} = \text{cost of 3 qrs. 1 na. at } £1 \ 15s. \text{ per yard.}$$

$$£172 \ 18 \ 5\frac{1}{2} = \text{cost of 98 yds. 3 qrs. 1 na. at } £1 \ 15s. \text{ per yd}$$

1s.

1d.

2 r.

1 r.

10 per

5 per.

£1394

3

£1398

5 dwt.

1 dwt.

12 grs.

4 grs.

1 gr.

\$6466.72½.

(13)

10s.

5s.

2s. 6d.

1s. 3d.

0 1½d.

8s. 10½d.

1s.	$\frac{1}{20}$	344	
		4	
<hr/>			
		£1376 0 0	= rent of 344 acres at £4.
1d.	$\frac{1}{12}$	17 4 0	= " " at 0 1s.
		1 8 8	= " " at 0 0 1d.
<hr/>			
		£1394 12 8	= rent of 344 acres at £4 1s. 1d.

2 r.	$\frac{1}{2}$	£4 1 1	
<hr/>			
1 r.	$\frac{1}{2}$	2 0 6½	= rent of 2 roods.
10 per.	$\frac{1}{2}$	1 0 3½	= " 1 rood.
5 per.	$\frac{1}{2}$	5 0 1½	= " 10 perches.
		2 6 1½	= " 5 perches.
<hr/>			
		£3 8 4¾	= " 3 roods 15 perches.

£1394 12 8 = rent of 344 acres at £4 1s. 1d.  
 3 8 4¾ = " 3 roods 15 per. at £4 1s. 1d. per ac.

£1398 1 0¾ = " 344 a. 3 r. 15 per. at £4 1s. 1d.

(14)

5 dwt.	$\frac{1}{2}$	5 10	
		5	
<hr/>			
		£1 9 2	= price of 5 oz. at 5s. 10d. per oz
1 dwt.	$\frac{1}{2}$	1 5½	= " 5 dwt. " "
12 grs.	$\frac{1}{2}$	3½	= " 1 dwt. " "
4 grs.	$\frac{1}{2}$	1½	= " 12 grs. " "
1 gr.	$\frac{1}{2}$	0 7½	= " 4 grs. " "
		0 7½	= " 1 gr. " "
<hr/>			
		£1 11 1¾	= " 5 oz. 6 dwt. 17 grs. at 5s. 10d. per oz.

(15)

2 qrs.	$\frac{1}{2}$	£1 2 4		
		4		
		£4 9 4	= price of 4 yards at £1 2 4 per yard	
2 na.	$\frac{1}{4}$	11 2	= " 2 qrs.	" "
1 na.	$\frac{1}{8}$	2 9 $\frac{1}{2}$	= " 2 na.	" "
		1 4 $\frac{1}{2}$	= " 1 na.	" "
		£5 4 8 $\frac{1}{2}$	= price of 4 yds. 2 qrs. 3 na.	" "

(16)

1 rood.	$\frac{1}{4}$	£1 16		
		32		
		£57 12 0	= price of 32 acres at £1 16s.	
10 per.	$\frac{1}{4}$	9 0	= " 1 rood.	"
2 per.	$\frac{1}{8}$	2 3	= " 10 per.	"
2 per.	$\frac{1}{8}$	5 $\frac{1}{2}$	= " 2 per.	"
		5 $\frac{1}{2}$	= " 2 per.	"
		£58 4 1 $\frac{1}{2}$	= price of 32 acres 1 rood 14 per.	

(17)

4 pts.	$\frac{1}{2}$	7 6		
		3		
		£1 2 6	= price of 3 gals. at 7s. 6d. per gal.	
1 pt.	$\frac{1}{4}$	3 9	= " 4 pts.	"
		11 $\frac{1}{2}$	= " 1 pt.	"
		£1 7 2 $\frac{1}{2}$	= price of 3 gals. 5 pts.	

(18)

$$\$1.67\frac{1}{2} \times 724 = \$1212.70.$$

(19)

$$\$1.93\frac{1}{2} \times 721 = \$1396.93\frac{1}{2}.$$

(20)

10s.	$\frac{1}{2}$	4514							
		2							
<hr/>									
		£9028	0	0	= cost of 4514 rods at £2.				
6s. 8d.	$\frac{1}{2}$	2257	0	0	=	"	"	at 0 10	
10d.	$\frac{1}{2}$	1504	13	4	=	"	"	at 0 6 8	
1d.	$\frac{1}{10}$	188	1	8	=	"	"	at 0 0 10	
$\frac{1}{2}$ d.	$\frac{1}{2}$	18	16	2	=	"	"	at 0 0 1	
		9	8	1	=	"	"	at 0 0 0 $\frac{1}{2}$	
<hr/>									
		£13005	19	3	=	"	"	at £2 17 7 $\frac{1}{2}$	

(21)

10s.	$\frac{1}{2}$	£3749	7	6					
		3							
<hr/>									
		£11248	2	6	= price of 3749 $\frac{1}{2}$ acres at £3				
5s.	$\frac{1}{2}$	1874	13	9	=	"	"	at 0 10	
6d.	$\frac{1}{10}$	937	6	10 $\frac{1}{2}$	=	"	"	at 0 5	
		93	14	8 $\frac{1}{2}$	=	"	"	at 0 0 6	
<hr/>									
		£14153	17	9 $\frac{1}{2}$	= price of 3749 $\frac{1}{2}$ acres at £3 15 6				

(22)

4s.	$\frac{1}{2}$	£17	0	0	= cost of 17 cwt. at £1				
8d.	$\frac{1}{2}$	3	8	0	=	"	"	at 0 4	
1d.	$\frac{1}{2}$	11	4	=	"	"	at 0 0 8		
		1	5	=	"	"	at 0 0 1		
<hr/>									
		£21	0	9	= cost of 17 cwt. at £1 4 9				
1 qr.	$\frac{1}{2}$	£1	4	9					
<hr/>									
16 lbs.	$\frac{1}{2}$	6	2 $\frac{1}{2}$	= cost of 1 qr.					
1 lb.	$\frac{1}{10}$	3	6 $\frac{3}{4}$	=	"	16 lbs.			
		0	2 $\frac{7}{11}$	=	"	1 lb.			
<hr/>									
		9	11 $\frac{37}{112}$	=	"	1 qr. 17 lbs.			
£21	0	9	= cost of 17 cwt.	at	£1 4s. 9d. per cwt.				
		9	11 $\frac{37}{112}$	=	"	1 qr. 17 lbs.	"	"	
<hr/>									
£21	10	8 $\frac{37}{112}$	=	"	17 cwt. 1 qr. 17 lbs.	"	"		



(23)

2 qrs.	$\frac{1}{2}$	\$11.55			
		78			
		<hr/>			
		9240			
		8085			
		<hr/>			
		\$900.90	= cost of 78 cwt. at \$11.55 per cwt.		
1 qr.	$\frac{1}{4}$	5.77 $\frac{1}{2}$	=	" 2 qrs.	" "
7 lbs.	$\frac{1}{8}$	2.88 $\frac{1}{2}$	=	" 1 qr.	" "
4 lbs.	$\frac{1}{16}$	.72 $\frac{3}{8}$	=	" 7 lbs.	" "
1 lb.	$\frac{1}{32}$	.41 $\frac{1}{2}$	=	" 4 lbs.	" "
		.10 $\frac{5}{16}$	=	" 1 lb.	" "
		<hr/>			
		\$910.80	= cost of 78 cwt. 3 qrs. 12 lbs.		

(24)

£10 10  
20

---

£210 0 = price of 20 tons at £10 10s.

19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs. = 1 ton. —  $\frac{1}{2}$  lb. The price of 1 ton is £10 10s., and the price of  $\frac{1}{2}$  lb. =  $\frac{1}{4480}$  of £10 10s. =  $\frac{63}{112}$  d. ∴ the price of 19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs. = £10 10s. —  $\frac{63}{112}$  d. = £10 9s. 11 $\frac{1}{2}$  d.

£210 0 0 = price of 20 tons at £10 10s.

10 9 11 $\frac{1}{2}$  = " 19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs.

£220 9 11 $\frac{1}{2}$  = " 20 tons 19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs. at £10 10s. per ton.

10 c

5 cwt

1 cwt

2 qrs

1 qr.

9 pair

6 pair

17 pair

23 pair

14 pair

18 pair

19 yards

75 $\frac{1}{2}$  lbs.

63 lbs.

126 lbs.

35 $\frac{1}{2}$  lbs.

17 lbs.

23 lbs. o

58 $\frac{1}{2}$  lbs.

(25)

per cwt.  
"  
"  
"  
"  
"

10 cwt.	1	\$45.50			
		219			
		40950			
		4550			
		9100			
		\$9964.50	= price of 219 tons at \$45.50 per ton.		
5 cwt.	1	22.75	= "	10 cwt.	" "
1 cwt.	1	11.37½	= "	5 cwt.	" "
2 qrs.	1	2.27½	= "	1 cwt.	" "
1 qr.	1	1.13½	= "	2 qrs.	" "
		58½	= "	1 qr.	" "
		\$10002.60½	= price of 219 tons 16 cwt. 3 qrs.		

S.

EXERCISE 91—Page 228.

BILLS OF PARCELS.

(No. 2.)

e of 1 ton is  
Ds. = 113 d.  
Ds. — 112 d.

	s.	d.		£	s.	d.
9 pair of worsted stockings, at.....	4	6	per pair	2	0	6
6 pair of silk ditto, at.....	15	9	"	4	14	6
17 pair of thread ditto, at.....	5	4	"	4	10	8
23 pair of cotton ditto, at.....	4	10	"	5	11	2
14 pair of yarn ditto, at.....	2	4	"	1	12	8
18 pair of women's silk gloves, at...	4	2	"	3	15	0
19 yards of flannel, at .....	1	7½	per yard	1	10	10½

Ans. £23 15 4½

(No. 3.)

7½ lbs. at  
.

75½ lbs. of sugar, at.....	7½ cents per lb.	\$5.85½
63 lbs. of tea, at.....	93	" 58.59
126 lbs. of butter, at.....	13	" 16.38
35½ lbs. of raisins, at.....	18½	" 6.71½
17 lbs. of sago, at.....	15	" 2.55
23 lbs. of rice, at.....	9	" 2.07
58½ lbs. of starch, at.....	22	" 12.87

Ans. \$105.02½

## (No. 4.)

198 Sangster's National Arithmetic, at.....	\$0.60	\$118.80
197 Robertson's Philosophy of Grammar, at...	0.50	98.50
83 Hodgins' Geography, at.....	1.00	83.00
57 Sangster's Algebraic Formula, at.....	0.12½	7.12½
217 Strachan's Canadian Penmanship, at.....	0.37½	81.37½
143 Hodgins' Geography of British Provinces, at	0.45	64.35
227 Sangster's First Arithmetic, at.....	0.30	68.10

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*Ans.* \$521.25

## (No. 5.)

	s.	d.	£	s.	d.
9½ yards of silk, at.....	12	9 per yard	6	1	1½
13 yards of flowered ditto, at...	15	6 "	10	1	6
11½ yards of lustring, at.....	6	10 "	4	0	3½
14 yards of brocade, at.....	11	3 "	7	17	6
12½ yards of satin, at.....	10	8 "	6	10	8
11½ yards of velvet, at.....	18	0 "	10	4	9

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*Ans.* £44 15 10

## (No. 6.)

14 oz. ipecacuanha, at.....	\$0.67	\$9.38
23 " laudanum, at.....	0.89	20.47
17 " emetic tartar, at.....	1.25	21.25
25 " cantharides, at.....	2.17	54.25
27 " gum mastic, at.....	0.61	16.47
56 " gum camphor, at.....	0.27	15.12

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*Ans.* \$136.94

## (No. 7.)

	s.	d.	£	s.	d.
15½ lbs. of currants, at.....	0	4 per lb.	5	2	
17½ lbs. of Malaga raisins, at.....	0	5½ "	7	10½	
19½ lbs. of sun raisins, at.....	0	6 "	9	10½	
17 lbs. of rice, at.....	0	3½ "	4	11½	
8½ lbs. of pepper, at.....	1	6 "	12	9	
3 loaves of sugar, weight 32½ lbs. at.	0	8½ "	1	3	0½
13 oz. of cloves, at.....	0	9 per oz.	9	9	

---

*Ans.* £3 13 5½

3 13 51

(2)

$$427.1 \div .0000637 = 427100000 \div 637 = 6704866.561 +.$$

(8)

[illegible]

$\text{£}379\ 19\ 7\frac{1}{2}$  = cost of 19 tons at  $\text{£}19\ 19\ 11\frac{1}{2}$

$19 \text{ cwt. } 3 \text{ qrs. } 27\frac{1}{2} \text{ lbs.} = 1 \text{ ton} - \frac{1}{2} \text{ lb.}$  The price of 1 ton is  
 $19 \text{ } 19\text{s. } 11\frac{1}{2}\text{d.}$ , and the cost of  $\frac{1}{2} \text{ lb.} = \frac{1}{4480}$  of  $\text{£}19 \text{ } 19\text{s. } 11\frac{1}{2}\text{d.}$   
 $= \frac{1}{17920} \text{d.}$ ;  $\therefore$  the cost of  $19 \text{ cwt. } 3 \text{ qrs. } 27\frac{1}{2} \text{ lbs.} = \text{£}19 \text{ } 19\text{s.}$   
 $11\frac{1}{2}\text{d.} - \frac{1}{17920} \text{d.} = \text{£}19 \text{ } 19\text{s. } 10\frac{1}{2}\text{d.}$

£379 19 7½ = cost of 19 tons.

19 19 10 17 10 = " 19 cwt. 3 qrs. 27½ lbs.

£399 19 5 ~~1994~~ = " 19 tons 19 cwt. 3 qrs. 27½ lbs.

(4)

Dissimilar.                      Similar.                      Similar and Coterminous.

$$\begin{array}{rclclcl}
 73 \cdot \overset{\cdot}{7}2\overset{\cdot}{3} & = & 73 \cdot 723\overset{\cdot}{7}2\overset{\cdot}{3} & = & 73 \cdot 723\overset{\cdot}{7}23\overset{\cdot}{7}2\overset{\cdot}{3} \\
 11 \cdot 342 & = & 11 \cdot 3422 & = & 11 \cdot 342222222 \\
 16 \cdot 713 & = & 16 \cdot 7130 & = & 16 \cdot 713000000 \\
 19 \cdot 034 & = & 19 \cdot 034034 & = & 19 \cdot 034034034 \\
 713 \cdot 213437 & = & 713 \cdot 213437 & = & 713 \cdot 213437437 \\
 12 \cdot 345678 & = & 12 \cdot 345678345 & = & 12 \cdot 345678345 \\
 & & & & \text{2 carried.}
 \end{array}$$

$$\text{Sum} = 846 \cdot 372095763$$

(5)

$$\left. \begin{array}{l}
 5 : 7 = 5 \div 7 = .714+ \\
 9 : 13 = 9 \div 13 = .692+ \\
 12 : 17 = 12 \div 17 = .705+ \\
 7 : 10 = 7 \div 10 = .7
 \end{array} \right\} \begin{array}{l}
 \text{Hence } 5 : 7 \text{ is the greatest,} \\
 \text{and } 9 : 13 \text{ least.}
 \end{array}$$

$$\frac{5}{7} \times \frac{9}{13} \times \frac{12}{17} \times \frac{7}{10} = \frac{54}{221} = 54 : 221.$$

(6)

1 acre = 160 rods, and 25 acres 2 roods 35 rods = 4115 rods.

$$\begin{array}{r}
 \text{rods.} \\
 160 : 4115 :: \$80 \cdot 50 : \frac{40 \cdot 25 \quad 823}{80 \cdot 50 \times 4115} = \$2070 \cdot 3593.
 \end{array}$$

(8)

$$\$3 \cdot 68\frac{1}{2} \times 7439 = \$27431 \cdot 31\frac{1}{2}.$$

EXERCISE 92.]

KEY.

(9)

$\frac{12345}{12345}$ . The G. C. M. of 135795 and 222210 is 12345; when both terms of the fraction are divided by 12345, it becomes  $\frac{11}{18}$ .  
 $\frac{714235}{999999}$ . Here 714235 and 999999 have no G. C. M.;  $\therefore$  the fraction cannot be reduced.

$\frac{109375}{100000}$ . The G. C. M. of 109375 and 100000 is 3125; when both terms of the fraction are divided by 3125, it becomes reduced to  $\frac{34}{32}$ .

$\frac{20301}{33633}$ . The G. C. M. of 20301 and 33633 is 303; when both terms of the fraction are divided by 303, it is reduced to its lowest terms, viz.,  $\frac{67}{111}$ .

(10)

34½ bushels turnips	= 17 bushels potatoes	} =
9 " potatoes	= 59½ lbs. tea	
6 lbs. tea	= 11½ stone flour	
13 stone flour	= 350 cents	
38 cents	= 12 lbs. bread	
119 lbs bread	= x bushels turnips	

$$\frac{34\frac{1}{2}}{17} \times \frac{9}{59\frac{1}{2}} \times \frac{6}{11\frac{1}{2}} \times \frac{13}{350} \times \frac{19}{12} \times \frac{119}{1} = \frac{3 \times 13 \times 19}{8\frac{1}{2} \times 40} = 23\frac{61}{40}$$

(11)

54 : 27 men	} :: 7 days :	$\frac{7 \times 27 \times 8 \times 77 \times 24 \times 22 \times 5}{54 \times 11 \times 42 \times 20 \times 16 \times 3}$
11 : 8 hours		
42 : 77 floors		
20 : 24 feet long		
16 : 22 feet wide		
3 : 5 coats paint		

$$= \frac{7 \times 11}{2 \times 3} = 12\frac{1}{2} \text{ days.}$$

(13)

IX.

12)72342

12)5403..2

12)407..0

12)30..7

2..3

IX.

6)72342

6)11806..2

6)1731..0

6)264..4

6)40..4

6)8..0

1..0

IX.

3)72342

3)23713..2

3)7234..0

3)2371..1

3)723..1

3)237..0

3)72..1

3)23..2

3)7..0

2..1

IX.

72342

9

65

9

588

9

5296

9

47666

XII.

23702

12

27

12

331

12

3972

12

47666

VI.

1004402

6

6

6

36

6

220

6

1324

6

7944

6

47666

III

2102101102

3

7

8

21

3

65

3

196

3

588

3

1765

1765

3

5296

3

15888

3

47666

4665

(14)

342

713..2

234..0

371..1

723..1

237..0

72..1

323..2

7..0

2..1

102101102

3

7

8

3

1765

3

5296

3

15888

3

47666

II.

111111

2

3

2

—

7

2

—

15

2

—

31

2

—

63 Greatest.

100000

2

2

2

—

4

2

—

8

2

—

16

2

—

32 Least.

IV.

333333

4

15

4

—

63

4

—

255

4

—

1023

4

—

4095 Greatest. 1024 Least.

IV.

100000

4

—

4

4

—

16

4

—

64

4

—

256

4

—

VI.

555555

6

—

35

6

—

215

6

—

1295

6

—

7775

6

—

46655 Greatest. 7776 Least. 262143 Greatest. 32768

VI.

100000

6

—

6

6

—

36

6

—

216

6

—

1296

6

—

VIII.

777777

8

—

63

8

—

511

8

—

4095

8

—

32767

8

—

VIII.

100000

8

—

8

8

—

64

8

—

512

8

—

4096

8

—

(Continued on next page.)



(14 continued.)

XII.	XII.
eeeeee	100000
12	12
—	—
143	12
12	12
—	—
1727	144
12	12
—	—
20735	1728
12	12
—	—
248831	20736
12	12
—	—
2985983 Greatest.	248832 Least.

(15)

$$1728 = 2^6 \times 3^3.$$

1..2..4..8..16..32..64

1 3..9..27

1..2..4..8..16..32..64..3..6..12..24..48..96..192..9..

18..36..72..144..288..576..27..54..108..216..432..864..

1728.

Therefore the divisors of 1728 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 96, 108, 144, 192, 216, 288, 432, 576, 864, 1728.

(16)

30	2..4..8..16..32..64..128..256..512..1024..2048..4096..8192..16384..32768..65536..131072..262144..524288..1048576..2097152..4194304..8388608..16777216..33554432..67108864..134217728..268435456..536870912..1073741824..2147483648..4294967296..8589934592..17179869184..34359738368..68719476736..137438953472..274877906944..549755813888..1099511627776..2199023255552..4398046511104..8796093022208..17592186044416..35184372088832..70368744177664..140737488355328..281474976710656..562949953421312..1125899906842624..2251799813685248..4503599627370496..9007199254740992..18014398509481984..36028797018963968..72057594037927936..144115188075855872..288230376151711744..576460752303423488..1152921504606846976..2305843009213693952..4611686018427387904..9223372036854775808..18446744073709551616..36893488147419103232..73786976294838206464..147573952589676412928..295147905179352825856..590295810358705651712..1180591620717411303424..2361183241434822606848..4722366482869645213696..9444732965739290427392..18889465931478580854784..37778931862957161709568..75557863725914323419136..151115727451828646838272..302231454903657293676544..604462909807314587353088..1208925819614629174706176..2417851639229258349412352..4835703278458516698824704..9671406556917033397649408..19342813113834066795298816..38685626227668133590597632..77371252455336267181195264..154742504910672534362390528..309485009821345068724781056..618970019642690137449562112..123794003928538027489924224..247588007857076054979848448..495176015714152109959696896..990352031428304219919393792..1980704062856608439838787584..3961408125713216879677575168..7922816251426433759355150336..15845632502852867518710300672..31691265005705735037420601344..63382530011411470074841202688..126765060022822940149682405376..253530120045645880299364810752..507060240091291760598729621504..1014120480182583521197459243008..2028240960365167042394918486016..4056481920730334084789836972032..8112963841460668169579673944064..16225927682921336339159347888128..32451855365842672678318695776256..64903710731685345356637391552512..129807421463370690713274783105024..259614842926741381426549566210048..519229685853482762853099132420096..103845937170696552570619826484096..207691874341393105141239652968192..415383748682786210282479305936384..830767497365572420564958611872768..1661534994731144841129917223745536..3323069989462289682259834447491072..6646139978924579364519668894982144..1329227995784915872903933778996448..2658455991569831745807867557992896..5316911983139663491615735115985792..10633823966279326983231470231971584..21267647932558653966462940463943168..42535295865117307932925880927886336..85070591730234615865851761855772672..170141183460469231731703523711545344..340282366920938463463407047423090688..680564733841876926926814094846181376..1361129467683753853853628189692362752..2722258935367507707707256379384725504..5444517870735015415414512758769451008..10889035741470030830829025517538902016..21778071482940061661658051035077804032..43556142965880123323316102070155608064..87112285931760246646632204140311216128..174224571863520493293264408280622432256..348449143727040986586528816561244864512..696898287454081973173057633122489729024..1393796574908163946346115266244995458048..2787593149816327892692230532489990916096..5575186299632655785384461064979981832192..11150372599265311570768922129959963664384..22300745198530623141537844259919927328768..44601490397061246283075688519839854657536..89202980794122492566151377039679709315072..178405961588244985132302754079359418630144..356811923176489970264605508158718837260288..713623846352979940529211016317437674520576..1427247692705959881058422032634875349041152..2854495385411919762116844065269750698082304..5708990770823839524233688130539501396164608..11417981541647679048467376261079002792329216..22835963083295358096934752522158005584658432..45671926166590716193869505044316011169316864..91343852333181432387739010088632022338633728..182687704666362864775478020177264044677267456..365375409332725729550956040354528089354534912..730750818665451459101912080709056178709069824..1461501637330902918203824161418112357418139648..292300327466180583640764832283622471483627296..584600654932361167281529664567244942967254592..1169201309864722334563059329134489988514510912..2338402619729444669126118658268979977029021824..4676805239458889338252237316537959954058043648..9353610478917778676504474633075919908116087296..18707220957835557353008949266151839816232174592..37414441915671114706017898532303679632464349184..74828883831342229412035797064607359264928698368..149657767662684458824071594129214718529857396736..299315535325368917648143188258429437059714793472..598631070650737835296286376516858874119429586944..1197262141301475670592572753033717748238859173888..2394524282602951341185145506067435496477718347776..4789048565205902682370291012134870992955436695552..9578097130411805364740582024269741985910873391104..19156194260823610729481164048539483971821746782208..38312388521647221458962328097078967943643493564416..76624777043294442917924656194157935887286987128832..153249554086588885835849312388315871777457394257664..306499108173177771671698624776637735549147985115328..612998216346355543343397249553275471092295970230656..1225996432692711086686794499106550942184591940461312..2451992865385422173373588998213101884369183880922624..4903985730770844346747177996426203768738367761845248..9807971461541688693494355992852407537476735523690496..196159429230833773869887119857048150749534710473780928..392318858461667547739774239714096301490694209447561952..784637716923335095479548479428192602981388418895123904..1569275433846670190959096958856385205962776837790247808..3138550867693340381918193917712770411925553675580495616..6277101735386680763836387835425540823851107351160991232..12554203470773361527672775670851081647702214702321982464..251084069415467230553455513417021632954044294046439648..502168138830934461106911026834043265908088588092879296..1004336277661868922213822053668065319816177176185758592..2008672555323737844427644107336130639632354352371517184..401734511064747568885528821467226129264470870474303424..803469022129495137771057642934452258528941740948606848..1606938044258990275542115285868904517057883481897213696..3213876088517980551084230571737809034115766893794427392..64277521770359611021684611434756180682315377875888546784..12855504354071922204336922286951236136462307575177709376..25711008708143844408673844573902472272924615150355418752..51422017416287688817347689147804945455849230300710837504..102844034832575377634695378295609890911698460601421675008..205688069665150755269390756591219781823397321202843350016..411376139330301510538781513182439563646794642405686700032..82275227866060302107756302636487912729358928481137340064..164550455732120604215512605272975825458717856962274680128..329100911464241208431025210545951650917435713924549360256..658201822928482416862050421091903301834871428278987200512..1316403645856964833724100842183806603669742857557974401024..26328072917139296674482016843676132073394857151159948802048..52656145834278593348964033687352264146789714302399976004096..105312291668557186697928067374704528293579428604799952008192..210624583337114373395856134749409056587158857209599904016384..421249166674228746791712269498818113174317711441998080032768..842498333348457493583424538997636226348635422883996160065536..1684996666696914987166849077995272452697270857767992320131072..3369993333393829974333698155990544905394541715535984640262144..6739986666787659948667396311981089810789083431071969280524288..13479973333575319897334792623962179621578166862143937761048576..26959946667150639794669585247924359243156333724287875522097152..53919893334301279589339170495848718486312667448575751044194304..107839786668602559178678340991697437726225334897151502088388608..215679573337205118357356681983394875452450697994303004777777216..431359146674410236714713363966789750904901395988060009554554432..862718293348820473429426727933579501809802791977612001910908864..1725436586697640946858853455867159003619605583955224003821817728..3450873173395281893717706911734318072392111167910448007643635456..6901746346790563787435413823468636144784222335820896015287270912..13803492693581127574870827646937272289568444671641792030574541824..27606985387162255149741655293874544579136889343283358401149083648..55213970774324510299483310587749089158273778686566706802298167296..110427941548649020598966621175498178316547557373133413604596334592..220855883097298041197933242350996356633095114746266827209192669184..441711766194596082395866484701992713266190229492533654418385338368..883423532389192164791732969403985426532380458985067308836770676736..1766847064778384329583465938807970853064760917970134617673541353504..3533694129556768659166931877615941706129521835940269235347082707008..7067388259113537318333863755231883412259043671880538470694165414016..14134776518227074636667727510463766824518087343761076941388330828032..28269553036454149273335455020927533649036164687522153892776661656064..56539106072908298546670910041855067298072329375043071785553323312128..113078212145816597093341820083710134596144598750060353711066466624256..22615642429163319418668364016742026919228919700012070742213293248512..45231284858326638837336728033484053838457839400024041484426586497024..90462569716653277674673456066968107676915678800048082968853136994048..180925139433306555349346912133936215353831357600096165977062739888096..361850278866613110698693824267872427076662715200192331954125497776192..723700557733226221397387648535744854153325430400384663908250995552384..1447401115466452442794775297071489708306650860800769327816501991104768..2894802230932904885589550594142979416613301721601538655633003982209536..5789604461865809771179101188285958833226603443203077311266007964419072..11579208923731619542358202376571917666453206886406154462520015928838144..23158417847463239084716404753143835332906413772812308885040031857676288..46316835694926478169432809506287670665812827545624617770080063715352576..9263367138985295633886561901257534133162565509124923554016012743070512..18526734277970591267773123802515068266325131018249467110032025486141024..37053468555941182535546247605030136532650262036498934220064050972282048..74106937111882365071092495210060273065300524072997868440128101944564096..148213874223764730142184990420120546130601048145957376880256203889128192..296427748447529460284369980840241092261202082891914753760512407778256384..592855496895058920568739961680482184522404165783829507520224815556512768..1185710993790117841137479823360964369044808331567659015040449631113025536..2371421987580235682274959646721928738089616663135318030080899262226051072..4742843975160471364549919293443857476179233326270636060161798524452102144..9485687950320942729099838586887714952384666625541272120323597048904204288..18971375900641885458199677173775429904769333251082544240647194097808408576..37942751801283770916399353547550859809538666502165088481294388195616817152..75885503602567541832798707095101719619077333004330176962586776391233634304..151771007205135083655597414190203439238154666008660353925173547822467268608..303542014410270167311194828380406878476309332017320707850347095644934537216..607084028820540334622389656760813756952618664034641415700694191289907444432..1214168057641080669244779313521627513905237328069282831401388382579814888864..2428336115282161338489558627043255027810474561385665662802776765159697777728..4856672230564322676979117254086510055620949122771331325605553530319395555456..971334446112864535
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(17)

Dissimilar.

Similar.

Similar and Coterminous.

$$\begin{array}{rcl}
 97 \cdot 91342 & = & 97 \cdot 913423 \\
 18 \cdot 1234567 & = & 19 \cdot 1234567 \\
 & & 18 \cdot 123456745674567
 \end{array}$$

$$\text{Difference} = 79 \cdot 789966677748855$$

(18)

20 ft. 7"

19 ft. 5 7"

$$\begin{array}{r}
 1 \quad 0 \quad 0 \quad 1'' \\
 8 \quad 6 \quad 11 \\
 391 \quad 1
 \end{array}$$

$$\begin{aligned}
 400 \quad 7 \quad 11 \quad 1 &= 44 \text{ sq. yds.} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = \\
 &44 \cdot 8053 \text{ sq. yds.} = 44 \cdot 517 \text{ sq. yds.} \\
 &\$2 \cdot 87\frac{1}{2} \times 44 \cdot 517 = \$127 \cdot 98 +.
 \end{aligned}$$

(19)

$$\begin{aligned}
 916 \text{ acres } 3 \text{ roods } 17 \text{ per. } 7 \text{ yds.} &= 4437591\frac{1}{4} \text{ sq. yds., and } 43 \\
 \text{acres } 1 \text{ rood } 2 \text{ per. } 17 \text{ yds.} &= 209407\frac{1}{4} \text{ sq. yds.} \\
 4437591\frac{1}{4} \div 209407\frac{1}{4} &= 4437591 \cdot 25 \div 207407 \cdot 5 = 21 \cdot 19117+.
 \end{aligned}$$

## EXERCISE 94---Page 233

(1)

(2)

$$\$742 \cdot 10 \times \cdot 05 = \$37 \cdot 10\frac{1}{2} \quad \$1000 \times \cdot 11 = \$110.$$

(3)

$$\$734 \cdot 19 \times \cdot 10 = \$73 \cdot 419.$$

(4)

$$\$1624 \cdot 50 \times \cdot 875 = \$1421 \cdot 4375.$$

(5)

$$\$994.70 \times .125 = \$124.3375.$$

(6)

$$\$777.50 \times .0875 = \$68.03125, \text{ or } \$68.03\frac{1}{2}.$$

(7)

(8)

$$\$7135.80 \times .0225 = \$160.5555.$$

$$2740 \times .20 = 548.$$

(9)

(10)

$$\$7490 \times .10 = \$749$$

$$\$740 \times .045 = \$33.30$$

$$\$7490 \times .17 = \$1273.30$$

$$\$1680 \times .025 = \$42.00$$

$$\$7490 \times .27 = \$2022.30$$

$$\$42.00 - \$33.30 = \$8.70$$

$$\$7490 \times .46 = \$3445.40$$

(11)

(12)

$$729 \times .11 = 80.19$$

$$\$763.22 \times .25 = \$190.8050$$

$$729 - 80.19 = 648.81 = 648\frac{81}{100}$$

$$\$847.16 \times .16 = 135.5456$$

$$\$1234.17 \times .0625 = 77.135625$$

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$$\text{Sum} = \$403.486225$$

(13)

(14)

$$\$17429.40 \times .43 = \$7494.64\frac{1}{2}$$

$$68978 \times .36 = 24832.08.$$

$$\$17429.40 \times .37 = 6448.87\frac{1}{2}$$

(15)

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$$\$13943.52$$

$$29800 \times .17 = 5066$$

$$\$17429.40 - \$13943.52 = \$3485.88.$$

$$29800 - 5066 = 24734$$

## EXERCISE 95—Page 235.

(1)

(2)

$$\$1000 \times .045 = \$45.$$

$$\$1678.30 \times .0225 = \$37.76175.$$

\$7531.1

\$7863.5

\$7193.1

\$7893.87

(3)

(4)

$$\$7531.19 \times .0375 = \$282.419625. \quad \$508.60 \times .0125 = \$6.3575$$

(5)

(6)

$$\$7863.50 \times .0175 = \$137.61125. \quad \$878.30 \times .025 = \$21.9575$$

(7)

(8)

$$\$7193.16 \times .03125 = \$224.78625. \quad \$6734.10 \times .17 = \$1144.797.$$

(9)

$$\$7.13 \times 718 \times .0425 = \$217.57195.$$

(10)

$$\$1.85 \times 9243 \times .05625 = \$857.7871875.$$

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 EXERCISE 96—Page 236.

(1)

(2)

$$\$7893.87 \times .02 = \$157.8774. \quad \$8000 \times .00875 = \$70.$$

(3)

$$\$8643.22 \times .0125 = \$108.04025.$$

(4)

$$\$78963.80 \times .00875 = \$690.93325.$$

(5)

$$\$1987.27 \times .0375 = \$74.522625.$$

## EXERCISE 97—Page 237.

(1)

$$\begin{aligned} \$4000 \div 1.0125 &= \$3950.61728 + = \text{sum to be invested.} \\ \$4000 - 3950.61728 &= \$49.38271 = \text{commission.} \end{aligned}$$

(2)

$$\begin{aligned} \$7500 \div 1.045 &= \$7177.03349 = \text{sum to be expended in laces,} \\ \$7500 - \$7177.03349 &= \$322.96651 = \text{commission.} \end{aligned}$$

(3)

$$\begin{aligned} \$8470 \div 1.05 &= \$8066.66\frac{2}{3} = \text{sum to be invested.} \\ \$8066.66\frac{2}{3} \div \$6.40 &= 1260\frac{1}{2} \text{ barrels.} \end{aligned}$$

(4)

$$\$11000 \div 1.00875 = \$10904.584882 = \text{sum to be invested.}$$

(5)

$$\begin{aligned} \$13000 \div 1.045 &= \$12440.1913 + = \text{sum to be invested.} \\ \$13000 - \$12440.1913 &= \$559.8086 + = \text{commission.} \\ \$12440.1913 + \div \$3.63 &= 3427.0499 \text{ yds.} \end{aligned}$$

## EXERCISE 98—Page 238.

(1)

(2)

$$\$9000 \div 0.83 = \$10843.373. \quad \$8500 \div 1.11 = \$7657.6576$$

(3)

$$\begin{aligned} \$17500 \div 1.0125 &= \$17283.951 = \text{amount to be invested.} \\ \$17283.951 \div 1.07 &= \$16153.22 = \text{stock.} \end{aligned}$$

(4)

$$\begin{aligned} \$20000 \div 1.0175 &= \$19656.01965 = \text{amount to be invested.} \\ \$19656.01965 \div 0.97 &= \$20263.937 = \text{stock remitted.} \end{aligned}$$

(5)

$$\begin{aligned} \$200 \times 100 &= \$20000 = \text{par value of 200 shares.} \\ \$1 \text{ stock costs } \$1.055. \quad \$1.055 \times 20000 &= \$21100 = \text{cost of stock.} \\ \$21100 \times .00875 &= \$184.625 = \text{brokerage.} \\ \$21100 + \$184.625 &= \$21284.625 = \text{whole cost.} \end{aligned}$$

## EXERCISE 99—Page 240.

(1)

$$\$7500 \times .0175 = \$131.25.$$

(2)

$$\$8375 \times .0075 = \$62.8125;$$

(3)

$$\$6000 \times .01875 = \$112.50$$

(4)

$$\$5000 \times .0117 = \$58.50.$$

(5)

$$\$6400 \times .0090 = \$57.60$$

(6)

$$\$4500 \times .0035 = \$15.75.$$

(7)

$$\$36000 \times .03 = \$1080.$$

(8)

$$\$27000 \times .0482 \times 4 = \$5205.60.$$

(9)

$$\$39000 \times .022 = \$858.$$

(10)

$$\$17800 \times .005 = \$89.$$

(11)

$$\$12350 \times .017 \times 7 = \$1235.$$

## EXERCISE 100—Page 241.

(1)

$$\$17000 \div .965 = \$17616.58. \quad \$22750 \div .94 = \$24202.127.$$

(2)

(3)

$$\$15000 \div .9775 = \$15345.2685. \quad \$33000 \div .9425 = \$35013.2625.$$

(4)

## EXERCISE 101—Page 243.

(1)

$$1347 \times 5 = 6735 \text{ lbs.} = \text{gross weight.}$$

$$6735 \times .06 = 404.1 \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 6330.9 \text{ lbs.} = \text{net at } 3\frac{1}{2} \text{ cents per lb.} = 6330.9 \\ \times .035 = \$221.58. \end{array}$$

(2)

$$127 \times 11 = 1397 \text{ lbs.} = \text{gross weight.}$$

$$1397 \times .03 = 41.91 \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 1355.09 \text{ lbs.} = \text{net at } \$0.12 \text{ per lb.} = 1355.09 \\ \times .012 = \$16.26. \end{array}$$

(3)

$$.29 \times .13 = \$16.77.$$

(4)

$$31 \times 207 = 6417 \text{ lbs.} = \text{gross weight.}$$

$$207 \times 2\frac{1}{4} = 465\frac{1}{4} \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 5951\frac{1}{4} \text{ lbs.} = \text{net at } 5\frac{1}{2} \text{ cents per lb.} = 5951\frac{1}{4} \times \\ .0575 = \$342.1968. \end{array}$$

(5)

$$214 \times .47 = \$100.58.$$


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## EXERCISE 102—Page 243.

(1)

(2)

$$\$17429.80 \times .21 = \$3660.2580. \quad \$2920.16 \times .075 = \$219.012.$$

(3)

(4)

$$\$71342.90 \times .25 = \$17835.725. \quad \$913.73 \times .2 = \$182.746.$$

(5)

$$\$14713.19 \times .33 = \$4855.3527.$$


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## EXERCISE 103—Page 244.

(1)

$$\$23900 \div 7142300 = \$0.0033462 = \text{rate per dollar.}$$

$$\$0.0033462 \times 14729.50 = \$49.2878 +.$$

(2)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 18600 = \$321.0732.$$

(3)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 7500 = \$129.465.$$

(4)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 11400 = \$196.7868.$$



## EXERCISE 104—Page 252.

(1)

Here  $P = \$723.19$ ,  $r = .067$ , and  $t = 7.32$ .Then  $I = Prt = 723.19 \times .067 \times 7.32 = \$354.6818036$ .

(2)

Here  $P = 857.19$ ,  $r = .065$ , and  $t = 6\frac{1}{2}$  or  $6.5$ .Then  $A = P(1 + rt) = \$857.19 \times 1.4225 = \$1219.352775$ .

(3)

Here  $t = 11$ , and  $r = .725$ .Then  $n = tr + 1 = 11 \times .725 + 1 = 8.975$ .

(4)

Here  $P = \$654.32$ ,  $I = \$234.56$ , and  $r = .07$ .Then  $t = \frac{I}{Pr} = \frac{234.56}{654.32 \times .07} = 5.12112$  or 5 years 1 m. 13 d.

(5)

Here  $A = \$1200$ ,  $P = \$700$ , and  $t = 5$ .Then  $r = \frac{A - P}{Pt} = \frac{1200 - 700}{700 \times 5} = \frac{1}{7} = \text{rate per unit} \therefore 14\frac{2}{7} = \text{rate per cent.}$ 

(6)

Here  $n = 4$ , and  $r = .23$ .Then  $t = \frac{n-1}{r} = \frac{4-1}{.23} = 13$  years 15 days.

(7)

Here  $P = \$270$ ,  $I = \$87$  and  $r = .07$ .Then  $t = \frac{I}{Pr} = \frac{87}{270 \times .07} = 4$  years  $7\frac{1}{2}$  months.

(8)

Here  $P = \$680$ ,  $t = 11\frac{1}{2}$ , and  $r = .11$ .Then  $A = P(1 + rt) = 680 \times 2.265 = \$1540.20$ 

(9)

Here  $A = \$2000$ ,  $t = 20$ , and  $r = .08$ .Then  $P = \frac{A}{1 + rt} = \frac{2000}{2.6} = \$769.23\frac{1}{3}$ .

(10)

Here  $n = 21$ , and  $t = 24$ .Then  $r = \frac{n-1}{t} = \frac{21-1}{24} = .83\frac{1}{3} = \text{rate per unit. } \therefore 83\frac{1}{3} = \text{rate per cent.}$ 

(11)

Here  $n = 23$ , and  $r = .16$ .Then  $t = \frac{n-1}{r} = \frac{23-1}{.16} = 137\frac{1}{2} \text{ years}$ 

(12)

Here  $P = \$679.18$ ,  $r = .0775$ , and  $t = 11.73$ .Then  $I = Prt = 679.18 \times .0775 \times 11.73 = \$617.4255$ .

(13)

Here  $P = \$950$ ,  $A = \$1763.42$ , and  $t = 10$ .Then  $r = \frac{A - P}{Pt} = \frac{950 \times 10}{1763.42 - 950} = .08562 = \text{rate per unit}$   
 $\therefore 8.562 = \text{rate per cent.}$ 

K

(14)

Here  $P = \$666$ ,  $A = \$1347.50$ , and  $r = .06$ .

$$\text{Then } t = \frac{A - P}{Pr} = \frac{1347.50 - 666}{666 \times .06} = 17.054 + \text{ years, or } 17 \text{ years } 19 \text{ days.}$$

(15)

Here  $P = \$273$ ,  $I = \$100$ , and  $r = .09$ 

$$\text{Then } t = \frac{I}{Pr} = \frac{100}{273 \times .09} = 4.07 \text{ years} = 4 \text{ years } 25 \text{ days.}$$

(16)

Here  $P = \$476.30$ ,  $A = \$500$ , and  $t = 2$ .

$$\text{Then } r = \frac{A - P}{Pt} = \frac{500 - 476.30}{476.30 \times 2} = .0248 = \text{rate per unit.}$$

$\therefore 2\frac{1}{2}\% = \text{rate per cent.}$

(17)

Here  $P = \$749.49$ ,  $I = \$257$ , and  $t = 7$ .

$$\text{Then } r = \frac{I}{Pt} = \frac{257}{749.49 \times 7} = .04898 = \text{rate per unit.}$$

$\therefore 4.898 = \text{rate per cent.}$

(18)

Here  $A = \$1111.11$ ,  $t = 11$ , and  $r = .11$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1111.11}{2.21} = \$502.7647.$$

(19)

 $P = £167.47$ ,  $r = .11$ , and  $t = 9$ .

$$I = Prt = 167.47 \times .11 \times 9 = £165.7953 = £165 \text{ } 15\text{s. } 10\frac{1}{2}\text{d.}$$

EXERCISE 105—Page 253.

(1)

(2)

$$11 \div 2 = 5\frac{1}{2} \text{ cents.}$$

$$16 \div 2 = 8 \text{ cents} = \$0.08.$$

(3)

$$9 \text{ years and 8 months} = 116 \text{ months, and } 116 \div 2 = 58 \text{ cents} \\ = \$0.58.$$

(4)

$$16 \text{ years and 3 months} = 195 \text{ months, and } 195 \div 2 = 97\frac{1}{2} \text{ cents} \\ = \$0.97\frac{1}{2}.$$

(5)

$$11 \text{ years and 7 months} = 139 \text{ months, and } 139 \div 2 = 69\frac{1}{2} \text{ cents} \\ = \$0.695.$$

(6)

$$12 \text{ years and 5 months} = 149 \text{ months, and } 149 \div 2 = 74\frac{1}{2} \text{ cents} \\ = \$0.745.$$

(7)

$$3 \text{ years and 2 months} = 38 \text{ months, and } 38 \div 2 = 19 \text{ cents} = \\ \text{interest of \$1 for given rate and time.} \\ \$0.19 \times 279.40 = \$53.086.$$

(8)

$$6 \text{ years and 7 months} = 79 \text{ months, and } 79 \div 2 = 39\frac{1}{2} \text{ cents} = \\ \text{interest of \$1 for given rate and time.} \\ \$0.395 \times 189.70 = \$74.9315.$$

(9)

3 years and 11 months = 47 months, and  $47 \div 2 = 23\frac{1}{2}$  cents =  
interest of \$1 for given rate and time.

$$\$0.235 \times 1463 = \$343.805.$$

(10)

11 years and 1 month = 133 months, and  $133 \div 2 = 66\frac{1}{2}$  cents =  
interest of \$1 for given rate and time.

$$\$0.665 \times 28967.50 = \$19263.3875.$$

---

EXERCISE 106—Page 254.

(1)

$$2 \div 6 = \frac{1}{3} \text{ mill} = \$0.0003.$$

(2)

$$7 \div 6 = 1\frac{1}{6} \text{ mills} = \$0.001\frac{1}{6}.$$

(3)

$$11 \div 6 = 1\frac{5}{6} \text{ mills} = \$0.001\frac{5}{6}.$$

(4)

$$27 \div 6 = 4\frac{1}{2} \text{ mills} = \$0.004\frac{1}{2}.$$

(5)

$$47 \div 6 = 7\frac{5}{6} \text{ mills} = \$0.007\frac{5}{6}.$$

(6)

$$8 \div 2 = 4 \text{ cents} = \$0.04.$$

$$12 \div 6 = 2 \text{ mills} = \$0.002 \text{ and } \$0.04 + \$0.002 = \$0.042$$

(7)

$$66 \div 6 = 11 \text{ mills} = \$0.011.$$

(8)

$$2 \text{ years } 2 \text{ m'ths} = 26 \text{ months, and } 26 \div 2 = 13 \text{ cents} = \$0.13.$$

$$19 \div 6 = 3\frac{1}{6} \text{ mills} = \$0.003\frac{1}{6} \text{ and } \$0.13 + \$0.003\frac{1}{6} = \$0.133\frac{1}{6}.$$

(9)

7 years 8 m'ths = 92 months, and  $92 \div 2 = 46$  cents = \$0.46.  
 $9 \div 6 = 1\frac{1}{2}$  mills = \$0.001 $\frac{1}{2}$  and  $\$0.46 + \$0.001\frac{1}{2} = \$0.461\frac{1}{2}$ .

(10)

17 years 11 months = 215 months, and  $215 \div 2 = 107\frac{1}{2}$  cents = \$1.075.  
 $23 \div 6 = 3\frac{5}{6}$  mills = \$0.003 $\frac{5}{6}$ , and  $\$1.075 + \$0.003\frac{5}{6} = \$1.078\frac{5}{6}$ .

(11)

12 years 7 months = 151 months, and  $151 \div 2 = 75\frac{1}{2}$  cents = \$0.755.  
 $17 \div 6 = 2\frac{5}{6}$  mills = \$0.002 $\frac{5}{6}$ , and  $\$0.755 + \$0.002\frac{5}{6} = \$0.757\frac{5}{6}$ .

## EXERCISE 107—Page 255.

(1)

Interest on \$1 for 7 months = \$0.035  
 Interest on \$1 for 17 days = 2 $\frac{5}{8}$

Therefore interest on \$1 for 7 months 17 days, = \$0.037 $\frac{5}{8}$   
 Then  $\$0.037\frac{5}{8} \times 917.30 = \$34.704516$ .

(2)

Interest on \$1 for 3 months = \$0.015  
 Interest on \$1 for 13 days = 2 $\frac{1}{2}$

Therefore interest on \$1 for 3 months 13 days = \$0.017 $\frac{1}{2}$   
 Then  $\$0.017\frac{1}{2} \times 842.50 = \$14.462916$ .

(3)

Interest on \$1 for 2 years 11 months = \$0.175  
 Interest on \$1 for 10 days = 1 $\frac{3}{4}$

Therefore interest on \$1 for 2 years 11 m'ths 10 days = \$0.176 $\frac{3}{4}$   
 Then  $\$0.176\frac{3}{4} \times 573.83 = \$101.3786$ .

(4)

Interest on \$1 for 6 years 9 months = \$0.405

Interest on \$1 for 19 days =  $3\frac{1}{2}$ Therefore interest on \$1 for 6 years 9 m'ths 19 days = \$0.408 $\frac{1}{2}$ Then  $\$0.408\frac{1}{2} \times 642.30 = \$262.16545$ .

(5)

Interest on \$1 for 5 years 5 months = \$0.325

Interest on \$1 for 7 days =  $1\frac{1}{2}$ Therefore interest on \$1 for 5 years 5 months 7 days = \$0.326 $\frac{1}{2}$ Then  $\$0.326\frac{1}{2} \times 1427.875 = \$465.7252$ .

(6)

Interest on \$1 for 4 years 7 months = \$0.275

Interest on \$1 for 16 days =  $2\frac{3}{4}$ Therefore interest on \$1 for 4 years 7 m'ths 16 days = \$0.277 $\frac{3}{4}$ Then  $\$0.277\frac{3}{4} \times 709.63 = 197.040596$ .

(7)

Interest on \$1 for 7 years 7 months = \$0.455

Interest on \$1 for 22 days =  $3\frac{3}{4}$ Therefore interest on \$1 for 7 years 7 m'ths 22 days = \$0.458 $\frac{3}{4}$ Then  $\$0.458\frac{3}{4} \times 2463.20 = \$1129.7877 + \$2463.20 = \$3592.9877$ .

(8)

Interest on \$1 for 9 years 9 months = \$0.585

Interest on \$1 for 9 days =  $1\frac{1}{2}$ Therefore interest on \$1 for 9 years 9 months 9 days = \$0.586 $\frac{1}{2}$ Then  $\$0.586\frac{1}{2} \times 999.99 = \$586.49135$ .

(9)

$$\begin{array}{r}
 = \$0.405 \\
 = \quad 3\frac{1}{2} \\
 \hline
 = \$0.408\frac{1}{2}
 \end{array}$$

$$\begin{array}{rcl}
 \text{Interest on \$1 for 3 years 4 months} & = & \$0.20 \\
 \text{Interest on \$1 for 27 days} & = & 4\frac{1}{2}
 \end{array}$$

$$\text{Therefore interest on \$1 for 3 years 4 m'ths 27 days} = \$0.204\frac{1}{2}$$

$$\text{Then } \$0.2045 \times 68.70 = \$14.04915.$$

(10)

$$\begin{array}{r}
 = \$0.325 \\
 = \quad 1\frac{1}{2} \\
 \hline
 = \$0.326\frac{1}{2}
 \end{array}$$

$$\begin{array}{rcl}
 \text{Interest on \$1 for 3 years} & = & \$0.18 \\
 \text{Interest on \$1 for 28 days} & = & 4\frac{1}{2}
 \end{array}$$

$$\text{Therefore interest on \$1 for 3 years 28 days} = \$0.184\frac{1}{2}$$

$$\text{Then } \$0.184\frac{1}{2} \times 742.63 = \$137.139.$$

(11)

$$\begin{array}{r}
 = \$0.275 \\
 = \quad 2\frac{3}{4} \\
 \hline
 = \$0.277\frac{3}{4}
 \end{array}$$

$$\begin{array}{rcl}
 \text{Interest on \$1 for 7 years 4 months} & = & \$0.44 \\
 \text{Interest on \$1 for 11 days} & = & 1\frac{1}{2}
 \end{array}$$

$$\text{Therefore interest on \$1 for 7 years 4 m'ths 11 days} = \$0.441\frac{1}{2}$$

$$\text{Then } \$0.441\frac{1}{2} \times 200 = \$88.366 + \$200 = \$288.366.$$

(12)

$$\begin{array}{r}
 = \$0.455 \\
 = \quad 3\frac{3}{4} \\
 \hline
 = \$0.458\frac{3}{4} \\
 3592.9877.
 \end{array}$$

$$\begin{array}{rcl}
 \text{Interest on \$1 for 9 years 3 months} & = & \$0.555 \\
 \text{Interest on \$1 for 9 days} & = & 1\frac{1}{2}
 \end{array}$$

$$\text{Therefore interest on \$1 for 9 years 3 months 9 days} = \$0.556\frac{1}{2}$$

$$\text{Then } \$0.5565 \times 743.63 = \$413.830095 + \$743.63 = \$1157.460095.$$

EXERCISE 108—Page 256.

(1)

$$\begin{array}{r}
 = \$0.585 \\
 = \quad 1\frac{1}{2} \\
 \hline
 = \$0.586\frac{1}{2}
 \end{array}$$

$$\text{Interest on \$1 at 6 per cent. for given time} = \$0.526\frac{3}{4}.$$

$$\text{Interest on \$1234.56 at 6 per cent. for given time} = \$0.526\frac{3}{4} \times 1234.56 = \$650.2016.$$

$$\text{Hence interest on \$1234.56 at 7 per cent. for given time} = \$650.2016 + \text{one sixth of } \$650.2016 = \$758.5685.$$



(2)

Interest on \$1 at 6 per cent. for given time = \$0.126 $\frac{1}{2}$ .

Interest on \$9876.54 at 6 per cent. for given time = \$0.126 $\frac{1}{2}$   $\times$  9876.54 = \$1252.67449.

Hence interest on \$9876.54 at 3 per cent. for given time = \$1252.67449  $\div$  2 = \$626.337245.

(3)

Interest on \$1 at 6 per cent. for given time = \$0.216 $\frac{1}{2}$ .

Interest on \$715.30 at 6 per cent. for given time = \$0.216 $\frac{1}{2}$   $\times$  715.30 = \$154.98166.

Hence interest on \$715.30 at 8 per cent. for given time = \$154.98166 + one third of \$154.98166 = \$206.6422.

(4)

Interest on \$1 at 6 per cent. for given time = \$0.141 $\frac{1}{2}$ .

Interest on \$555.55 at 6 per cent. for given time = \$0.141 $\frac{1}{2}$   $\times$  555.55 = \$78.51773.

Hence interest on \$555.55 at 12 per cent. for given time = \$78.51773  $\times$  2 = \$157.03546; amount = \$157.03546 + \$555.55 = \$712.58546.

(5)

Interest on \$1 at 6 per cent. for given time = \$0.016 $\frac{1}{2}$ .

Interest on \$7766.55 at 6 per cent. for given time = \$0.016 $\frac{1}{2}$   $\times$  7766.55 = \$129.4425.

Hence interest on \$7766.55 at 5 per cent. for given time = \$129.4425 — one sixth of \$129.4425 = \$107.86875.

Amount = \$107.86875 + \$7766.55 = \$7874.41875.

(6)

Interest on \$1 at 6 per cent. for given time = \$0.521 $\frac{1}{2}$ .

Interest on \$500 at 6 per cent. for given time = \$0.521 $\frac{1}{2}$   $\times$  500 = \$260.666 $\frac{1}{2}$ .

Hence interest on \$500 at 16 per cent. for given time = \$260.666 $\frac{1}{2}$   $\times$  2 $\frac{1}{2}$  = \$695.111; amount = \$695.111 + \$500 = \$1195.111.

(7)

Interest on \$1 at 6 per cent. for given time = \$0.206 $\frac{1}{2}$ .

Interest on \$576 at 6 per cent. for given time = \$0.206 $\frac{1}{2}$   $\times$  576  
= \$118.752.

Hence interest on \$576 at 5 per cent. for given time = \$118.752  
—one sixth of \$118.752 = \$98.96.

(8)

Interest on \$1 at 6 per cent. for given time = \$0.151 $\frac{5}{6}$ .

Interest on \$2478.91 at 6 per cent. for given time = \$0.151 $\frac{5}{6}$   $\times$   
2478.91 = \$376.38116.

Hence interest on \$2478.91 at 4 $\frac{1}{2}$  per cent. for given time =  
\$376.38116 — one fourth of \$376.38116 = \$282.285.

(9)

From May 9th to December 11th = 216 days. Interest on \$1 at  
6 per cent. for 216 days = \$0.036.

Interest on \$780 at 6 per cent. for 216 days = \$0.036  $\times$  780 =  
\$28.08.

(10)

From August 16th 1851 to June 19th 1852 = 308 days.

Interest on \$1 at 6 per cent. for given time = \$0.051 $\frac{1}{2}$ .

Interest on \$1830.63 at 6 per cent. for given time = \$0.051 $\frac{1}{2}$   $\times$   
1830.63 = \$93.97234.

Hence interest on \$1830.63 at 7 per cent. for given time =  
\$93.97234 + one sixth of \$93.97234 = \$109.63439.

(11)

From September 3rd 1858 to January 9th 1859 = 128 days.

Interest on \$1 at 6 per cent. for given time = \$0.021 $\frac{1}{2}$ .

Interest on \$6200 at 6 per cent. for given time = \$0.021 $\frac{1}{2}$   $\times$  6200  
= \$132.266.

Amount = \$132.266 + \$6200 = \$6332.266.

## EXERCISE 109.—Page 258.

(1)

From June	2nd to July	17th there are 45 days.
" July	17th to October	6th " 81 "
" October	6th to December	11th " 66 "
" December	11th to March	29th " 109 "
" March	29th to October	7th " 192 "

Whole sum \$1217·30 for 45 days = \$54778·50 for 1 day.  
 1st endorsement 207·80

Balance \$1009·50 for 81 days = \$81769·50 for 1 day.  
 2nd endorsement 209·60

Balance \$799·90 for 66 days = \$52793·40 for 1 day.  
 3rd endorsement 320·90

Balance \$479·00 for 109 days = \$52211·00 for 1 day.  
 4th endorsement 421·83

Balance \$57·17 for 192 days = \$10976·64 for 1 day.

Whole interest = that of \$252529·04 for 1 day.

Interest on \$252529·04 at 6 per cent. for 1 year = \$15151·7424.

Hence interest for 1 day =  $\$15151\cdot7424 \div 365 = \$41\cdot5116$ .

Then interest due = \$41·5116

Balance on Note = \$57·17

Principal and interest due = \$98·6816

(2)

From 17th June to 5th September there are 80 days.

" 5th September to 7th December " 93 "

" 7th December to 11th June " 186 "

" 11th June to 7th February " 241 "

" 7th February to 19th December " 315 "

" 19th December to 1st May " 133 "

Whole sum \$7348.25 for 80 days = \$587860.00 for 1 day.  
1st endorsement 2463.80

Balance \$4884.45 for 93 days = \$454253.85 for 1 day.  
2nd endorsement 392.20

Balance \$4492.25 for 186 days = \$835558.50 for 1 day.  
3rd endorsement 982.20

Balance \$3540.05 for 241 days = \$845922.05 for 1 day.  
4th endorsement 2842.90

Balance \$667.15 for 315 days = \$210152.25 for 1 day.  
5th endorsement 317.23

Balance \$349.92 for 133 days = \$46539.36 for 1 day.

Whole interest = that of \$2980286.01 for 1 day.

Interest on \$2980286.01 at 8 per cent. for 1 year = \$238422.8808.  
Hence interest for 1 day =  $\$238422.8808 \div 365 = \$653.2133$ .

Then interest due = \$653.2133

Balance on Note = \$349.92

Principal and interest due = \$1003.1333

## EXERCISE 110.—Page 259.

(1)

\$1800	Principal.
108	Interest for 1st year.
<hr/>	
\$1908	Amount for 1 year = principal for 2nd year.
114.48	Interest for 2nd year.
<hr/>	
\$2022.48	Amount for 2 years = principal for 3rd year.
121.3488	Interest for 3rd year.
<hr/>	
\$2143.8288	Amount for 3 years = principal for 4th year.
128.629728	Interest for 4th year.
<hr/>	
\$2272.458528	Amount for 4 years = principal for 5th year.
136.347511	Interest for 5th year.
<hr/>	
\$2408.806039	Amount for 5 years.
1800	Given Principal.
<hr/>	
\$608.806 =	Compound interest required.

(2)

\$700	Principal.
49	Interest for 1st half year.
<hr/>	
\$749	Amount for 1 half y. = principal for 2nd half y.
52.43	Interest for 2nd half year.
<hr/>	
\$801.43	Amount for 1 year = principal for 3rd half y.
56.1001	Interest for 3rd half year.
<hr/>	
\$857.5501	Amount for 1½ years = principal for 4th half y.
60.027107	Interest for 4th half year.
<hr/>	
\$917.557207	Amount for 2 years = principal for 5th half y.
64.229004	Interest for 5th half year.
<hr/>	
\$981.786211	Amount for 2½ years = principal for 6th half y.
68.725034	Interest for 6th half year.
<hr/>	
\$1050.511245	Amount for 3 years = principal for 7th half y.
73.535787	Interest for 7th half year.
<hr/>	
\$1124.047032	Amount for 3½ years.
700	Given Principal.
<hr/>	
\$424.047 =	Compound interest required.

Exercise 110.—Page 259.

\$673  
20

\$693  
20

\$714  
21

\$735  
22

\$757  
22

\$780  
23

\$804  
24

\$828  
24

\$853  
673

\$179

\$860  
34

\$894  
35

\$930  
37

\$967  
38

\$1006  
40

\$1046  
41

\$1088  
860

\$22

(3)

	<u>\$673.40</u> 20-202	Principal. Interest for 1st quarter.
2nd year.	<u>\$693.602</u> 20-80806	Amount for 1 quar. = principal for 2nd quarter. Interest for 2nd quarter.
3rd year.	<u>\$714.41006</u> 21-4323018	Amount for 1 half y. = principal for 3rd quart. Interest for 3rd quarter.
4th year.	<u>\$735.8423618</u> 22-0752708	Amount for 3 quarters = principal for 4th quar. Interest for 4th quarter.
5th year.	<u>\$757.9176326</u> 22-7375289	Amount for 1 year = principal for 5th quarter. Interest for 5th quarter.
	<u>\$780.6551615</u> 23-4196548	Amount for 5 quarters = principal for 6th quar. Interest for 6th quarter.
	<u>\$804.0748163</u> 24-1222444	Amount for 3 half y. = principal for 7th quarter. Interest for 7th quarter.
	<u>\$828.1970607</u> 24-8459118	Amount for 7 quarters = principal for 8th qr. Interest for 8th quarter.
2nd half y.	<u>\$853.0429</u> = 673.40	Amount for 2 years required. Given Principal.
rd half y.	<u>\$179.6429</u> =	Compound Interest required.

(4)

	<u>\$860</u> 34.4	Principal. Interest for 1st half year.
4th half y	<u>\$894.4</u> 35.776	Amount for 1 half year = principal for 2nd half y. Interest for 2nd half year.
5th half v	<u>\$930.176</u> 37-20704	Amount for 1 year = principal for 3rd half year. Interest for 3rd half year.
6th half y.	<u>\$967.38304</u> 38-69532	Amount for 3 half years = principal for 4th half y. Interest for 4th half year.
7th half y	<u>\$1006.07836</u> 40-24313	Amount for 2 years = principal for 5th half year. Interest for 5th half year.
	<u>\$1046.32149</u> 41-85285	Amount for 5 half years = principal for 6th half y. Interest for 6th half year.
	<u>\$1088.17434</u> = 860	Amount for 3 years required. Given Principal.
	<u>\$228.1743</u> =	Compound Interest required.

## EXERCISE 111—Page 261.

(1)

By the table the am't of \$1 at 6 per cent. for 11 years = \$1.8983.

Then  $\$1.8983 \times 875 = \$1661.0125 = \text{Amount.}$ 

875	Principal.
<hr style="width: 100px; margin: 0 auto;"/>	

 $\$786.0125 = \text{Interest.}$ 

(2)

By the table the am't of \$1 for the given time and rate = \$2.77247.

Then  $\$2.77247 \times 643.98 = \$1785.41523 = \text{Amount.}$ 

643.98	Principal.
<hr style="width: 100px; margin: 0 auto;"/>	

 $\$1141.43523 = \text{Interest.}$ 

(3)

By the table the am't of \$1 at 6 per cent. for 45 years = \$13.76461.

Then  $\$13.76461 \times .01 = \$137646 = \text{Amount.}$ 

.01	Principal.
<hr style="width: 100px; margin: 0 auto;"/>	

 $\$127646 = \text{Interest.}$ 

(4)

By the table the am't of \$1 for the given time and rate = \$2.28793.

Then  $\$2.28793 \times 78.2 = \$178.916 = \text{Amount.}$ 

78.2	Principal.
<hr style="width: 100px; margin: 0 auto;"/>	

 $\$100.716 = \text{Interest.}$ 

(5)

By the table the am't of \$1 for the given rate and time = \$2.40662

Then  $\$2.40662 \times 777.77 = \$1871.7968 = \text{Amount.}$ 

777.77	Principal.
<hr style="width: 100px; margin: 0 auto;"/>	

 $\$1094.0268 = \text{Interest.}$

(6)

$$£44 \text{ 5s. 9d.} = £44.2875.$$

By the table the am't of £1 at 6 per cent. for 11 years = £1.8983.

Then  $£1.8983 \times 44.2875 = £84.07096 = £84 \text{ 1 5} = \text{Amount.}$

$\underline{44 \text{ 5 9}}$  Principal.

$\underline{£39 \text{ 15 8}} = \text{Interest.}$

(7)

$$£32 \text{ 4s. 9½d.} = £32.240625.$$

By the table the amount of £1 for the given time and rate =

$£1.26532$ . Then  $£1.26532 \times 32.240625 =$

$£40.7947076 = £40 \text{ 15 10½}$  nearly = Amount.

$\underline{32 \text{ 4 9½}}$  Principal.

$\underline{£8 \text{ 11 1}} = \text{Interest.}$

EXERCISE 112—Page 262.

(1)

Amount of \$1 for 7 years at 4 per cent. = \$1.31593.

$\$7439.87 \div 1.31593 = \$5653.697.$

(2)

Amount of \$1 at 5 per cent. for 20 years = \$2.6533.

$\$9193.90 \div 2.6533 = \$3465.081.$

(3)

$$£595 \text{ 10s. 2½d.} = £595.51$$

Amount of £1 at 6 per cent. for 3 years = £1.19102.

$£595.51 \div 1.19102 = £500.$

(4)

Amount of \$1 at 6 per cent. for 7 years = \$1.50363.

$\$7111.11 \div 1.50363 = \$4729.295.$



(5)

$$£268 \text{ Os. } 4\frac{1}{2}\text{d.} = £268.02.$$

Amount of £1 at 5 per cent for 6 years = £1.3401.

$$£268.02 \div 1.3401 = £200.$$

## EXERCISE 113—Page 263.

(1)

Here  $A = \$962$ ,  $r = .04$ , and  $t = 1$ . Whence  $1 + rt = 1.04$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{962}{1.04} = \$925.$$

(2)

Here  $A = \$2202$ ,  $r = .06$ , and  $t = 5.75$ . Whence  $1 + rt = 1.345$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2202}{1.345} = \$1637.174.$$

(3)

Here  $A = \$1003.50$ ,  $r = .06$ , and  $t = \frac{3}{4}$  year. Whence  $1 + rt = 1.04$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1003.50}{1.04} = \$964.9038.$$

(4)

Here  $A = \$716$ ,  $r = .08$ , and  $t = \frac{7}{12}$  year. Whence  $1 + rt = 1.04\frac{2}{3}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{716}{1.04\frac{2}{3}} = \$684.0764.$$

(5)

Here  $A = \$1342.50$ ,  $r = .065$ , and  $t = \frac{11}{12}$  year. Whence  $1 + rt = 1.022\frac{11}{12}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1342.50}{1.022\frac{11}{12}} = \$1313.266.$$

(6)

Here  $A = \$2400$ ,  $r = .05$ , and  $t = \frac{33}{8}$  year. Whence  $1 + rt = 1.03\frac{3}{8}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2400}{1.03\frac{3}{8}} = \$2324.84.$$

(7)

Here  $A = \$2202$ ,  $r = .05$ , and  $t = .75$  year. Whence  $1 + rt = 1.0375$ .

$$\$2202 \div 1.0375 = \$2122.40963+ = \text{Present worth.}$$

$$\$2202 - \$2122.40963+ = \$79.59036 = \text{Discount.}$$

(8)

Here  $A = \$4360$ ,  $r = .06$ , and  $t = 1\frac{5}{8}$ . Whence  $1 + rt = 1.085$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{4360}{1.085} = \$4018.43317.$$

(9)

Here  $A = \$1647$ ,  $r = .06$ , and  $t = \frac{11}{12}$  year. Whence  $1 + rt = 1.055$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1647}{1.055} = \$1561.13744.$$

(10)

Here  $A = \$2000$ ,  $r = .06$ , and  $t = 3\frac{1}{8}$ . Whence  $1 + rt = 1.215$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2000}{1.215} = \$1646.09053.$$

L

(11)

Here  $A = \$2070.90$ ,  $r = .05$ , and  $t = 1\frac{1}{2}$ . Whence  $1 + rt = 1.07\frac{1}{2}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2070.90}{1.07\frac{1}{2}} = \$1918.9806.$$

$$\$2070 - \$1918.9806 = \$151.019 = \text{Discount required.}$$

(12)

Here  $A = \$970.63$ ,  $r = .08$ , and  $t = \frac{1}{2}$  year. Whence  $1 + rt = 1.07\frac{1}{2}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{970.63}{1.07\frac{1}{2}} = \$904.313.$$

(13)

Here in first case  $A = \$1512$ ,  $r = .07$ , and  $t = .5$  year. Whence  $1 + rt = 1.035$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.035} = \$1460.8695.$$

Also  $A = 1512$ ,  $r = .07$ , and  $t = 1$ . Whence  $1 + rt = 1.07$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.07} = \$1413.0841.$$

$$\$1460.8695 + \$1413.0841 = \$2873.9536 = \text{Present worth of whole amount.}$$

$$\$3024 - \$2873.9536 = \$150.0464 = \text{Discount required.}$$

(14)

Here in first case  $A = \$440$ ,  $r = .08$ , and  $t = 1.25$ . Whence  $1 + rt = 1.1$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{440}{1.1} = \$400.$$

In second case  $A = \$896$ ,  $r = .08$ , and  $t = 1.5$ . Whence  $1 + rt = 1.12$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{896}{1.12} = \$800.$$

$$\$400 + \$800 = \$1200.$$

## EXERCISE 114—Page 265.

(1)

Here the time the note has to run is 2 years 3 months 3 days.  
 Interest of \$1 at 7 per cent. for 2 yrs., 3 m., 3 days = \$0.1580 $\frac{1}{2}$ .  
 Interest of \$986 at 7 per cent. for 2 years, 3 months, 3 days =  
 $\$0.1580\frac{1}{2} \times 986 = \$155.8701$ .

(2)

Here the time the note has to run is 103 days = 3 months 13 days.  
 Interest of \$1 at 8 per cent. for 3 months 13 days = \$0.022 $\frac{1}{2}$ .  
 Interest of \$640 at 8 per cent. for 3 months, 13 days =  
 $\$0.022\frac{1}{2} \times 640 = \$14.6488$ .

(3)

Here the time the note has to run is 94 days = 3 months 4 days.  
 Interest of \$1 at 6 per cent. for 3 months 4 days = \$0.015 $\frac{1}{2}$ .  
 Interest of \$563.80 at 6 per cent. for 3 months 4 days =  
 $\$0.015\frac{1}{2} \times 563.80 = \$8.8328$  and  $\$563.80 - \$8.8328 = \$554.967$ .

## EXERCISE 115—Page 266.

(1)

Interest on \$1 for 93 days at 7 p. c. = \$0.0180 $\frac{1}{2}$ , and this taken  
 from \$1 gives a remainder of \$0.9819 $\frac{1}{2}$  = present worth of \$1.  
 Then  $\$3755 \div 0.9819\frac{1}{2} = \$3824.15$ .

(2)

Interest on \$1 for 6 months 3 days at 5 per cent. = \$0.0254 $\frac{1}{2}$   
 and this taken from \$1 gives a remainder \$0.9745 $\frac{1}{2}$  = present  
 worth of \$1.

Then  $\$1147.80 \div 0.9745\frac{1}{2} = \$1177.734$ .

(3)

Interest on \$1 for 48 days at  $3\frac{1}{2}$  per cent. =  $\$0.004\frac{1}{2}$ , and this taken from \$1 gives a remainder  $\$0.9953\frac{1}{2}$  = present worth of \$1.

$$\text{Then } \$713.90 \div 0.995\frac{1}{2} = \$717.2471.$$

## EXERCISE 116—Page 268.

(1)

$$\begin{array}{r} \$200 \times 3 = 600 \\ 150 \times 4 = 600 \\ 250 \times 6 = 1500 \\ \hline 600 \quad 600) 2700 (4\frac{1}{2} \text{ months.} \\ \underline{2400} \\ 300 \\ \underline{600} \end{array} \Bigg\} = \frac{1}{2}$$

(2)

$$\begin{array}{r} \frac{1}{2} \times 0 = 0 \\ \frac{1}{2} \times 3 = \frac{3}{2} \\ \frac{1}{2} \times 6 = 1\frac{1}{2} \\ \frac{1}{2} \times 9 = 2\frac{1}{2} \\ \hline 1 \quad 1) 4\frac{1}{2} \\ \hline 4\frac{1}{2} \text{ months.} \end{array}$$

(3)

$$\begin{array}{r} \$50 \times 2 = 100 \\ 40 \times 5 = 200 \\ 30 \times 7 = 210 \\ \hline 120 \quad 120) 510 (4\frac{1}{2} \text{ months.} \\ \underline{480} \\ 30 \\ \underline{120} \end{array} \Bigg\} = \frac{1}{2}$$

(4)

$$\begin{array}{r} \$1000 \times 0 = 0 \\ 1500 \times 1 = 1500 \\ 600 \times 3 = 1800 \\ 700 \times 5 = 3500 \\ 1400 \times 7 = 9800 \\ \hline 5200 \quad 5200) 16600 (3\frac{5}{8} \text{ months.} \\ \underline{15600} \\ 1000 \\ \underline{5200} \end{array} \Bigg\} = \frac{5}{8}$$

(5)

Six months from 15th January = 15th July, and from 1st July to 15th July there are 14 days.

Six months from 10th February = 10th August, and from 1st July to 10th August there are 40 days.

Six months from 6th March = 6th September, and from 1st July to 6th September there are 67 days.

Six months from 8th June = 8th December, and from 1st July to 8th December there are 160 days.

$$\begin{array}{r} \$3750 \times 14 = 52500 \\ 3000 \times 40 = 120000 \\ 2400 \times 67 = 160800 \\ 2250 \times 160 = 360000 \\ \hline 11400 \end{array}$$

$$11400 \quad 11400) 693300 (60\frac{3}{4} \text{ days.}$$

$$\begin{array}{r} 684000 \\ \hline 9300 \end{array}$$

$$\begin{array}{r} 9300 \\ \hline 11400 \end{array} \} = 3$$

4½ months.

Therefore the note must be made payable on the 61st day from the 1st of July, which is the 31st of August.

EXERCISE 117.—Page 269.

(1)

Whole stock : A's stock :: whole profit : A's profit.

$$1117 \times 3000$$

$$\text{That is, } \$4300 : \$3000 :: \$1117 : \frac{1117 \times 3000}{4300} = \$779.302 = \text{A's sh.}$$

$$\$1117 - \$779.302 = \$337.697 = \text{B's share.}$$

(2)

Whole stock = \$6470 + \$3780 + \$9860 = \$20110.

Whole stock : A's stock :: whole profit : A's profit,

$$7890 \times 6470$$

That is, \$20110 : \$6470 :: \$7890 :  $\frac{7890 \times 6470}{20110}$  = \$2538.453+ = A's sh.

Again, whole stock : B's stock :: whole profit : B's profit.

$$7890 \times 3780$$

That is, \$20110 : \$3780 :: \$7890 :  $\frac{7890 \times 3780}{20110}$  = \$1483.053+ = B's sh.

Lastly, whole stock : C's stock :: whole profit : C's profit.

$$7890 \times 9860$$

That is, \$20110 : \$9860 :: \$7890 :  $\frac{7890 \times 9860}{20110}$  = \$3868.493+ = C's sh.

(3)

Whole stock : B's stock :: whole gain : B's gain.

$$80 \times 120$$

That is, \$320 : \$120 :: \$80 :  $\frac{80 \times 120}{320}$  = \$30 = B's gain.

Again, whole stock : C's stock :: whole gain : C's gain.

$$80 \times 200$$

That is, \$320 : \$200 :: \$80 :  $\frac{80 \times 200}{320}$  = \$50 = C's share.

(4)

Whole stock : B's stock :: whole gain : B's gain.

$$728 \times 1200$$

That is, \$2800 : \$1200 :: \$728 :  $\frac{728 \times 1200}{2800}$  = \$312 = B's gain.

Again, whole stock : C's stock :: whole gain : C's gain.

$$728 \times 1600$$

That is, \$2800 : \$1600 :: \$728 :  $\frac{728 \times 1600}{2800}$  = \$416 = C's gain.

(5)

Whole stock : B's stock :: whole amount to be divided : B's share

$$\text{That is, } \$3 : \$2 :: \$100 : \frac{100 \times 2}{3} = \$66.66\frac{2}{3} = \text{B's share.}$$

Again, whole st'k : C's st'k :: whole amo't to be divided : C's sh'e

$$\text{That is, } \$3 : \$1 :: \$100 : \frac{100 \times 1}{3} = \$33.33\frac{1}{3} = \text{C's share.}$$

(6)

$$£1400 : £500 :: £1100 : \frac{1100 \times 500}{1400} = £392\frac{2}{7} = \text{B's share.}$$

$$£1100 - £392\frac{2}{7} = £707\frac{1}{7} = \text{C's share.}$$

(7)

casks. casks.  $180 \times 200$

$$900 : 200 :: 180 : \frac{180 \times 200}{900} = 40 \text{ casks} = \text{B's loss.}$$

$$900 : 300 :: 180 : \frac{180 \times 300}{900} = 60 \text{ casks} = \text{C's loss.}$$

$$180 - (40 + 60) = 80 \text{ casks} = \text{D's loss.}$$

(8)

$$\$1800 : \$800 :: \$100 : \frac{100 \times 800}{1800} = \$44.44\frac{4}{9} = \text{B's share.}$$

$$\$1800 : \$600 :: \$100 : \frac{100 \times 600}{1800} = \$33.33\frac{1}{3} = \text{C's share.}$$

$$\$44.44\frac{4}{9} + \$33.33\frac{1}{3} = \$77.77\frac{7}{9}, \text{ and } \$100 - \$77.77\frac{7}{9} = \$22.22\frac{2}{9} = \text{D's share.}$$



(9)

$$6 : 1 :: 120 : \frac{120 \times 1}{6} = 20$$

$$6 : 2 :: 120 : \frac{120 \times 2}{6} = 40$$

$$6 : 3 :: 120 : \frac{120 \times 3}{6} = 60$$

(10)

$$\text{Whole loss} = \$900 - \$540 = \$360.$$

$$8 : 1 :: \$360 : \frac{360}{8} = \$45 = \text{B's loss.}$$

$$8 : 2 :: \$360 : \frac{360 \times 2}{8} = \$90 = \text{C's loss.}$$

$$\$45 + 90 = \$135, \text{ and } \$360 - 135 = \$225 = \text{D's loss.}$$

(11)

$$\$12 : \$6 :: \$1320 : \frac{1320 \times 6}{12} = \$660 = \text{B's gain.}$$

$$\$12 : \$4 :: \$1320 : \frac{1320 \times 4}{12} = \$440 = \text{C's gain.}$$

$$\$12 : \$2 :: \$1320 : \frac{1320 \times 2}{12} = \$220 = \text{D's gain.}$$

(12)

$$£35 + £29 = £64, \text{ and } £110 - £64 = £46 = \text{D's profit.}$$

$$\text{D's profit} : \text{B's profit} :: \text{D's stock} : \text{B's stock.}$$

$$\text{That is, } £46 : £35 :: £1090 : \frac{1090 \times 35}{46} = £829 \text{ 6s. } 11\frac{1}{2}\text{d.} = \text{B's st.}$$

$$\text{Again, D's profit} : \text{C's profit} :: \text{D's stock} : \text{C's stock.}$$

$$\text{That is, } £46 : £29 :: £1090 : \frac{1090 \times 29}{46} = £687 \text{ 3s. } 5\frac{1}{2}\text{d.} = \text{C's st.}$$

## EXERCISE 118.—Page 271.

(1)

$$\left. \begin{array}{l} \$357 \times 5 = \$1785 \text{ for one month} \\ 371 \times 7 = 2597 \text{ for one month} \\ 154 \times 11 = 1694 \text{ for one month} \end{array} \right\} = \$6076 \text{ for one month.}$$

$$\$6076 : \$1785 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1785}{6076} = \$102$$

$$\$6076 : \$2597 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 2597}{6076} = \$148 \cdot 40.$$

$$\$6076 : \$1694 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1694}{6076} = \$96 \cdot 80$$

(2)

$$\left. \begin{array}{l} 40 \times 6 = 240 \text{ for one month} \\ 30 \times 5 = 150 \text{ for one month} \\ 50 \times 1 = 50 \text{ for one month} \end{array} \right\} = 440 \text{ for one month.}$$

$$440 : 240 :: \$160 : \frac{160 \times 240}{440} = \$87 \cdot 27_{11}^3; \text{ B's share.}$$

$$440 : 150 :: \$160 : \frac{160 \times 150}{440} = \$54 \cdot 54_{11}^6; \text{ C's share.}$$

$$440 : 50 :: \$160 : \frac{160 \times 50}{440} = \$18 \cdot 18_{11}^2; \text{ D's share.}$$

(3)

$$\left. \begin{array}{l} £150 \times 6 = £900 \text{ for one month} \\ 200 \times 3 = 600 \text{ for one month} \\ 125 \times 16 = 2000 \text{ for one month} \end{array} \right\} = £3500 \text{ for one month.}$$

$$£3500 : £900 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 900}{3500} = £75.$$

$$£3500 : £600 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 600}{3500} = £50.$$

$$£3500 : £2000 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 2000}{3500} = £166 \text{ 13s. 4d.}$$

(4)

$$\begin{array}{l} \$4000 \times 12 = \$48000 \text{ for one month} \\ 3000 \times 15 = 45000 \text{ for one month} \\ 5000 \times 8 = 40000 \text{ for one month} \end{array} \} = \$133000 \text{ for one month}$$

$$\$133000 : \$48000 :: \$665 : \frac{665 \times 48000}{133000} = \$240; \text{ B's share.}$$

$$\$133000 : \$45000 :: \$665 : \frac{665 \times 45000}{133000} = \$225; \text{ C's share.}$$

$$\$133000 : \$40000 :: \$665 : \frac{665 \times 40000}{133000} = \$200; \text{ D's share.}$$

(5)

$$\begin{array}{l} 56 \times 12 = 672 \text{ for one day} \\ 64 \times 15 = 960 \text{ for one day} \\ 80 \times 18 = 1440 \text{ for one day} \end{array} \} = 3072 \text{ for one day.}$$

$$3072 : 672 :: \$320 : \frac{320 \times 672}{3072} = \$70 = \text{rent to be paid by 1st troop.}$$

$$3072 : 960 :: \$320 : \frac{320 \times 960}{3072} = \$100 = \text{" " " 2nd "}$$

$$3072 : 1440 :: \$320 : \frac{320 \times 1440}{3072} = \$150 = \text{" " " 3rd "}$$

(7)

$$\text{Sum of profits} = 240 + 800 + 400 = \$1440.$$

$$\text{Whole profit : A's profit :: Whole stock for 1 m. : A's st. for 1 m.}$$

$$\text{That is, } 1440 : 240 :: 34560 : \frac{34560 \times 240}{1440} = 5760 = \text{A's stock}$$

for one month. Hence, since A's stock was in for 6 months, it will be  $5760 \div 6 = \$960$

(Continued on next page.)

(7 Continued.)

Whole profit : B's profit :: Whole stock for 1 m. : B's st. for 1 m.

$$34560 \times 800$$

$$1440 : 800 :: 34560 : \frac{34560 \times 800}{1440} = 19200 = \text{B's stock for one}$$

month. And, since B's stock was in for 12 months,  $19200 \div 12 = \$1600$  will be his stock.

Whole profit : C's profit :: whole stock for 1 m. : C's st. for 1 m.

$$34560 \times 400$$

$$1440 : 400 :: 34560 : \frac{34560 \times 400}{1440} = \$9600 = \text{C's stock for one}$$

month, and hence his stock will be  $\$9600 \div 15 = \$640$ .

(8)

A's profit was \$240 for 6 months = \$40 for 1 month.

B's profit was \$800 for 12 months = \$66 $\frac{2}{3}$  for 1 month.C's profit was \$400 for 15 months = \$26 $\frac{2}{3}$  for 1 month.Sum of profits for 1 month = \$133 $\frac{1}{3}$ 

Whole profit for 1 m. : A's profit for 1 m. :: whole stock : A's st.

$$3200 \times 40$$

$$133\frac{1}{3} : 40 :: 3200 : \frac{3200 \times 40}{133\frac{1}{3}} = \$960 = \text{A's stock.}$$

$$3200 \times 66\frac{2}{3}$$

$$133\frac{1}{3} : 66\frac{2}{3} :: 3200 : \frac{3200 \times 66\frac{2}{3}}{133\frac{1}{3}} = \$1600 = \text{B's stock.}$$

$$3200 + 26\frac{2}{3}$$

$$133\frac{1}{3} : 26\frac{2}{3} :: 3200 : \frac{3200 + 26\frac{2}{3}}{133\frac{1}{3}} = \$640 = \text{C's stock.}$$

## EXERCISE 119—Page 275.

(1)

\$0.12 $\frac{1}{2}$  = selling price.

\$0.09 = buying price.

\$0.03 $\frac{1}{2}$  = gain per lb.

$$\$0.03\frac{1}{2} \times 317 = \$11.095.$$

(2)

\$1.00 = selling price

\$0.71 = buying price.

\$0.32 $\frac{1}{2}$  = gain per bushel

$$\$0.32\frac{1}{2} \times 213 = \$69.485.$$

(3)

$\$0.15 \times 317 \times 13 = \$618.15 = \text{cost of 13 barrels at } \$0.15 \text{ per lb.}$   
 $\$735 - 618.15 = \$116.85 \text{ gain.}$

(4)

$\$3.15 \times 22 \times 17 = \$1178.10 = \text{price of 17 kegs at } \$3.15 \text{ per gal.}$   
 $\$0.37\frac{1}{2} \times 1178.1 = \$441.7875 = \text{ad valorem duty.}$   
 $\$1178.10 + \$441.7875 + \$26.33 = \$1646.2175 = \text{whole cost.}$   
 $\$1646.2175 - \$1625 = \$21.2175 \text{ loss.}$

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EXERCISE 120—Page 276.

(1)

Here for every \$1 I expend I wish to receive \$1.30, and hence the selling price will be  $\$3.25 \times 1.30 = \$4.22\frac{1}{2}$ .

(2)

Here for every \$1 I expend I wish to receive \$1.05, and hence the selling price will be  $\$1.05 \times 13420 = \$14091$ .

(3)

Here for every \$1 I expend I desire to receive \$1.15, and hence the selling price will be  $\$1.15 \times .11 = \$0.1265 = 12\frac{1}{2}$  cents.

(4)

Here for every \$1 I expend I wish to receive \$1.23, and hence the selling price will be  $\$1.23 \times 15.25 = \$18.75\frac{1}{2}$ .

(5)

Here for every \$1 I expend I am willing to receive \$0.89, and hence the selling price will be  $\$0.89 \times 7890 = \$7022.10$ .

## EXERCISE 121—Page 277.

(1)

Here the whole gain is  $\$0.87\frac{1}{2} - \$0.60 = \$0.27\frac{1}{2}$ .

That is,  $\$0.60$  gains  $\$0.27\frac{1}{2}$ , and therefore 1 cent gains  $\frac{27\frac{1}{2}}{60} =$

$$\frac{11}{12} = \frac{1}{4}\frac{1}{2} \text{ of a cent.}$$

And hence, the gain per cent  $= \frac{1}{4}\frac{1}{2} \times 100 = \frac{1125}{100} = 45\frac{1}{2}$  per cent.

(2)

Here the loss on each lb. is 2 cents.

That is, every 13 cents invested gives a loss of 2 cents.

Therefore every cent invested loses  $\frac{1}{13}$  of 2  $= \frac{2}{13}$  cents.

And hence, the loss per cent  $= \frac{2}{13} \times 100 = \frac{200}{13} = 15\frac{1}{13}$  per c.

(3)

Here the gain on each barrel is  $\$1.60$ .

That is, every  $\$6.20$  invested gives a gain of  $\$1.60$ .

Therefore every  $\$1$  invested gains  $\frac{160}{620}$  of 100  $= \frac{80}{31}$  of a \$.

And hence, the gain per cent  $= \frac{80}{31} \times 100 = \frac{8000}{31} = 258\frac{2}{31}$  p. c.

(4)

Here the gain on each yard is 35 cents.

That is, every  $\$2.75$  invested gives a gain of 35 cents.

Therefore every  $\$1$  invested gains  $\frac{35}{275}$  of 100  $= \frac{14}{11}$  of a dollar.

And hence the gain per cent  $= \frac{14}{11} \times 100 = \frac{1400}{11} = 127\frac{4}{11}$  p. c.

(5)

Here the gain on every bushel is 9 cents.

That is, every 47 cents invested gives a gain of 9 cents.

Therefore every cent invested gains  $\frac{9}{47}$  of 100  $= \frac{900}{47}$  cents.

And hence the gain per cent  $= \frac{900}{47} \times 100 = \frac{90000}{47} = 1914\frac{24}{47}$  p. c.

(6)

Here the loss on each lb. is  $1\frac{1}{2}$  cents.

That is every 12 cents invested gives a loss of  $1\frac{1}{2}$  cents.

Therefore every cent invested loses  $\frac{1}{2}$  of  $1\frac{1}{2} = \frac{1}{2}$  of a cent.

And hence, the loss per cent  $= \frac{1}{2} \times 100 = \frac{100}{2} = 50$  p. c.

(7)

Here the whole gain is  $\$127 - \$93 = \$34$ .

That is,  $\$93$  gain  $\$34$ , and therefore  $\$1$  gains  $\frac{34}{93}$  of a dollar.

Hence, gain per cent  $= \frac{34}{93} \times 100 = \frac{3400}{93} = 36\frac{52}{93}$  per cent.

(8)

Here the loss is  $\$6742.50 - \$6000 = \$742.50$ .

That is,  $\$6742.50$  loses  $\$742.50$ , and therefore  $\$1$  loses  $\frac{742.50}{6742.50}$  of a dollar.

Hence loss per cent  $= \frac{742.50}{6742.50} \times 100 = \frac{74250}{67425} = 11\frac{11}{67425}$  per cent.

(9)

Here  $\$5700 + \$275 + \$1987.32 = \$7962.32 =$  whole sum expended.

Whole gain  $= \$8750 - \$7962.32 = \$787.68$ .

That is,  $\$7962.32$  gains  $\$787.68$ , and therefore  $\$1$  gains  $\frac{787.68}{7962.32}$  of a \$.

Hence gain per cent  $= \frac{787.68}{7962.32} \times 100 = \frac{78768}{796232} = 9.89$  or nearly 10 per cent.

(10)

$\$4.25 \times 723 = \$3072.75 =$  price of 723 yds. @  $\$4.25$ .

$\$3072.75 \times .07 = \$215.0925 =$  amount for Insurance.

$\$3072.75 \times .22 = \$676.005 =$  amount for ad valorem duty.

Then whole cost  $= \$3072.75 + \$215.0925 + \$23.70 + \$2.70 + \$3.16 + \$676.005 = \$3993.4075$ .

Whole gain  $= \$5270 - \$3993.4075 = \$1276.5925$ .

That is,  $\$3993.4075$  gains  $\$1276.5925$   $\therefore$   $\$1$  gains  $\frac{1276.5925}{3993.4075}$  of a \$.

Hence gain per cent  $= \frac{1276.5925}{3993.4075} \times 100 = 31.96749$  or nearly 32 per cent.

## EXERCISE 122—Page 278.

(1)

Loss on \$1 is 4 cents, or for every \$1 paid I receive \$0.96.  
Hence cost =  $\$24.60 \div 0.96 = \$25.625$ .

(2)

Loss on \$1 is 10 cents, or for every \$1 paid he receives \$0.90.  
Hence cost =  $\$2360 \div .90 = \$2622.22$ .

(3)

Gain on \$1 is 11 cents, or for every \$1 paid he receives \$1.11.  
Hence cost =  $\$7400 \div 1.11 = \$6666.666$ .

(4)

Gain on \$1 is 17 cents, or for every \$1 paid he receives \$1.17.  
 $\$117 : \$100 :: \$3789.40 : \frac{3789.40 \times 100}{117} = \$3238.803$ .

(5)

Loss on \$1 is 13 cents, or for every \$1 paid I receive \$0.87.  
 $\$87 : \$100 :: \$2740 : \frac{2740 \times 100}{87} = \$3149.425$ .

## EXERCISE 123—Page 279.

(1)

\$2 gains 50 cents.

Hence  $\$0.50 : \$0.10 :: \$2.00 : \frac{2.00 \times 10}{50} = 40 \text{ cents}$ .



(2)

$$\$2.00 : \$2.80 :: \$2.50 : \frac{2.50 \times 2.80}{2.00} = \$3.50.$$

(3)

8 cents gain 5 cents in 9 months.

$$\text{Hence } 9 \text{ mo's} : 6 \text{ mo's} :: 5 \text{ cents} : \frac{5 \times 6}{9} = 3\frac{1}{3} = \text{gain for } 6 \text{ mo's.}$$

$$8 \text{ cts.} : 12 \text{ cts.} :: 3\frac{1}{3} : \frac{3\frac{1}{3} \times 12}{8} = 5 \text{ cts. gain on } 12 \text{ cts. for } 6 \text{ mo's.}$$

Therefore  $12 + 5 = 17$  cents = his selling price.

(4)

$$\$1.60 : \$1.85 :: \$0.55 : \frac{1.85 \times .55}{1.60} = \$0.6359375 = \text{what L}$$

ought to get in order to sell at the same profit as K.

But L only gets 60 cents, therefore K has the advantage.

$$70 \text{ yds. of cloth at } \$1.85 = \$1.85 \times 70 = \$129.50.$$

$$\$129.50 \div \$ .60 = 215\frac{1}{2}.$$

(5)

$$5 \text{ tons of butter at } \$102 = \$102 \times 5 = \$510$$

$$10\frac{1}{2} \text{ tons of tallow at } \$135 = \$135 \times 10\frac{1}{2} = \$1417.50$$

$$\text{Total value} = \$1927.50$$

$$\text{Deduct ready money, } \$600.30$$

$$\$1327.20$$

$$\$1327.20 \div \$4.20 = 316 \text{ barrels.}$$

## EXERCISE 124—Page 281.

(1)

$$7 \text{ oz.} \times 22 = 154 \text{ carats.}$$

$$12\frac{1}{2} \text{ " } \times 21 = 262\frac{1}{2} \text{ "}$$

$$17 \text{ " } \times 9 = 153 \text{ "}$$

$$\begin{array}{r} 36\frac{1}{2} \\ 36\frac{1}{2} 569\frac{1}{2} \text{ "} \\ 2 \quad 2 \text{ "} \end{array}$$

$$73) 1139(15\frac{1}{3} \text{ carats.}$$

73

409

365

44

(2)

$$2 \text{ gallons @ } 14\text{s.} = 28\text{s.}$$

$$1 \text{ " @ } 12\text{s.} = 12\text{s.}$$

$$2 \text{ " @ } 9\text{s.} = 18\text{s.}$$

$$4 \text{ " @ } 8\text{s.} = 32\text{s.}$$

$$9) 90\text{s.}$$

10s.

(3)

$$15 \text{ bushels @ } \$1.20 = \$18.00$$

$$30 \text{ " @ } \$1.50 = \$45.00$$

$$60 \text{ " @ } \$1.10 = \$66.00$$

$$83 \text{ " @ } \$1.75 = \$145.25$$

188

$$188) \$274.25(\$1.458$$

188

86.2

75.2

11.05

9.40

1.650

1.504

.146

M

(4)

$$\begin{array}{rcl}
 12 \text{ lbs. @ } 50 \text{ cents} & = & 600 \text{ cents.} \\
 16 \text{ " @ } 72 \text{ " } & = & 1152 \text{ " } \\
 22 \text{ " @ } 65 \text{ " } & = & 1430 \text{ " } \\
 18 \text{ " @ } 85 \text{ " } & = & 1530 \text{ " } \\
 100 \text{ " @ } 42 \text{ " } & = & 4200 \text{ " } \\
 \hline
 168 & & 168)8912 \text{ cents (53}\frac{1}{2}\text{ cents.} \\
 & & \underline{840} \\
 & & 512 \\
 & & \underline{504} \\
 & & 8 \\
 & & \underline{168} \} = \frac{1}{2}.
 \end{array}$$

## EXERCISE 125—Page 283.

(1)

Prices. Differences. Prices.

$$125 = \left\{ \begin{array}{l} 160 - 35 \quad \overbrace{\quad\quad\quad} \quad 15 + 110 \\ 140 - 15 \quad \quad \quad 25 + 100 \end{array} \right\} = 125$$

Prices. Differences. Prices.

$$125 = \left\{ \begin{array}{l} 160 - 35 \quad \overbrace{\quad\quad\quad} \quad 15 + 110 \\ 140 - 15 \quad \quad \quad 25 + 100 \end{array} \right\} = 125.$$

Ans. 35 bush. @ \$1.10, 15 @ \$1.60, 15 @ \$1, and 25 @ \$1.40.  
 35 bush. @ \$1.00, 15 @ \$1.40, 15 @ \$1.10, and 25 @ \$1.60.

(2)

Prices. Differences. Prices.

$$45 = \left\{ \begin{array}{l} 60 - 15 \quad 3 + 42 \\ 50 - 5 \quad 7 + 38 \\ \quad \quad 15 + 30 \end{array} \right\} = 45.$$

Prices. Differences. Prices.

$$45 = \left\{ \begin{array}{l} 60 - 15 \quad 3 + 42 \\ 50 - 5 \quad 7 + 38 \\ \quad \quad 15 + 30 \end{array} \right\} = 45.$$

*Ans.* 15 quarts @ 42 cents, 3 @ 60 cents, 5 @ 38 cents, 5 @ 30 cents, and  $7 + 15 = 22$  @ 50 cents.

15 quarts @ 38 cents, 3 @ 50 cents, 5 @ 42 cents, 15 @ 30 cents, and  $7 + 15 = 22$  @ 60 cents.

(3)

Prices. Differences. Prices.

$$12\frac{1}{2} = \left\{ \begin{array}{l} 18 - 5\frac{1}{2} \\ 17 - 4\frac{1}{2} \\ 16 - 3\frac{1}{2} \\ 15 - 2\frac{1}{2} \\ 14 - 1\frac{1}{2} \end{array} \right\} = 12\frac{1}{2}.$$

*Ans.*  $\frac{1}{2}$  lb. @ 18 cents,  $\frac{1}{2}$  @ 17 cents,  $\frac{1}{2}$  @ 16 cents,  $2\frac{1}{2}$  @ 15 cents,  $2\frac{1}{2}$  @ 14 cents,  $5\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} = 13\frac{1}{2}$  @ 12 cents, and  $2\frac{1}{2} + 1\frac{1}{2} = 4$  @ 10 cents.

(4)

Prices. Differences. Prices.

$$10 = \left\{ \begin{array}{l} 13 - 3 \quad 3 + 7 \\ 12 - 2 \quad 5 + 5 \end{array} \right\} = 10.$$

*Ans.* 3 lbs. @ 7d., 3 @ 13d., 2 @ 5d., and 5 @ 12d.

## EXERCISE 126—Page 284.

(1)

By Case I we find that 17 quarts @ 31 cents, 6 @ 16 cents, 6 @ 19 cents, and 6 @ 23 cents will make a mixture worth 25 cents per quart.

$$\text{Therefore } 17 \text{ qts.} : 87 \text{ qts.} :: 6 \text{ qts.} : \frac{6 \times 87}{17} = 30\frac{1}{2} \text{ quarts @}$$

16 cents, and as there are 6 lbs. at each of the other prices, the same statement may be used, and the answer is therefore  $30\frac{1}{2}$  quarts @ each price.

(2)

To produce a mixture worth 75 cents per bushel, we require 45 bushels @ 80 cents, 5 @ 37 cents, and 5 @ 68 cents.

$$\text{Therefore } 45 \text{ bush.} : 70 \text{ bush.} :: 5 \text{ bush.} : \frac{5 \times 70}{45} = 7\frac{2}{3} \text{ bush.}$$

oats @ 37 cents.

$$45 \text{ bush.} : 70 \text{ bush.} :: 5 \text{ bush.} : \frac{5 \times 70}{45} = 7\frac{2}{3} \text{ bush.}$$

barley @ 68 cents.

(3)

To produce a mixture worth 1s. per lb., we require  $1\frac{1}{2}$  lbs. @ 16d.,  $1\frac{1}{2}$  @ 14d., and 6 @ 10½d.

Then  $1\frac{1}{2}$  lbs. : 50 lbs. ::  $1\frac{1}{2}$  lbs. : 50 lbs. brass @ 14d.

$1\frac{1}{2}$  lbs. : 50 lbs. :: 6 lbs. : 200 lbs. pewter @ 10½d.

(4)

By Case I we find that 1 oz. of 20 carats fine, 1 of 21 carats fine and 3 of 23 carats fine, will make a mixture 22 carats fine.

Then 1 oz. : 30 oz. :: 1 oz. : 30 oz. of 21 carats fine.

1 oz. : 30 oz. :: 3 oz. : 90 oz. of 23 carats fine.

## EXERCISE 127—Page 285.

(1)

To produce a mixture worth \$1.40 per lb., we require 20 lbs. @ \$1.00, 40 @ \$1.20, 40 @ \$1.60, and 20 @ \$1.80. But all of these added together, will make 120 lbs.

$$\begin{array}{ccccccc} & \text{lbs.} & \text{lbs.} & \text{lbs.} & \text{lbs.} & & \\ & & & & & & \\ \text{Therefore } 120 : 20 :: 168 : & \frac{168 \times 20}{120} & = & 28 \text{ lbs., the required} & & & \\ & & & \text{quantity @ \$1.00.} & & & \end{array}$$

$$\begin{array}{ccccccc} & & & & \text{lbs.} & & \\ & & & & & & \\ 120 : 40 :: 168 : & \frac{168 \times 40}{120} & = & 56 \text{ lbs., the required} & & & \\ & & & \text{quantity @ \$1.20.} & & & \end{array}$$

$$\begin{array}{ccccccc} & & & & \text{lbs.} & & \\ & & & & & & \\ 120 : 40 :: 168 : & \frac{168 \times 40}{120} & = & 56 \text{ lbs., the required} & & & \\ & & & \text{quantity @ \$1.60.} & & & \end{array}$$

$$\begin{array}{ccccccc} & & & & \text{lbs.} & & \\ & & & & & & \\ 120 : 20 :: 168 : & \frac{168 \times 20}{120} & = & 28 \text{ lbs., the required} & & & \\ & & & \text{quantity @ \$1.80.} & & & \end{array}$$

(2)

To produce a mixture worth 4s. 4d. per lb. we require 10 lbs. @ 5s. and 8 @ 3s. 6d. But these added together make 18 lbs.

$$\begin{array}{ccccccc} & \text{lbs.} & \text{lbs.} & \text{lbs.} & \text{lbs.} & & \\ & & & & & & \\ \text{Therefore } 18 : 10 :: 27 : & \frac{27 \times 10}{18} & = & 15 \text{ lbs., the required quan-} & & & \\ & & & \text{tity of tea @ 5s.} & & & \end{array}$$

$$\begin{array}{ccccccc} & & & & \text{lbs.} & & \\ & & & & & & \\ 18 : 8 :: 27 : & \frac{27 \times 8}{18} & = & 12 \text{ lbs., the required quan-} & & & \\ & & & \text{tity of tea @ 3s. 6d.} & & & \end{array}$$

(3)

To produce a mixture worth \$2.70 per gallon, we require 20 gallons @ \$2.40, 10 @ \$2.60, 10 @ \$2.80, and 30 @ \$2.90. But all of these added together will make 70 gallons. Therefore

$$70 : 20 :: 63 : \frac{63 \times 20}{70} = 18 \text{ gallons, the required quantity of brandy @ \$2.40.}$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ \$2.60.}$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ \$2.80.}$$

$$70 : 30 :: 63 : \frac{63 \times 30}{70} = 27 \text{ gallons, the required quantity of brandy @ \$2.90.}$$

## EXERCISE 128—Page 289.

(1)

$$1974.80 \times \frac{1}{2} = £740.55 = £740 \text{ 11s.}$$

(2)

$$765.43 \times \frac{1}{2} = £306.172 = £306 \text{ 3s. } 5\frac{1}{2}\text{d.}$$

(3)

$$8172.19 \times \frac{1}{4} = £2043.0475 = £2043 \text{ 0s. } 11\frac{3}{4}\text{d.}$$

## EXERCISE 129—Page 289.

(1)

$$£743 \text{ 18s. 11d.} = £743.94583 \text{ and } 743.94583 \div \frac{3}{10} = \$2479.8194.$$

(2)

$$£119 \text{ 9s. } 8\frac{1}{2}\text{d.} = £119.484375 \text{ and } 119.484375 \div \frac{1}{4} = \$318.625.$$

(3)

$$£473 \text{ 17s. } 1\frac{1}{2}\text{d.} = £473.8572916, \text{ and } 473.8572916 \div \frac{1}{4} = \$2030.816964.$$

---

EXERCISE 130—Page 290.

(1)

$$1006.90 \div 4.867 = £206.88309 = £206 \text{ 17s. } 7\frac{1}{2}\text{d.}$$

(2)

$$916.87 \div 4.867 = £188.38504 = £188 \text{ 7s. } 8\frac{1}{2}\text{d.}$$

(3)

$$2114.81 \div 4.867 = £434.52023 = £434 \text{ 10s. } 4\frac{1}{2}\text{d.}$$

---

EXERCISE 131—Page 290.

(1)

$$£2043 \text{ 11s. } 3\text{d.} = £2043.5625 \text{ and } 2043.5625 \times 4.867 = \$9946.01868.$$

(2)

$$£777 \text{ 7s. } 7\text{d.} = £777.37916 \text{ and } 777.37916 \times 4.867 = \$3783.50437.$$

(3)

$$£557 \text{ 19s. } 5\frac{1}{2}\text{d.} = £557.972916 \text{ and } 557.972916 \times 4.867 = \$2715.65418.$$



## EXERCISE 132—Page 294.

(1)

$$\$16785.25 \times 5.04 = 84597 \text{ francs } 66 \text{ centimes.}$$

(2)

Commercial value of the marc banco = 35 cents.

Add 1 per cent

35

3535

$$\text{Then } \$0.3535 \times 4000 = \$1414.$$

(3)

$$\$35678 \times 1.0225 = \$36480.755.$$

(4)

The par value of 1 ruble = 75 cents.

Deduct 2 per cent

15

735

$$\text{Then } \$0.735 \times 2560 = \$1881.60.$$

(5)

Old commercial par of £1 sterling = \$4.444 = \$4.44444

Add 8 per cent

.35555

4.79999

$$\text{Then } \$4.79999 \times 800 = \$3839.999 = \$3840.00.$$

## EXERCISE 133—Page 295.

$$£1 = 420d.$$

$$19\frac{1}{2}d. = 1 \text{ franc.}$$

$$300 \text{ francs} = 60 \text{ ducats.}$$

$$1 \text{ ducat} = 360 \text{ maravedis.}$$

$$x = £1000.$$

$$x = \frac{300 \times 1 \times 50 \times 360 \times 1000}{19\frac{1}{2} \times 300} = 1564138 \text{ maravedis ex.}$$

$$42\frac{1}{2}d. : £1000 :: 272 \text{ maravedis} : \frac{272 \times 1000}{42\frac{1}{2}} =$$

$$\frac{272 \times 1000 \times 8 \times 17}{42\frac{1}{2}} = 1536000 \text{ maravedis by direct exchange.}$$

$$\text{Difference} = 1564138 - 1536000 = 28138 \text{ maravedis.}$$

$$34)28138$$

$$8)827 \text{ reals } 20 \text{ maravedis}$$

$$103 \text{ piastres } 3 \text{ reals } 20 \text{ maravedis.}$$

(2)

$$\text{Old commercial par of } £1 \text{ sterling} = \$4.444$$

$$\text{To which add 10 per cent. of itself} = .4444$$

$$\text{Gives price of } £1 \text{ sterling} = \$4.8884$$

$$\$4888.40 \div \$4.8884 = £1000 = \text{amount of bill he receives if he remits direct to London.}$$

$$\left. \begin{array}{l} \$1 = 515 \text{ centimes.} \\ 2580 \text{ cen.} = £1 \text{ sterling.} \end{array} \right\} x = \frac{515 \times 4888.40}{2580} = £975.78526.$$

$$x = \$4888.40$$

$$= £975 \text{ } 15s. \text{ } 8\frac{1}{2}d. + = \text{amount of bill he receives if he remits through Paris.}$$

$$35 \text{ cents} = 1 \text{ marc.}$$

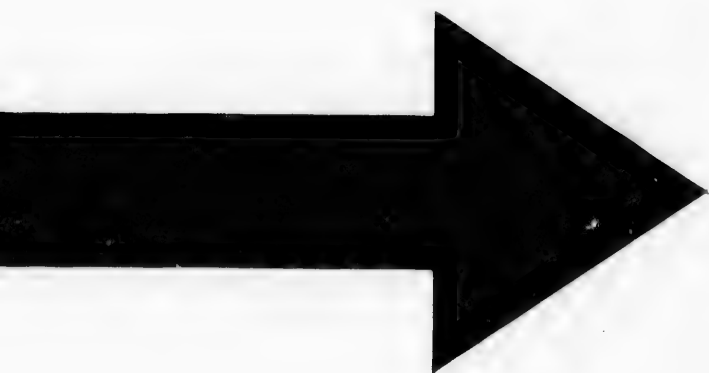
$$3\frac{3}{4} \text{ marcs} = £1 \text{ sterling.}$$

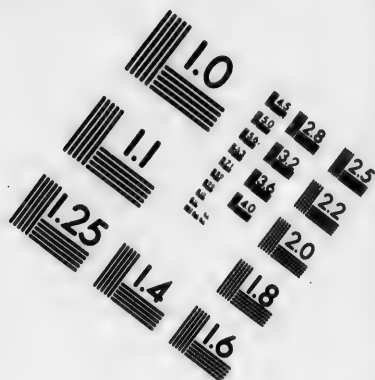
$$x = \$4888.40.$$

$$x = \frac{4888.40}{35 \times 13\frac{1}{4}} = \frac{391072}{385} = £1015.77142 = £1015 \text{ } 15s. \text{ } 5d. + =$$

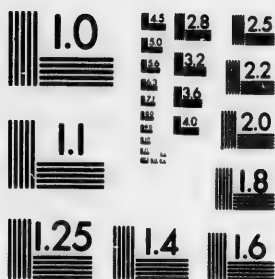
$$\text{amount of bill he receives by remitting through Hamburg.}$$



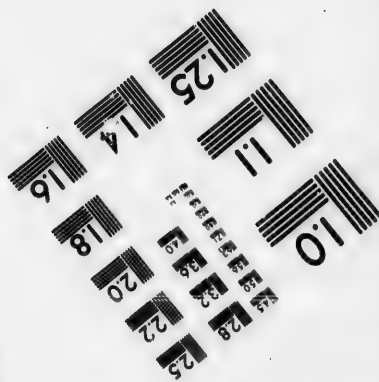




# IMAGE EVALUATION TEST TARGET (MT-3)



6"



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(3)

$$\begin{array}{lcl}
 18 \text{ cents.} & = & 1 \text{ franc.} \\
 25 \text{ francs.} & = & 240 \text{d.} \\
 180 \text{d.} & = & 3 \text{ milrees,} \\
 5 \text{ milrees.} & = & 18 \text{ marcs ban.} \\
 1200 \text{ marcs ban.} & = & x
 \end{array}
 \left. \vphantom{\begin{array}{lcl} 18 \text{ cents.} \\ 25 \text{ francs.} \\ 180 \text{d.} \\ 5 \text{ milrees.} \\ 1200 \text{ marcs ban.} \end{array}} \right\} x = \frac{18 \times 25 \times 180 \times 1200 \times 5}{240 \times 3 \times 18}$$

= \$375 = circuitous exchange or sum he pays for 1200 marks.

1200  $\times$  .35 = \$420 = direct exchange or sum paid for 1200 marks, \$420 - \$375 = \$45 = gain by circuitous exchange.

## EXERCISE 134—Page 298.

(1)

$$(3)^4 = 3 \times 3 \times 3 \times 3 = 81.$$

(2)

$$(20)^{10} = 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 = 10240000000000.$$

(3)

$$(1.05)^4 = 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.216708515625.$$

(4)

$$(2)^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128.$$

(5)

$$(5)^4 = 5 \times 5 \times 5 \times 5 = 625.$$

(6)

$$11\frac{1}{2} = \frac{23}{2}. \quad (\frac{23}{2})^3 = \frac{23}{2} \times \frac{23}{2} \times \frac{23}{2} = \frac{12167}{8} = 1520\frac{7}{8}.$$

EXERCISE 135—Page 292.

$$\begin{array}{r} 0 \quad 5 \\ Q \times 12QQ \times 5 \end{array}$$

$\times 18$   
200 marks.  
aid for 1200  
exchange.

(1)

$$4^2 \times 4^4 \times 4^5 \times 4^7 = 4^{2+4+5+7} = 4^{18}.$$

(2)

(3)

$$13^{11} \div 13^3 = 13^{11-3} = 13^8. \quad (3^3)^5 = 3^{3 \times 5} = 3^{15}.$$

(4)

$$\begin{aligned} \{ (7^4 \times 7^3) \div (7^2 \times 7^2) \}^6 &= \{ (7^4 + 3) \div (7^2 + 2) \}^6 = \\ \{ 7^7 \div 7^4 \}^6 &= (7^{7-4})^6 = (7^3)^6 = 7^{3 \times 6} = 7^{18}. \end{aligned}$$

(5)

$$\times 20 \times 20$$

$$\begin{aligned} \{ (5^3 \times 5^4 \times 5^{11} \times 5^9) \div (5^3 \times 5^2 \times 5^7 \times 5^5) \}^3 &= \\ \{ (5^{3+4+11+9}) \div (5^{3+2+7+5}) \}^3 &= \{ 5^{27} \div 5^{17} \}^3 \\ (5^{27-17})^3 &= (5^{10})^3 = 5^{10 \times 3} = 5^{30}. \end{aligned}$$

$$1.05 =$$

EXERCISE 137—Page 304.

(1)

(2)

(3)

$$\begin{array}{r} \dots \\ 195384(442 \\ 18 \\ \hline 84)353 \\ 336 \\ \hline 882)1764 \\ 1764 \end{array}$$

$$\begin{array}{r} \dots \\ .0676(.26 \\ 4 \\ \hline 46)276 \\ 276 \end{array}$$

$$\begin{array}{r} \dots \\ 984064(992 \\ 81 \\ \hline 189)1740 \\ 1701 \\ \hline 1982)3964 \\ 3964 \end{array}$$

$$1481.64$$



(4)

$$\begin{array}{r}
 5\cdot0000000000(2\cdot23808 \\
 4 \\
 \hline
 42)1\cdot00 \\
 \cdot84 \\
 \hline
 443)1600 \\
 \cdot1329 \\
 \hline
 4466)27100 \\
 26796 \\
 \hline
 447206)3040000 \\
 2683236 \\
 \hline
 356764
 \end{array}$$

(5)

$$\begin{array}{r}
 5\cdot0000000000(-707108 \\
 49 \\
 \hline
 1407)10000 \\
 9849 \\
 \hline
 14141)15100 \\
 14141 \\
 \hline
 1414206)9590000 \\
 8485236 \\
 \hline
 1104764
 \end{array}$$

(6)

$$\begin{array}{r}
 60\cdot487129(7\cdot777 \\
 49 \\
 \hline
 147)1148 \\
 1029 \\
 \hline
 1547)11971 \\
 10829 \\
 \hline
 15547)114229 \\
 108829 \\
 \hline
 5400
 \end{array}$$

(7)

$$\begin{array}{r}
 79792266297612001(282475249 \\
 4 \\
 \hline
 48)397 \\
 384 \\
 \hline
 562)1392 \\
 1124 \\
 \hline
 5644)26826 \\
 22576 \\
 \hline
 56437)425062 \\
 395409 \\
 \hline
 564945)2965397 \\
 2824725
 \end{array}$$

(8)

$$\begin{array}{r}
 0\cdot0000012321(-00111 \\
 1 \\
 \hline
 21)23 \\
 21 \\
 \hline
 221)221 \\
 221
 \end{array}$$

$$\begin{array}{r}
 5649502)14067261 \\
 11299004 \\
 \hline
 56495044)276825720 \\
 225980176 \\
 \hline
 564950489)5084554401 \\
 5084554401
 \end{array}$$

## EXERCISE 138—Page 304.

(1)

$$1 = \frac{1}{1} \text{ and } \sqrt{\frac{1}{1}} = \frac{1}{1}.$$

(3)

$$5\frac{1}{2} = 5.142857142857 \text{ and } \sqrt{5.142857142857} = 2.267786.$$

(4)

$$3\frac{1}{3} = 3.4033457249 \text{ and } \sqrt{3.4033457249} = 1.84509.$$

(5)

$$13\frac{1}{2} = 13.2 \text{ and } \sqrt{13.2} = 3.63318$$

## EXERCISE 139—Page 305.

(1)

(2)

$$\begin{array}{r} 11333311(2626 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 46)433 \\ 411 \\ \hline \end{array}$$

$$\begin{array}{r} 552)2233 \\ 1434 \\ \hline \end{array}$$

$$\begin{array}{r} 5546)46611 \\ 46611 \\ \hline \end{array}$$

$$\begin{array}{r} 33233344(4344 \\ 24 \\ \hline \end{array}$$

$$\begin{array}{r} 123)523 \\ 413 \\ \hline \end{array}$$

$$\begin{array}{r} 1304)11033 \\ 10024 \\ \hline \end{array}$$

$$\begin{array}{r} 13124)100544 \\ 100544 \\ \hline \end{array}$$

(3)

$$\begin{array}{r}
 \cdot\cdot\cdot\cdot\cdot \\
 4234 \cdot 101230 (43 \cdot 412 \\
 \underline{31} \\
 133) 1134 \\
 \underline{1004} \\
 1414) 130 \cdot 10 \\
 \underline{122 \cdot 21} \\
 14231) 2 \cdot 3412 \\
 \underline{1 \cdot 4231} \\
 142322) 413130 \\
 \underline{\cdot 340144} \\
 22431
 \end{array}$$

(4)

$$\begin{array}{r}
 \cdot\cdot\cdot\cdot\cdot \\
 888888 \cdot 88880 (888 \cdot 88 \\
 \underline{71} \\
 178) 1788 \\
 \underline{1601} \\
 1878) 18788 \\
 \underline{16801} \\
 18878) 1887 \cdot 88 \\
 \underline{1688 \cdot 01} \\
 188878) 188 \cdot 8780 \\
 \underline{168 \cdot 8801} \\
 18 \cdot 8878
 \end{array}$$

(5)

$$\begin{array}{r}
 \cdot\cdot\cdot\cdot\cdot \\
 248664e \cdot 169 (54373 \\
 \underline{21} \\
 14) 386 \\
 \underline{354} \\
 183) 3264 \\
 \underline{2809} \\
 1867) 657e \cdot t \\
 \underline{62et \cdot 1} \\
 18723) 281969 \\
 \underline{281969}
 \end{array}$$

EXERCISE 140—Page 307.

(1)

$$\begin{array}{r}
 100^2 = 10000 \\
 60^2 = 3600 \\
 \hline
 \end{array}$$

$$\text{Difference} = 6400 \text{ and } \sqrt{6400} = 80.$$

(2)

$$50^2 = 2500$$

$$80^2 = 6400$$


---

$$\text{Sum} = 8900 \text{ and } \sqrt{8900} = 94.34 \text{ nearly}$$

(3)

$$24^2 = 576 \div 2 = 288 \text{ and } \sqrt{288} = 16.97.$$

(4)

$$36^2 = 1296$$

$$20^2 = 400$$


---

$$\text{Difference} = 896 \text{ and } \sqrt{896} = 29.933.$$

(5)

$$40^2 = 1600$$

$$14^2 = 196$$


---

$$\text{Difference} = 1404 \text{ and } \sqrt{1404} = 37.469.$$

$$40^2 = 1600$$

$$26^2 = 676$$


---

$$\text{Difference} = 924 \text{ and } \sqrt{924} = 30.397.$$

$$37.469 + 30.397 = 67.866 \text{ and } 67.866 \div 3 = 22.622.$$

(6)

$$1760 \text{ sq. yds.} = 15840 \text{ sq. ft. and } \sqrt{15840} = 125.857.$$

(7)

$$\sqrt{141376} = 376.$$

(8)

$$3^2 = 9$$

$$3^2 = 9$$


---

$$\text{Sum} = 18 \text{ and } \sqrt{18} = 4.24264.$$

(9)

$$16^2 = 256$$

$$12^2 = 144$$

$$\text{Sum} = 400 \text{ and } \sqrt{400} = 20$$

(10)

$$3^2 + 3^2 + 3^2 = 27 \text{ and } \sqrt{27} = 5.196.$$

(11)

$$(\frac{1}{10})^2 = \frac{1}{100} \text{ and } (1)^2 = 1.$$

$$\text{Then } \frac{1}{100} : 1 :: 450 : \frac{450}{\frac{1}{100}} = 45000.$$

(12)

$$1 \text{ sq. acre} = 160 \text{ sq. perches. } 160 \div 3.1416 = 50.929462 \text{ and } \sqrt{50.929462} = 7.136.$$

EXERCISE 141—Page 311.

(1)

$$\begin{array}{r} 62712728317(3973 \\ 27 \\ \hline \end{array}$$

$$\begin{array}{r} 3^2 \times 300 = 2700 \\ 3 \times 9 \times 30 = 810 \\ 9^2 = 81 \\ \hline \end{array}$$

$$35712$$

$$3591$$

$$32319$$

$$\begin{array}{r} 39^2 \times 300 = 456300 \\ 39 \times 7 \times 30 = 8190 \\ 7^2 = 49 \\ \hline \end{array}$$

$$3393728$$

$$464539$$

$$3251773$$

$$\begin{array}{r} 397^2 \times 300 = 47282700 \\ 397 \times 3 \times 30 = 35730 \\ 3^2 = 9 \\ \hline \end{array}$$

$$141955317$$

$$47318439$$

$$141955317$$

(3)

		1953125(125
		I
		-
$1^2 \times 300 =$	300	953
$1 \times 2 \times 30 =$	60	
$2^2 =$	4	
	<hr/>	
	364	728
	<hr/>	
$12^2 \times 300 =$	43200	225125
$12 \times 5 \times 30 =$	1800	
$5^2 =$	25	
	<hr/>	
	45025	225125

0-929462 and

(3)

		1076890625(1025
		I
		-
$1^2 \times 300 =$	300	76
$10^2 \times 300 =$	30000	76890
$10 \times 2 \times 30 =$	600	
$2^2 =$	4	
	<hr/>	
	30604	61208
	<hr/>	
$102^2 \times 300 =$	3121200	15682625
$102 \times 5 \times 30 =$	15300	
$5^2 =$	25	
	<hr/>	
	3126525	15682625

73

N

(4)

697864103(·887

512

$$8^2 \times 300 = 19200$$

$$8 \times 8 \times 30 = 1920$$

$$8^2 = 64$$

21184

185864

169472

$$88^2 \times 300 = 2323200$$

$$88 \times 7 \times 30 = 18480$$

$$7^2 = 49$$

2341729

16392108

16392108

(5)

102503·232(46·8

64

$$4^2 \times 300 = 4800$$

$$4 \times 6 \times 30 = 720$$

$$6^2 = 36$$

5556

38503

33336

$$46^2 \times 300 = 634800$$

$$46 \times 8 \times 30 = 11040$$

$$8^2 = 64$$

645904

5167·232

5167·232

(6)

$$\begin{array}{r}
 5^2 \times 300 = 7500 \\
 5 \times 6 \times 30 = 900 \\
 6^2 = 36 \\
 \hline
 8436 \\
 56^2 \times 300 = 940800 \\
 56 \times 4 \times 30 = 6720 \\
 4^2 = 16 \\
 \hline
 947536 \\
 564^2 \times 300 = 95428800 \\
 564 \times 2 \times 30 = 33840 \\
 2^2 = 4 \\
 \hline
 95462644
 \end{array}$$

$$\begin{array}{r}
 179597 \cdot 069288(56 \cdot 42 \\
 125 \\
 \hline
 54597
 \end{array}$$

$$\begin{array}{r}
 50616 \\
 \hline
 3981 \cdot 069
 \end{array}$$

$$\begin{array}{r}
 3790 \cdot 144 \\
 \hline
 100 \cdot 925288
 \end{array}$$

$$190 \cdot 925288$$

(7)

$$\begin{array}{r}
 7^2 \times 300 = 14700 \\
 7 \times 8 \times 30 = 1680 \\
 8^2 = 64 \\
 \hline
 16444 \\
 78^2 \times 300 = 1825200 \\
 78 \times 5 \times 30 = 11700 \\
 5^2 = 25 \\
 \hline
 1836925
 \end{array}$$

$$\begin{array}{r}
 483 \cdot 736625(7 \cdot 85 \\
 343 \\
 \hline
 140 \cdot 736
 \end{array}$$

$$\begin{array}{r}
 131 \cdot 552 \\
 \hline
 9 \cdot 184625
 \end{array}$$

$$9 \cdot 184625$$



(9)

$$\begin{array}{r} \cdot 636056 \\ \cdot 86 \end{array}$$

512

$$8^2 \times 300 = 19200$$

$$124056$$

$$8 \times 6 \times 30 = 1440$$

$$6^2 = 36$$

$$20676$$

$$124056$$

EXERCISE 142.—Page 312.

(1)

$$\sqrt[3]{r} = \cdot 105263157894+ \text{ and } \sqrt[3]{\cdot 105263157894} = \cdot 4721.$$

(2)

$$\sqrt[3]{r} = \cdot 176470588235+ \text{ and } \sqrt[3]{\cdot 176470588235} = \cdot 5609.$$

(3)

$$\frac{1}{3} \text{ of } 2\frac{1}{2} = \frac{5}{6} = \cdot 833333333+ \text{ and } \sqrt[3]{\cdot 833333333} = \cdot 941.$$

(4)

$$28\frac{1}{2} = 28.75 \text{ and } \sqrt[3]{28.75} = 3.063$$

(5)

$$32.72 = 32.\ddot{7}2 \text{ and } \sqrt[3]{32.\ddot{7}2} = 3.198.$$

EXERCISE 143.—Page 313.

(1)

One million = 83233344 senary.

83233344 (244.  
12

$$\begin{array}{rcl} 2^2 & = 4 \times 300 & = 2000 \\ 2 \times 30 & = 100 \times 4 & = 400 \\ 4^2 & & = 24 \end{array}$$

21233

2424

14544

$$\begin{array}{rcl} 24^2 & = 1104 \times 300 & = 332000 \\ 24 \times 30 & = 1200 \times 4 & = 5200 \\ 4^2 & & = 24 \end{array}$$

2245344

341224

2245244

(2)

6131271.000000 (165.32.  
1

$$\begin{array}{rcl} 1^2 \times 300 & = & 300 \\ 1 \times 30 \times 6 & = & 220 \\ 6^2 & = & 44 \end{array}$$

5131

564

4270

$$\begin{array}{rcl} 16^2 & = 304 \times 300 & = 111400 \\ 16 \times 30 & = 520 \times 5 & = 3220 \\ 5^2 & = & 31 \end{array}$$

641271

114651

600115

$$\begin{array}{rcl} 165^2 & = 32571 \times 300 & = 12015300 \\ 165 \times 30 & = 5370 \times 3 & = 20350 \\ 3^2 & = & 11 \end{array}$$

41154.000

12035661

36131.423

$$\begin{array}{rcl} 1653^2 & = 3272071 \times 300 & = 1205625300 \\ 1653 \times 30 & = 54010 \times 2 & = 130020 \\ 2^2 & = & 4 \end{array}$$

3022.355000

1205755324

2413.732650

406.422130

(3)

10221012 · 102000000

1 | 112 · 012 = root.

$1 \times 1000 =$	1000	2221
$1 \times 1 \times 100 =$	100	
$1^2 =$	1	
	1101	1101
$11^2 = 121 \times 1000 =$	121000	1120012
$11 \times 100 = 1100 \times 2 =$	2200	
$2^2 =$	11	
	200211	1101122
$112^2 = 21021 \times 1000 =$	21021000	11120 · 102
$1120^2 = 2102100 \times 1000 =$	2102100000	11120 · 102000
$1120 \times 100 = 112000 \times 1 =$	112000	
$1^2 =$	1	
	2102212001	2102 · 212001
$11201^2 = 211010101 \times 1000 =$	211010101000	2010 · 112222000
$11201 \times 100 = 1120100 \times 2 =$	10010200	
$2^2 =$	11	
	211020111211	1122 · 111000122
		111 · 001221101

440<sup>2</sup> =  
4400<sup>2</sup> =  
4400 × 3

(4)

11-00000(7-12.  
92e

$$\begin{array}{rcl} e^2 & = & t1 \times 300 = 26300 \\ e \times 30 & = & 290 \times 7 = 1730 \\ 7^2 & = & 41 \end{array}$$

18ccet

27t71

167217

$$\begin{array}{rcl} e^{7^2} & = & e^{221} \times 300 = 2966300 \\ e^7 \times 30 & = & 2190 \times t = 24e60 \\ t^2 & = & 84 \end{array}$$

249 t3-000

**298e324**

24154-7e4

$$\begin{aligned} e7t^2 &= e39544 \times 300 = 29e441000 \\ e7t \times 30 &= 2te60 \times 2 = 59e00 \\ 2^2 &= 4 \end{aligned}$$

84t-408000

29e49 t e 04

**57t-979t08**

**28c-64t1e4**

(b)

421030-441200000(44-004  
224

$$\begin{array}{r} 4^2 = 31 \times 300 = 14300 \\ 4 \times 30 = 220 \times 4 = 1430 \\ 4^2 = 31 \end{array}$$

142030

21311

141244

$$44^2 = 4301 \times 300 = \underline{2340300}$$

231-441

$$440^2 = 430100 \times 300 = 234030000$$

**231·441200**

$$\begin{array}{rcl} 4400^2 & = & 43010000 \times 300 = 23403000000 \\ 4400 \times 30 & = & 242000 \times 4 = 2123000 \\ & & 4^2 = 31 \end{array}$$

231-441200000

23410123031

210-141102224

**21-300043221**

## EXERCISE 144.—Page 314.

(2)

$$3^3 : 6^3 :: 416 : \text{Ans.} = 321 \text{ lbs.}$$

(3)

$$1^3 : \left(\frac{1}{2}\right)^3 :: \$120 : \text{Ans.} = \$5145.$$

(4)

$$\begin{aligned} (70)^3 : (422)^3 &:: 180 \text{ lbs.} : \text{Ans.} \\ 343000 : 21180432 &:: 180 : \text{Ans.} = \\ 180 \times \frac{21180432}{343000} &= 1015.1 \text{ lbs.} \end{aligned}$$

(5)

$$973^3 = 921167317$$

$$45^3 = 91125$$

$$62^3 = 238328$$

$$30^3 = 27000$$

$$80^3 = 512000$$

$$20^3 = 8000$$

$$\begin{aligned} 921167317 - (91125 + 238328 + 27000 + 512000 + 8000) &= \\ 920290864 \text{ and } \sqrt[3]{920290864} &= 972.69. \end{aligned}$$

(6)

$$\begin{aligned} 8 \text{ feet } 3 \text{ inches} &= 99 \text{ inches, } 3 \text{ feet} = 36 \text{ inches, and } 2 \text{ feet } 7 \\ &\text{ inches} = 31 \text{ inches.} \end{aligned}$$

$$99 \times 36 \times 31 = 110484 \text{ and } \sqrt[3]{110484} = 47.9843.$$

(7)

After the first has wound off her portion, there will remain  $\frac{1}{2}$  of the thread.

Then the whole ball : part remaining :: cube of diameter of whole ball : cube of diameter of part remaining.

That is,  $1 : \frac{1}{4} :: 3^3 : x^3$ , and hence  $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.75} = .90856 \times 3 = 2.72568 =$  diameter of the ball after the first has wound off her portion.

Similarly after the second has wound off her portion, there will remain  $\frac{1}{4}$  of the ball, and after the third has taken her portion,  $\frac{1}{4}$  of the ball.

Hence  $1 : \frac{1}{4} :: 3^3 : x^3$ , whence  $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.5} = 3 \times .79370 = 2.38110 =$  diameter after the second has taken her portion.

$1 : \frac{1}{4} :: 3^3 : x^3$ , whence  $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.25} = 3 \times .62996 = 1.88988 =$  diameter after the third has taken her portion.

Hence 1st takes off 3	— 2.72568 =	.27432 inches.
2nd " "	2.72568 — 2.38110 =	.34458 "
3rd " "	2.38110 — 1.88988 =	.49122 "
4th " "	remaining	1.88988 "

EXERCISE 145—Page 315.

(1)

$$\sqrt{19987173376} = 141376, \text{ and } \sqrt{141376} = 376.$$

(2)

$$\sqrt[3]{308915776} = 676, \text{ and } \sqrt{676} = 26.$$

(3)

$$\sqrt[3]{40353607} = 343, \text{ and } \sqrt[3]{343} = 7.$$

(4)

$$\sqrt[3]{387420489} = 729, \sqrt[3]{729} = 9, \text{ and } \sqrt{9} = 3.$$

(5)

$$\sqrt[4]{134217728} = 512, \sqrt[4]{512} = 8, \text{ and } \sqrt[4]{8} = 2.$$

## EXERCISE 148—Page 321.

(1)

The mantissa of the logarithm of 8193 (the first four digits) = .913443, and the next following mantissa is .913496.

Then from .913496

Subtract.. .913443

Difference, 53; and  $53 \times 217$  (remaining digits of given number) = 11501, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 12.

Then mantissa of logarithm of first four digits .913443  
Add, 12

Mantissa of logarithm of given number, .913455

To which attach the characteristic 6 and required logarithm = 6.913455.

The mantissa of the logarithm of 7392 (the first four digits) = .868762, and the next following mantissa is .868821.

Then from .868821

Subtract.. .868762

Difference, 59; and  $59 \times 45$  (remaining digits of given number) = 2655, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 27.

Then mantissa of logarithm of first four digits, .868762  
Add, 27

Mantissa of logarithm of given number, .868789

(Continued on next page.)

(1 continued.)

To which attach the characteristic 1 and required logarithm =  
1.868789.

The mantissa of the logarithm of 8437 (the first four digits)  
= .926188, and the next following mantissa is .926240.

Then from .926240

Subtract.. .926188

Difference,  $52$ ; and  $52 \times 42$  (remaining digits of given  
number) = 2184, from which we cut off two digits, since we  
multiplied by a number of two digits, and since the highest digit  
cut off is not less than 5, we add unity to the part retained,  
which becomes 22.

Then mantissa of logarithm of first four digits .926188

Add,

22

Mantissa of logarithm of given number, .926210

To which attach the characteristic  $\bar{1}$  and required logarithm =  
 $\bar{1}.926210$ .

(2)

The mantissa of the logarithm of 2345 = .370143, and the next  
following mantissa is .370328.

Then from .370328

Subtract.. .370143

Difference, 185; and  $185 \times 64 = 11840$ , from which we  
cut off two digits, since we multiplied by a number of two  
digits, which gives us 118.

Then mantissa of logarithm of 2345 = .370143

Add,

118

Mantissa of logarithm of given number = .370261

To which attach the characteristic  $\bar{4}$  and required logarithm =  
 $\bar{4}.370261$ .

(Continued on next page.)



(3 continued.)

The mantissa of the logarithm of 1007 = .003029, and the next following mantissa is .003461.

Then from .003461

Subtract.. .003029

---

Difference, 432; and  $432 \times 013 = 5616$ , from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 6.

Then mantissa of logarithm of 1007 = .003029

Add, 6

---

Mantissa of logarithm of given number .003035

To which attach the characteristic 3, and required logarithm =  
3.003035.

(3)

Mantissa of logarithm of 5237 ..... .719083  
Difference from column D = 83; and  $83 \times 6 = 498$   
from which we cut off 1 digit and add..... 50

---

And also attach the characteristic 1, and required  
logarithm = ..... 1.719133  
Mantissa of logarithm of 1294..... .111934  
Difference from column D = 335; and  $335 \times 76 =$   
25460 from which we cut off two digits and add, 255

---

And also attach the characteristic 2 and required  
logarithm = ..... 2.111939

(4)

Mantissa of logarithm of	·0004713	=	·673297
P. P. corresponding to	·00000009	=	83
P. P. " to	·000000008	=	74

Sum, = ·6733874

Therefore required mantissa = ·673387 and required logarithm  
= 4·673387.

Mantissa of logarithm of	9136000	=	·960756
P. P. corresponding to	700	=	33
P. P. " to	10	=	5
P. P. " to	2	=	9

Sum, = ·96078959

Therefore required mantissa = ·960790 and required logarithm  
= 6·960790.

(5)

Mantissa of logarithm of	4·23400	=	·626751
P. P. corresponding to	20	=	20
P. P. " to	9	=	92

Sum, = ·6267802

Therefore required logarithm is 0·626780.

Mantissa of logarithm of	763·1	=	·882581
P. P. corresponding to	·02	=	11
P. P. " to	·009	=	51
P. P. " to	·0008	=	46
P. P. " to	·00009	=	40

Sum, = ·882597600

Therefore required logarithm is 2·882598.

## EXERCISE 149.—Page 323.

(1)

Given logarithm,  $\cdot 137139$ Next lower in table,  $\cdot 137037 = \log.$  of 1371.Difference  $\quad\quad\quad 102$ , Tabular difference  $= 316$ .Then  $1020000 \div 316$  gives 3227 for digits in 5th, 6th, 7th, and 8th places.

Hence the digits of the natural number are 13713227; and since the characteristic is 4, i.e., one less than the number of digits to the left of the decimal point the required number is 13713·227.

Given logarithm,  $\cdot 718134$ Next lower in table,  $\cdot 718086 = \log.$  of 5225.Difference,  $\quad\quad\quad 48$ , Tabular difference  $= 83$ .Then  $48000 \div 83$  gives 578 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 5225578, and since the characteristic is 0, i.e., one less than the number of digits to the left of the decimal point, the required number is 5·225578.

Given logarithm,  $\cdot 635421$ Next lower in table,  $\cdot 635383 = \log.$  of 4319.Difference,  $\quad\quad\quad 38$ , Tabular difference  $= 101$ .

Then  $38000 \div$  gives 376 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 4319376, and since the characteristic is  $\bar{4}$ , i.e., one more than the number of ciphers between the decimal point and the first figure to the right, the required number is  $\cdot 0004319376$ .

(2)

Given log.  $\cdot 921686 = \log.$  of 8350.

And since the characteristic is 2, i.e., one less than the number of digits to the left of the decimal point, the required number is 835.

Given logarithm,  $\cdot 922165$

Next lower in table,  $\cdot 922154 = \log.$  of 8359.

Difference = 11, Tabular difference = 52.

Then  $11000 \div 52$  gives 211 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 8359211; and since the characteristic is  $\bar{1}$ , i.e., one more than the number of ciphers between the decimal point and first figure to the right, the required number is  $\cdot 8359211$ .

(3)

Given logarithm,  $\cdot 407968$

Next lower in table,  $\cdot 407901 = \log.$  of 2558.

Difference, = 67

Highest P. P. not greater than 67 = 51 corresponds to 3  
for 5th place.

Highest P. P. not greater than 160 = 160  
153 corresponds to 9  
for 6th place.

Highest P. P. not greater than 70 = 70  
68 corresponds to  
4 for 7th place.  
2

Therefore digits of required number are 2558394; and since the characteristic is 5, there must be six digits to the left of the decimal point.

Hence required number is 2558394.

(Continued on next page.)

(3 continued.)

Given logarithm,  $\cdot 408368$   
 Next lower in table,  $\cdot 408240 = \log. \text{ of } 2560.$

Difference, =  $148$

Highest P.P. not greater than 148 = 136 corresponds to 8  
 in 5th place.

100

Highest P.P. not greater than 100 = 85 corresponds to 5  
 in 6th place.

150

Highest P.P. not greater than 150 = 136 corresponds to 8  
 in 7th place.

140

Highest P.P. not greater than 140 = 136 corresponds to 8  
 in 8th place.

Therefore digits of required number are 25608588; and since the characteristic is 7, there must be eight digits to the left of the decimal point.

Hence required number is 25608588.

Given logarithm,  $\cdot 416369$   
 Next lower in table,  $\cdot 416308 = \log. \text{ of } 2608.$

Difference, =  $61$

Highest P.P. not greater than 61 = 49 corresponds to 3  
 in 5th place.

12

Therefore digits of required number are 26083; and since the characteristic is  $\bar{3}$ , there must be two ciphers between the decimal point and first figure.

Hence required number is  $\cdot 0026083.$

(4)

Given logarithm,

 $\cdot 877777$ 

Next lower in table,

 $\cdot 877774 = \log. \text{ of } 7547.$ 

Difference, =

 $\underline{3}$ 

There is no P.P. not greater than 3

0 corresponds to 0 in  
5th place. $\underline{30}$ 

Highest P.P. not greater than 30 =

29 corresponds to 5 in  
6th place. $\underline{10}$ 

Highest P.P. not greater than 10 =

6 corresponds to 1  
in 7th place. $\underline{40}$ 

Highest P.P. not greater than 40 =

35 corresponds to 6  
in 8th place. $\underline{50}$ 

Highest P.P. not greater than 50 =

 $\cdot 46$  corresponds to  
8 in 9th place. $\underline{4}$ 

Therefore digits of required number are 754705168; and since the characteristic is 4, there must be five digits to the left of the decimal point.

Hence required number is 75470.5168.

Given logarithm,

 $\cdot 555555$ 

Next lower in table,

 $\cdot 555457 = \log. \text{ of } 3593.$ 

Difference, =

 $\underline{98}$ 

Highest P.P. not greater than 98 =

98 corresponds to 8  
in 5th place.

Therefore digits of required number are 35938; and since the characteristic is 0, there must be one digit to the left of the decimal point.

Hence required number is 3.5938.

## EXERCISE 150.—Page 324.

(1)

(2)

$$\begin{array}{ll}
 10 - 5.031643 = 4.968358. & 10 - 3.123456 = 12.876544. \\
 10 - 0.714000 = 9.286000. & 10 - 7.213149 = 16.786851.
 \end{array}$$

(3)

$$10 - 6.124357 = 3.875643 \text{ and } 10 - 2.000837 = 11.999163.$$

## EXERCISE 151.—Page 325.

(1)

$$\begin{array}{ll}
 \text{Logarithm of } 61 = 1.785330 \\
 \text{" } 22 = 1.342423 \\
 \text{" } 65 = 1.812913
 \end{array}$$

$$\text{Sum} = 4.940666 = \text{logarithm of } 87230.$$

(2)

$$\begin{array}{ll}
 \text{Logarithm of } 52 = 1.716003 \\
 \text{" } 734 = 2.865696 \\
 \text{" } 6 = 0.778151
 \end{array}$$

$$\begin{array}{l}
 \text{Sum} = 5.359850 \\
 5.359835 = \text{logarithm of } 229000
 \end{array}$$

15 =

8

Ans, 2290

(3)

$$\text{Logarithm of } 35.86 = 1.554610$$

$$" \quad 2.1046 = 0.373189$$

$$" \quad .8372 = 1.922929$$

$$" \quad .00294 = 3.468347$$

$$= \text{Sum} = 1.268955$$

$$1.268912 = \text{logarithm of } .185700$$

$$143 =$$

61

Ans. .185761

(4)

$$\text{Log. of } .00008764 = 5.942702$$

$$" \quad .86359 = 1.936308$$

$$\text{Sum} = 5.879010$$

$$5.878991 = \text{logarithm of } .000075680$$

$$29 =$$

5

Ans. .000075685

87230.

## EXERCISE 152.—Page 326.

(1)

$$\text{Logarithm of } .8734 = 1.938273$$

$$" \quad .0009278 = 4.967454$$

$$\text{Difference} = 2.860819$$

$$2.860817 = \text{logarithm of } .725.8000$$

$$2 =$$

33

Ans. 725.8033

229000

8

2290



(2)

$$\text{Logarithm of } 437.89 = 2.641365$$

$$\text{" } 62.735 = 1.797510$$

$$\text{Difference} = .843855 = \text{logarithm of } 6.98$$

(3)

$$\text{Logarithm of } 93.217 = 1.969495$$

$$\text{" } .0007132 = \overline{4.853211}$$

$$\text{Difference} = 5.116284$$

$$5.116276 = \text{logarithm of } 130700.0$$

$$8 = \quad \quad \quad 2.4$$

$$\text{Ans. } 130702.4$$

(4)

$$\text{Logarithm of } 23 = 1.361728$$

$$\text{" } 189 = 2.276462$$

$$\text{" } 2.748 = 0.439017$$

$$\text{Sum} = 4.077207$$

$$\text{Logarithm of } 9835267 = 6.992786$$

$$4.077207$$

$$\text{Difference} = 2.915579$$

$$2.915558 = \text{logarithm of } 823.300$$

$$21 = \quad \quad \quad 39$$

$$\text{Ans. } 823.339$$

EXERCISE 153.—Page 326.

(1)

$$\text{Logarithm of } 5 = 0.698970.$$

$$\text{Then } 0.698970 \times 5 = 3.494850 = \text{logarithm of } 312.$$

(2)

Logarithm of 1.073 = .030600.

Then .030600  $\times$  6 = .183600 = logarithm of 1.5261.

(3)

Logarithm of .0279 =  $\bar{2}$ .445604.

Then  $\bar{2}$ .445604  $\times$  4 =  $\bar{7}$ .782416 = logarithm of .00000060592.

(4)

Logarithm of 1.111 = .045714.

Then .045714  $\times$  11 = .502854 = logarithm of 3.1831.

---

EXERCISE 154.—Page 327.

(1)

Logarithm of 913426000 = 8.960673.

8.960673  $\div$  7 = 1.2800961 = logarithm of 19.0588.

(2)

Logarithm of 1.91342 = .207747.

.207747  $\div$  11 = .01888609 = logarithm of 1.0444.

(3)

Logarithm of .000007139 =  $\bar{6}$ .853637 =  $\bar{10}$  + 4.853637.

( $\bar{10}$  + 4.853637)  $\div$  5 =  $\bar{2}$ .970727 = logarithm of .0934817.

(4)

Logarithm of .002147 =  $\bar{3}$ .331832 =  $\bar{7}$  + 4.331832.

( $\bar{7}$  + 4.331832)  $\div$  7 =  $\bar{1}$ .6188331 = logarithm of .41575,

## EXERCISE 155.—Page 328.

(1)

$$14000 = 7 \times 2 \times 1000 \therefore \log. 14000 = (\log. 7) + (\log. 2) + (\log. 1000).$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Log. } 2 = 0.301030$$

$$\text{Log. } 1000 = 3$$

$$\text{Sum,} \quad \underline{\hspace{1cm}} = 4.146128 = \log. 14000$$

$$4.9 = 7^2 \div 10 \therefore \log. 4.9 = (\log. 7) \times 2 - (\log. 10).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 10 = \underline{\hspace{1cm}} 1$$

$$\text{Difference} = \underline{\hspace{1cm}} .690196 = \log. 4.9$$

$$.00196 = 49 \times 4 \div 100000 = 7^2 \times 2^2 \div 100000$$

$$\therefore \log .00196 = (\log. 7) \times 2 + (\log. 2) \times 2 - (\log. 100000).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 2 = 0.602060$$

$$\text{Sum} = \underline{\hspace{1cm}} 2.292256$$

$$\text{Log. of } 100000 = 5 \text{ and } 2.292256 - 5 = \bar{3}.292256 = \log \text{ of } .00196.$$

$$\text{Since } 5 = 10 \div 2, \text{ the logarithm of } 5 = \log. 10 - \log. 2 = 1 - 0.301030 = 0.698970.$$

$$1750 = 5^2 \times 7 \times 10 \therefore \log. 1750 = (\log. 5) \times 2 + (\log. 7) + (\log. 10)$$

$$\text{Log. } 5 = 0.698970 \times 2 = 1.397940$$

$$\text{Log. } 7 = \underline{\hspace{1cm}} .845098$$

$$\text{Log. } 10 = \underline{\hspace{1cm}} 1$$

$$\text{Sum,} = \underline{\hspace{1cm}} 3.243038 = \log. \text{ of } 1750.$$

$$1428.571428 = 4 \times 10000 \therefore \log. 1428.571428 = (\log. 4) + \log. 10000.$$

(1 continued.)

$$\text{Log. } \frac{1}{4} = (\text{log. } 1) - (\text{log. } 7) = 0 - 0.845098 = \overline{1.154902}$$

$$\text{Log. } 10000 = 4$$

$$\therefore \text{log. of } 1428.571428 = \text{sum} = \overline{3.154902}$$

$$.00000112 = 2^4 \times 7 \div 100000000 \therefore \text{log. } .00000112 =$$

$$(\text{log. } 2) \times 4 + (\text{log. } 7) - (\text{log. } 100000000).$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Sum} = 2.049218 = \text{and log. } 100000000 = 8$$

$$2.049218 - 8 = \overline{6.049218} = \text{log. } .00000112$$

$$3.0625 = \frac{1}{2} \therefore \text{log. } 3.0625 = (\text{log. } 49) - (\text{log. } 16) =$$

$$(\text{log. } 7) \times 2 - (\text{log. } 2) \times 4.$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Difference} = 0.486076 = \text{log. of } 3.0625.$$

(2)

$$49\frac{1}{2} = 3^2 \times 11 \times \frac{1}{2} \therefore \text{log. } 49\frac{1}{2} = (\text{log. } 3) \times 2 + (\text{log. } 11)$$

$$+ (\text{log. } \frac{1}{2}).$$

$$\text{Log. } 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Log. } 11 = 1.041393$$

$$\text{Log. } \frac{1}{2} = \overline{1.698970}$$

$$\text{Sum} = 1.694605 = \text{log. of } 49\frac{1}{2}.$$

$$363 = 11^2 \times 3 \therefore \text{log. } 363 = (\text{log. } 11) \times 2 + (\text{log. } 3).$$

$$\text{Log. } 11 = 1.041393 \times 2 = 2.082786$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 2.559907 = \text{log. of } 363.$$

Log. .5 or  $\frac{1}{2}$  = 1.698970, and by altering the characteristic we get 0.698970 for log. of 5.

(Continued on next page.)

(2 continued.)

$$4.09 = 4\frac{1}{11} = \frac{44}{11} = 3^3 \times 5 \div 11 \therefore \log. 4.09 = (\log. 3) \times 2 + (\log. 5) - (\log. 11).$$

$$\log. 3 = .477121 \times 2 = 0.954242$$

$$\log. 5 = .698970$$

$$\hline 1.653212$$

$$\log. 11 = 1.041393 \text{ and } 1.653212 - 1.041393 = 0.611819 = \log. \text{ of } 4.09.$$

$$2.4 = 2\frac{2}{5} = \frac{11}{5} = 11 \times 2 \div 5 \therefore \log. 2.4 = (\log. 11) + (\log. 2) - (\log. 5) \times 2.$$

$$\log. 2 = (\log. 10) - (\log. 5) = 1 - 0.698970 = 0.301030.$$

$$\log. 11 = 1.041393$$

$$\log. 2 = 0.301030$$

$$\hline 1.342423$$

$$\log. 3 = 0.477121 \times 2 = 0.954242 \text{ and } 1.342423 - 0.954242 = 0.388181 = \log. \text{ of } 2.4.$$

$$392.72 = 392\frac{72}{100} = \frac{4382}{125} = 2^4 \times 3^3 \times 10 \div 11 \therefore \log. 392.72 = (\log. 2) \times 4 + (\log. 3) \times 3 + (\log. 10) - (\log. 11).$$

$$\log. 2 = 0.301030 \times 4 = 1.204120$$

$$\log. 3 = 0.477121 \times 3 = 1.431363$$

$$\log. 10 = 1$$

$$\hline \text{Sum} = 3.635483$$

$$\log. 11 = 1.041393 \text{ and } 3.635483 - 1.041393 = 2.594090 = \log. \text{ of } 392.72.$$

$$293333\frac{1}{3} = \frac{880000}{3} = 2^3 \times 11 \times 10000 \div 3 \therefore \log. 293333\frac{1}{3} = (\log. 2) \times 3 + (\log. 11) + (\log. 10000) - (\log. 3).$$

$$\log. 2 = 0.301030 \times 3 = 0.903090$$

$$\log. 11 = 1.041393$$

$$\log. 10000 = 4$$

$$\hline \text{Sum} = 5.944483$$

(Continued on next page.)

(2 continued.)

$$\text{Log. } 3 = 0.477121 \text{ and } 5.944483 - 0.477121 = 5.467362 = \text{log. of } 293333\frac{1}{3}.$$

$$19.965 = 11^3 \times 5 \times 3 \div 1000 \therefore \text{log. } 19.965 = (\text{log. } 11) \times 3 + (\text{log. } 5) + (\text{log. } 3) - (\text{log. } 1000).$$

$$\text{Log. } 11 = 1.041393 \times 3 = 3.124179$$

$$\text{Log. } 5 = 0.698970$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 4.300270$$

$$\text{Log. } 1000 = 3 \text{ and } 4.300270 - 3 = 1.300270 = \text{log. of } 19.965.$$

## EXERCISE 156—Page 336.

(1)

Here we have given the first term 4, the number of terms 17 and the sum of the series 884, to find  $l$ , the last term.

$$\text{Then } l = \frac{2r}{n} - a = \frac{884 \times 2}{17} - 4 = 104 - 4 = 100.$$

(2)

Here we have given the first term 21, the last term 497 and the number of terms 41, to find the common difference.

$$\text{Then } d = \frac{l - a}{n - 1} = \frac{497 - 21}{41 - 1} = \frac{476}{40} = \frac{119}{10} = 11 \frac{9}{10}.$$

(3)

Here we have given  $a$ ,  $l$ , and  $d$ , to find  $n$ , and since  $a = 12$ ,  $l = 96$ , and  $d = 6$ , we have

$$n = \frac{l - a}{d} + 1 = \frac{96 - 12}{6} + 1 = \frac{84}{6} + 1 = 14 + 1 = 15.$$

(4)

Here we have given  $l$ ,  $d$ , and  $s$ , to find  $n$ , and since  $l = 14$ ,  $d = 1$ , and  $s = 105$ , we have

$$n = \frac{2l + d}{2d} + \sqrt{\left(\frac{2l + d}{2d}\right)^2 - \frac{2s}{d}} = \frac{2 \times 14 + 1}{2 \times 1} + \sqrt{\left(\frac{2 \times 14 + 1}{2 \times 1}\right)^2 - \frac{2 \times 105}{1}} = 14\frac{1}{2} + \sqrt{\left(\frac{29}{2}\right)^2 - 210} = 14\frac{1}{2} + \sqrt{24\frac{1}{4} - 210} = 14\frac{1}{2} + \sqrt{\frac{1}{4}} = 14\frac{1}{2} + \frac{1}{2} = 15.$$

(5)

Here we have given  $a$ ,  $d$ , and  $s$ , to find  $l$ , and since  $a = \frac{2}{3}$ ,  $d = \frac{2}{3}$ , and  $s = 1180$ , we have

$$l = -\frac{1}{2}d + \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2} = -\frac{1}{2} \text{ of } \frac{2}{3} + \sqrt{2 \times \frac{2}{3} \times 1180 + \left(\frac{2}{3} - \frac{1}{2} \times \frac{2}{3}\right)^2} = -\frac{1}{3} + \sqrt{1773\frac{2}{3} + \left(\frac{1}{3}\right)^2} = -\frac{1}{3} + \sqrt{1773\frac{2}{3} + \frac{1}{9}} = -\frac{1}{3} + \sqrt{1773\frac{2}{3} + \frac{1}{9}} = -\frac{1}{3} + \frac{1}{3} = \frac{1}{3} = 39\frac{1}{3}.$$

(6)

Here we have given  $a$ ,  $l$ , and  $s$ , to find  $d$ , and since  $a = 8$ ,  $l = 170$ , and  $s = 4895$ , we have

$$d = \frac{(l + a)(l - a)}{2s - l - a} = \frac{(170 + 8)(170 - 8)}{2 \times 4895 - 170 - 8} = \frac{178 \times 162}{9790 - 178} = \frac{28836}{9612} = 3.$$

(7)

Here we have given  $a$ ,  $l$ , and  $d$ , to find  $n$ , and since  $a = 5$ ,  $l = 27\frac{1}{2}$ , and  $d = 2\frac{1}{2}$ , we have

$$n = \frac{l - a}{d} + 1 = \frac{27\frac{1}{2} - 5}{2\frac{1}{2}} + 1 = \frac{22\frac{1}{2}}{2\frac{1}{2}} + 1 = \frac{45}{5} + 1 = 10 + 1 = 11$$

and since  $l = 14$ ,

$$= \frac{2 \times 14 + 1}{2 \times 1} +$$

$$9)^2 - 210 =$$

15.

and since  $a = \frac{2}{3}$ ,

$$- \frac{1}{3} \text{ of } \frac{2}{3} +$$

$$\frac{4790}{3} + (\frac{1}{3})^2 =$$

$$\frac{1}{3} = \frac{1}{3} = 39\frac{1}{3}.$$

and since  $a = 8$ ,

$$\frac{178 \times 162}{9790 - 178} =$$

and since  $a = 5$ ,

$$= 10 + 1 = 11$$

(8)

Here we have given  $a$ ,  $l$ , and  $n$ , to find  $s$ , and since  $a = 2$ ,  $l = 478$ , and  $n = 86$ , we have

$$s = (a + l)\frac{n}{2} = (2 + 478)\frac{86}{2} = 480 \times 43 = 20640.$$

(9)

Here we have given  $a$ ,  $l$ , and  $d$ , to find  $s$ , and since  $a = 2$ ,  $l = 998$ , and  $d = 6$ , we have

$$s = \frac{(l+a)(l-a)}{2d} + \frac{l+a}{2} = \frac{(998+2)(998-2)}{2 \times 6} + \frac{998+2}{2} =$$

$$\frac{1000 \times 996}{12} + \frac{1000}{2} = 83000 + 500 = 83500.$$

(10)

Here we have given  $a$ ,  $n$ , and  $d$ , to find  $l$ , and since  $a = 5$ ,  $n = 11$ , and  $d = 2\frac{1}{4}$ , we have

$$l = a + (n-1)d = 5 + (11-1)2\frac{1}{4} = 5 + (10 \times 2\frac{1}{4}) = 5 + 4\frac{1}{2} = 4\frac{1}{2} = 27\frac{1}{2}.$$

(11)

Here we have given  $l$ ,  $d$ , and  $n$ , to find  $s$ , and since  $l = 199$ ,  $d = 11$ , and  $n = 19$ , we have

$$s = \left\{ 2l - (n-1)d \right\} \frac{n}{2} = \left\{ 2 \times 199 - (19-1)11 \right\} \frac{19}{2} =$$

$$\left\{ 398 - (18 \times 11) \right\} \frac{19}{2} = 200 \times \frac{19}{2} = 1900.$$

(12)

Here we have given  $s$ ,  $a$ , and  $l$ , to find  $n$ , and since  $s = 39840$ ,  $a = 2$ , and  $l = 478$ , we have

$$n = \frac{2s}{l+a} = \frac{2 \times 39840}{478+2} = \frac{79680}{480} = 166.$$



(13)

Here we have given  $s$ ,  $l$ , and  $a$ , to find  $d$ , and since  $s = 83500$ ,  $l = 998$ , and  $a = 2$ , we have

$$l = \frac{(l+a)(l-a)}{2s-l-a} = \frac{(998+2)(998-2)}{(2 \times 83500) - 998 - 2} = \frac{1000 \times 996}{167000 - 1000} = \frac{996000}{166000} = 6$$

(14)

Here we have given  $s$ ,  $a$ , and  $d$ , to find  $n$ , and since  $s = 360$ ,  $a = 2$ , and  $d = 2$ , we have

$$n = \frac{d-2a}{2d} + \sqrt{\frac{2s}{d} + \left(\frac{2a-d}{2d}\right)^2} = \frac{2-(2 \times 2)}{2 \times 2} +$$

$$\sqrt{\frac{2 \times 360}{2} + \left(\frac{(2 \times 2) - 2}{2 \times 2}\right)^2} = -\frac{1}{2} + \sqrt{360 + \left(\frac{1}{2}\right)^2} =$$

$$-\frac{1}{2} + \sqrt{360\frac{1}{4}} = -\frac{1}{2} + 18.7228 = 18.2228 \text{ days} = 18 \text{ days, } 5 \text{ hours, } 10 \text{ minutes, } 27.264 \text{ seconds.}$$

(15)

Here we have given  $s$ ,  $a$ , and  $d$ , to find  $l$ , and since  $s = 83500$ ,  $a = 2$ , and  $d = 6$ , we have

$$l = -\frac{1}{2}d + \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2} = -\frac{1}{2} \times 6 + \sqrt{2 \times 6 \times 83500 + \left(2 - \frac{1}{2} \times 6\right)^2} = -3 + \sqrt{1002000 + (2-3)^2} = -3 + \sqrt{1002001} = -3 + 1001 = 998.$$

(16)

Here we have given  $s$ ,  $n$ , and  $l$ , to find  $a$ , and since  $s = \$1125$ ,  $n = 18$ , and  $l = 120$ , we have

$$a = \frac{2s}{n} - l = \frac{2 \times 1125}{18} - 120 = 125 - 120 = 5.$$

(17)

Here we have given  $a$ ,  $l$ , and  $n$ , to find  $d$ , and since  $a = 5$ ,  $l = 27\frac{1}{2}$ , and  $n = 11$  we have

$$d = \frac{l - a}{n - 1} = \frac{27\frac{1}{2} - 5}{11 - 1} = \frac{22\frac{1}{2}}{10} = 2\frac{1}{4}.$$

(18)

Here we have  $a$ ,  $d$ , and  $n$  given, to find  $s$ , and since to deposit one stone he must walk 5 yards, and the distance travelled for each succeeding stone is 5 yards, therefore  $a = 5$ ,  $d = 5$ , and  $n = 220$ .

$$\begin{aligned} \text{Then } s &= \left\{ 2a + (n - 1)d \right\} \frac{n}{2} = \left\{ 2 \times 5 + (220 - 1)5 \right\} \frac{1}{2} \times 220 \\ &= \left\{ 10 + (219 \times 5) \right\} 110 = \\ 1105 \times 110 &= 121550 \text{ yards} = 69\frac{1}{6} \text{ miles.} \end{aligned}$$

(19)

Here we have  $s$ ,  $n$ , and  $l$  given, to find  $a$ , and since  $s = 39840$ ,  $n = 166$ , and  $l = 478$ , we have

$$a = \frac{2s}{n} - \frac{l}{n} = \frac{2 \times 39840}{166} - \frac{478}{166} = 480 - 478 = 2.$$

(20)

Here we have  $n$ ,  $a$ , and  $d$  given, to find  $s$ , and since  $n = 12$ ,  $a = 4$ , and  $d = 2$ , we have

$$s = \left\{ 2a + (n - 1)d \right\} \frac{n}{2} = \left\{ 2 \times 4 + (12 - 1)2 \right\} \frac{1}{2} \times 12 = \left\{ 8 + (11 \times 2) \right\} 6 = 30 \times 6 = 180.$$

(21)

Here we have given  $a$ ,  $l$ , and  $n$ , to find  $s$ , and  $a = 1$ ,  $l = 24$ , and  $n = 24$ .

$$\text{Then } s = (a + l) \frac{n}{2} = (1 + 24) \frac{1}{2} \times 24 = 25 \times 12 = 300.$$

## EXERCISE 157—Page 342.

(1)

Here  $n = 11$ ,  $a = £1024$ , and  $r = 1\frac{1}{2}$ .Then  $l = ar^{n-1} = 1024 \times (\frac{3}{2})^{10} = 1024 \times \frac{59049}{1024} = £59049$ 

$$r = \frac{rl - a}{r - 1} = \frac{\frac{3}{2} \times 59049 - 1024}{\frac{3}{2} - 1} = \frac{177147 - 1024}{\frac{1}{2}} = \frac{176123}{\frac{1}{2}} = £176123 = \text{whole fortune.}$$

(2)

Here  $a = 7$ ,  $l = 1240029$  and  $s = 1860040$ .

$$\text{Then } r = \frac{s - a}{s - l} = \frac{1860040 - 7}{1860040 - 1240029} = \frac{1860033}{620011} = 3.$$

(3)

Here  $n = 12$ ,  $a = £1$ , and  $l = £2048$ .

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{2048}{1}\right)^{\frac{1}{11}} = \sqrt[11]{2048} = 2.$$

$$s = \frac{rl - a}{r - 1} = \frac{(2 \times 2048) - 1}{2 - 1} = 4096 - 1 = £4095.$$

(4)

Here  $r = \frac{3}{2}$ ,  $n = 8$ , and  $l = 106\frac{2}{3}$ .

$$\text{Then } s = \frac{l(r^n - 1)}{(r - 1)r^{n-1}} = \frac{106\frac{2}{3} \times [(\frac{3}{2})^8 - 1]}{(\frac{3}{2} - 1)(\frac{3}{2})^7} = \frac{4197\frac{1}{2} \times 6305}{1 \times 121\frac{1}{2}} = \frac{2645625}{512} = 5167\frac{1}{8}.$$

(5)

Here  $a = 1$ ,  $n = 7$ , and  $r = 3$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (3^7 - 1)}{3 - 1} = \frac{2187 - 1}{2} = 1093.$$

(6)

Here  $a = 1$ ,  $l = 10077696$ , and  $n = 10$ .

$$\text{Then } s = \frac{l^{\frac{n}{2}} - a^{\frac{n}{2}}}{l^{\frac{1}{2}} - a^{\frac{1}{2}}} = \frac{(10077696)^{5} - 1}{(10077696)^{1} - 1} = \frac{\sqrt[5]{(10077696)^{10}} - 1}{\sqrt[5]{10077696} - 1} = \frac{\sqrt[5]{(216)^{10}} - 1}{\sqrt[5]{216} - 1} = \frac{6^{10} - 1}{6 - 1} = \frac{60466176 - 1}{5} = 12093235.$$

(7)

Here  $a = 6$ ,  $l = 3072$ , and  $s = 6138$ .

$$\text{Then } r = \frac{s - a}{s - l} = \frac{6138 - 6}{6138 - 3072} = \frac{6132}{3066} = 2.$$

(8)

Here  $r = 2$ ,  $n = 11$ , and  $s = 20470$ .

$$\text{Then } l = \frac{(r - 1)sr^{n-1}}{r^n - 1} = \frac{(2 - 1) \times 20470 \times 2^{10}}{2^{11} - 1} = \frac{20470 \times 1024}{2047} = 10240.$$

(9)

Here  $a = 1s.$ ,  $n = 12$ , and  $r = 2$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{12} - 1)}{2 - 1} = 4095s. = £204 \text{ } 15s.$$

(10)

Here  $a = 1$  farthing,  $r = 2$  and  $n = 32$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{32} - 1)}{2 - 1} = 4294967295 \text{ far.} = £4473924 \text{ } 5s. \text{ } 3\frac{1}{2}d.$$

(11)

Here  $a = 4$ ,  $l = 78732$ , and  $n = 10$ .

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{78732}{4}\right)^{\frac{1}{10-1}} = \sqrt[9]{19683} = 3.$$

(12)

Here  $a = 5$ ,  $r = 2$ , and  $n = 7$ .

$$\text{Then } l = ar^{n-1} = 5 \times 2^{7-1} = 5 \times 2^6 = 5 \times 64 = 320.$$

(13)

Here  $a = 5$ ,  $l = 327680$ , and  $r = 4$ .

$$\text{Then } s = \frac{rl - a}{r - 1} = \frac{(327680 \times 4) - 5}{4 - 1} = 131071\frac{1}{3} = 43690\frac{1}{3}.$$

(14)

Here  $a = 1$ ,  $r = 2$ , and  $n = 64$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{64} - 1)}{2 - 1} = 18446744073709551615 \text{ gr.}$$

$$18446744073709551615 \div (7680 \times 64) = 37529996894754 \text{ bush.}$$

$$\$1.50 \times 37529996894754 = \$56294995342131$$

(15)

Here  $r = 3$ ,  $n = 10$ , and  $s = 295240$ .

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(3-1) \times 295240 \times 3^9}{3^{10} - 1} = \frac{2 \times 295240 \times 19683}{59048} = 196830.$$

(16)

Here  $a = 1$ ,  $l = 2048$ , and  $n = 12$ .

$$\text{Then } s = \frac{l^{\frac{n}{2}} - a^{\frac{n}{2}}}{l^{\frac{1}{2}} - a^{\frac{1}{2}}} = \frac{2048^{\frac{12}{2}} - 1^{\frac{12}{2}}}{2048^{\frac{1}{2}} - 1^{\frac{1}{2}}} =$$

$$\frac{\sqrt{(2048)^{12}} - 1}{\sqrt{2048} - 1} = \frac{2^{12} - 1}{2 - 1} = 2^{12} - 1 = 4095.$$

(17)

Here  $a = 5$ ,  $r = 4$ , and  $n = 9$ .

$$\text{Then } l = ar^{n-1} = 5 \times 4^{9-1} = 5 \times 4^8 = 5 \times 65536 = 327680.$$

P

## EXERCISE 158.—Page 344.

(1)

Here  $a = \frac{1}{2}$ , and  $r = \frac{1}{2}$ .

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1.$$

(2)

Here  $a = 4$ , and  $r = \frac{1}{2}$ ,

$$\text{Then } s = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8.$$

(3)

Here  $a = \frac{72}{100}$ , and  $r = \frac{1}{100}$ .

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{72}{100}}{1-\frac{1}{100}} = \frac{\frac{72}{100}}{\frac{99}{100}} = \frac{72}{99} = \frac{8}{11}.$$

(4)

Here  $a = \frac{1234}{10000}$ , and  $r = \frac{1}{10000}$ .

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{1234}{10000}}{1-\frac{1}{10000}} = \frac{\frac{1234}{10000}}{\frac{9999}{10000}} = \frac{1234}{9999} = \frac{1234}{10000}.$$

## EXERCISE 159.—Page 345.

(1)

Since there are 9 means and 2 extremes the number of terms is 11.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{92-2}{11-1} = \frac{90}{10} = 9.$$

1st term = 2; 2nd =  $2 + 9 = 11$ ; 3rd =  $11 + 9 = 20$ ; 4th =  $20 + 9 = 29$ ; 5th =  $29 + 9 = 38$ ; 6th =  $38 + 9 = 47$ ; and so on.

And series is 2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92.

(2)

Since there are 4 means and 2 extremes the number of terms is 6.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{50-7}{6-1} = \frac{43}{5} = 8\frac{3}{5}.$$

1st term = 7; 2nd =  $7 + 8\frac{3}{5} = 15\frac{3}{5}$ ; 3rd =  $15\frac{3}{5} + 8\frac{3}{5} = 24\frac{1}{5}$ ; 4th =  $24\frac{1}{5} + 8\frac{3}{5} = 32\frac{4}{5}$ ; 5th =  $32\frac{4}{5} + 8\frac{3}{5} = 41\frac{2}{5}$ ; and 6th =  $41\frac{2}{5} + 8\frac{3}{5} = 50$ .

And series is 7,  $15\frac{3}{5}$ ,  $24\frac{1}{5}$ ,  $32\frac{4}{5}$ ,  $41\frac{2}{5}$ , 50.

(3)

Since there are 8 means and 2 extremes the number of terms is 10.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{4096}{4096}\right)^{\frac{1}{10-1}} = \left(\frac{1}{1}\right)^{\frac{1}{9}} = 1.$$

1st term = 4096; 2nd =  $4096 \times \frac{1}{2} = 2048$ ; 3rd =  $2048 \times \frac{1}{2} = 1024$ ; 4th =  $1024 \times \frac{1}{2} = 512$ ; 5th =  $512 \times \frac{1}{2} = 256$ , and so on.

And the means are 2048, 1024, 512, 256, 128, 64, 32, and 16.



(4)

Since there are 7 means and 2 extremes the number of terms is 9.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{1679616}{14}\right)^{\frac{1}{8-1}} = (1679616)^{\frac{1}{7}} = 6.$$

1st term = 14; 2nd =  $14 \times 6 = 84$ ; 3rd =  $84 \times 6 = 504$ ; 4th =  $504 \times 6 = 3024$ ; 5th =  $3024 \times 6 = 18144$ , and so on.

And the means are 84, 504, 3024, 18144, 108864, 653184, and 3919104.

## EXERCISE 160.—Page 347

(1)

Assume 4 to be the number of men.

Then  $2 \times 4 = 8 =$  number of women.

And  $8 \times 3 = 24 =$  number of children.

6d.  $\times 4 = 24$ d. = amount received by the men.

4d.  $\times 8 = 32$ d. = " " " women.

2d.  $\times 24 = 48$ d. = " " " children.

Sum, = 104d., but it should, by question, = 78d.

$$\text{Then } 104 : 78 :: 4 : \frac{78 \times 4}{104} = 3 = \text{number of men.}$$

$3 \times 2 = 6 =$  number of women, and  $6 \times 3 = 18 =$  number of children. [ren.]

(2)

Assume £8 to be the price of the harness.

Then  $\text{£}8 \times 2 = 16 =$  price of horse.

And  $\text{£}8 + \text{£}16 = \text{£}24 \times 2 = 48 =$  " chaise.

Sum, = £ 72, but it should by question = £60.

$$\text{Then } \text{£}72 : \text{£}60 :: \text{£}8 : \frac{8 \times 60}{72} = \text{£}6 \ 13 \ 4 = \text{price of harness.}$$

$\text{£}6 \ 13 \ 4 \times 2 = 13 \ 6 \ 8 =$  " horse.  
 $\text{£}6 \ 13 \ 4 + \text{£}13 \ 6 \ 8 = \text{£}20 \ 0 \ 0 =$  " chaise.

One  
One  
should

A ca  
B  
C

Then

There

A and B  
10 of  
A c  
There  
Then  
whole.

\*The m  
cannot tr  
them by s

number of terms

$$(1679616)^{\frac{1}{4}} = 6.$$

$$= 84 \times 6 = 504;$$

3144, and so on.

864, 653184, and

(3)

Assume 20 as C's age.

$$\text{Then } 20 \times 3 = 60 = \text{B's age.}$$

$$\text{And } 60 \times 2 = 120 = \text{A's age.}$$

$$\text{Sum} = 200, \text{ but by question it should} = 140.$$

$$20 \times 140$$

$$\text{Then } 200 : 140 :: 20 : \frac{200}{140} = 14 = \text{C's age.}$$

$$200$$

$$14 \times 3 = 42 = \text{B's age, and } 42 \times 2 = 84 = \text{A's age.}$$

(4)

Assume 100.

$$\text{One fourth of } 100 = 25 \text{ and remainder} = 100 - 25 = 75.$$

$$\text{One fifth of } 75 = 15 \text{ and remainder} = 75 - 15 = 60, \text{ but it should by the question} = 72.$$

$$100 \times 72$$

$$\text{Then } 60 : 72 :: 100 : \frac{100 \times 72}{60} = 120.$$

$$60$$

the men.

" women.

" children.

tion, = 78d.

r of men.

number of child-

[ren.

of the harness.

e.

e.

question = £60.

price of harness.

(5)

A can do the work in 7 days  $\therefore$  he will do  $\frac{1}{7}$  of it in 1 day.B " " 5 "  $\therefore$  "  $\frac{1}{5}$  " "C " " 6 "  $\therefore$  "  $\frac{1}{6}$  " "Then all working together will do  $\frac{1}{7} + \frac{1}{5} + \frac{1}{6} = \frac{17}{105}$  in 1 day.Therefore to do the whole work it will take them  $\frac{1}{\frac{17}{105}} = \frac{105}{17} =$ 12 $\frac{3}{17}$  days.

(6)\*

A and B working together can do it in 10 days  $\therefore$  they will do  $\frac{1}{10}$  of it in 1 day.A can do it in 15 days  $\therefore$  he will do  $\frac{1}{15}$  of it in 1 day.Therefore  $\frac{1}{10} - \frac{1}{15} = \frac{1}{30}$  = amount done by B in 1 day.Then if he does  $\frac{1}{30}$  in 1 day, it will take him 30 days to do the whole.

" horse.

" chaise.

\*The mode of working these questions by position is so simple that they cannot trouble any one; it has therefore been thought advisable to work them by simple analysis.

(7)\*

The first pipe empties the whole of it in 1 hour.

The second pipe empties  $\frac{1}{2}$  of it in 1 hour.

The third pipe empties  $\frac{1}{3}$  of it in 1 hour.

Then all these pipes running together will empty  $1 + \frac{1}{2} + \frac{1}{3}$   
 $= \frac{11}{6}$  in 1 hour.

Therefore to empty the cistern it will take  $1 \div \frac{11}{6} = \frac{6}{11}$  hours.

(8)

Assume 84

One third of 84 = 28

One sixth of 84 = 14

One seventh of 84 = 12

Sum = 54, but by question it should = 27.

$$\text{Then } 54 : 27 :: 84 : \frac{84 \times 27}{54} = 42.$$

(9)

All 5 mills working together will grind  $7 + 5 + 4 + 3 + 1$   
 $= 20$  bushels in 1 hour.

Therefore to grind 500 bushels it will take them  $500 \div 20 =$   
 25 hours.

(10)\*

One pipe fills  $\frac{1}{3}$  of the cistern in 1 hour, and the other  
 empties  $\frac{1}{6}$  of it in 1 hour.

Then  $\frac{1}{3} - \frac{1}{6} = \frac{1}{6} =$  part of the cistern filled in 1 hour  
 when both are left open.

And if  $\frac{1}{6}$  of it is filled in 1 hour, the whole will be filled in  
 $\frac{1}{\frac{1}{6}} = 6$  hours.

$\frac{1}{6}$

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\* See note on page 227.

EXERCISE 161.—Page 352.

(1)

Assume 60 for father's age, then 15 = son's.

$$\begin{array}{r} 5 \\ \hline 5)55 \\ \hline 11 \\ 10 \\ \hline -1 \end{array}$$

Assume 100 for father's age, then 25 = son's.

$$\begin{array}{r} 5 \\ \hline 5)95 \\ \hline 19 \\ 20 \\ \hline +1 \end{array}$$

Errors. Assumed numbers.

$$\begin{array}{rclcl} -1 & \times & 100 & = & 100 \\ +1 & \times & 60 & = & 60 \\ \hline \end{array}$$

Sum of errors = 2      Sum of products = 160

Therefore result required =  $160 \div 2 = 80$  = father's age, and  
 $\frac{1}{2}$  of 80 = 20 = son's age.

(2)

Assume 80

Assume 44

$$\begin{array}{r} 34 \\ \hline 46 \\ 3 \\ \hline 138 \\ 80 \\ \hline 58 \\ \frac{1}{2} \text{ of } 80 = 20 \\ \hline + 38 \end{array}$$

$$\begin{array}{r} 34 \\ \hline 10 \\ 3 \\ \hline 30 \\ 44 \\ \hline -14 \\ \frac{1}{2} \text{ of } 44 = 11 \\ \hline -25 \end{array}$$

(Continued on next page.)

(2 continued.)

Errors.	Assumed numbers.	
- 25	× 80	= 2000
+ 38	× 44	= 1672
Sum of errors = 63		3672
Therefore result required = $3672 \div 63 = 58\frac{2}{3}$ .		

(4)

$$\begin{array}{r} \text{Assume 18} \quad \text{and 7} \\ \text{One half of 18} = \frac{9}{14} \quad 2 \times 7 = \frac{14}{14} \\ \hline 14 \\ - 5 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Assume 22} \quad \text{and 3} \\ \text{One half of 22} = \frac{11}{6} \quad 2 \times 3 = \frac{6}{6} \\ \hline 6 \\ + 5 \\ \hline \end{array}$$

Errors.	Assumed numbers.	
- 5	× 22	= 110
+ 5	× 18	= 90
Sum of errors = 10	Sum of products = 200	
Then $200 \div 10 = 20$ = one number, and $25 - 20 = 5$ = other number.		

(5)

A.	B.
Suppose 8	6
224	9
180	12
132	15
8)48	18
24	21
27	24
+ 6	27
6	132
36	
72	
108	
132	

A 3)36

12

9 - 6 = 3 = difference of errors.

A.	B.
Suppose 6	6
224	9
135	12
81	15
6)54	18
27	21
36	24
54	27
+ 9	36
9	132
18	
27	
36	
45	
54	

Suppose A  
Then 9 +  
fir  
 $6 + 1 = 7$ ,

Suppose A  
Then 11 +  
fir  
 $7 + 1 = 8$ ,

(7)

Assume 30.

$$\begin{aligned} \frac{1}{2} \text{ of } 30 &= 15; \frac{1}{3} \text{ of } 30 = 10; \\ \frac{1}{4} \text{ of } 30 &= 7\frac{1}{2}; \text{ and } \frac{1}{5} \text{ of } 30 = 6; \\ 15 \times 7\frac{1}{2} \times 6 \times 5 &= 3375; \\ 3375 - 6998\frac{1}{2} &= -3623\frac{1}{2} = \text{error.} \end{aligned}$$

Assume 60.

$$\begin{aligned} \frac{1}{2} \text{ of } 60 &= 30; \frac{1}{3} \text{ of } 60 = 20; \\ \frac{1}{4} \text{ of } 60 &= 15; \text{ and } \frac{1}{5} \text{ of } 60 = 12. \\ 30 \times 20 \times 15 \times 12 &= 54000. \\ 54000 - 6998\frac{1}{2} &= +47001\frac{1}{2} = \text{error.} \\ 30^4 &= 810000, \text{ and } 60^4 = 12960000 \\ -3623\frac{1}{2} \times 12960000 &= 46959264000 \\ +47001\frac{1}{2} \times 810000 &= 38071296000 \end{aligned}$$

$$\text{Sum} = 50625$$

$$\text{Sum} = 85030560000$$

$$85030560000 \div 50625 = 1679616$$

4th root = square root of square root.

$$\sqrt{1679616} = 1296, \text{ and } \sqrt{1296} = 36 = \text{required number.}$$

NOTE.—For reason why we multiply by the 4th powers of the assumed numbers and then take the 4th root of the quotient, see Arith. page 353, Example 11.

It may, however, perhaps be clearer from the following illustration:

Let  $x$  = the number required.

$$\begin{array}{ccccccccc} x & x & x & x & x & x^4 & & & \\ \text{Then } - & \times & - & \times & - & \times & - & - & = 6998\frac{1}{2} \\ 2 & 4 & 5 & 6 & & 240 & & & \\ \therefore x^4 & = & 1679616 & & & & & & \\ \therefore x & = & \sqrt[4]{1679616} = 36. & & & & & & \end{array}$$

(8)

Suppose A had 9s. at first.

Then  $9 + 1 = 10$ ;  $10 \div 2 = 5$ ;  $5 + 1 = 6$  = what B had at first.

$6 + 1 = 7$ , but should  $= 9 - 1 = 8$ .

$$\text{Error} = 7 - 8 = -1.$$

Suppose A had 11s. at first.

Then  $11 + 1 = 12$ ;  $12 \div 2 = 6$ ;  $6 + 1 = 7$  = what B had at first.

$7 + 1 = 8$ , but should  $= 11 - 1 = 10$ .

$$\text{Error} = 8 - 10 = -2.$$

(Continued on next page.)

(8 continued.)

Errors.

$$- 2 \times 9 = 18$$

$$- 1 \times 11 = 11$$

$$\text{Diff.} = 1 \quad \text{diff.} = 7$$

$7 \div 1 = 7 =$  shillings A had at first.

$7 + 1 = 8$ ;  $8 \div 2 = 4$ ;  $4 + 1 = 5 =$  shillings B had at first.

(9)

Assume 24 and 6.

$$\frac{24}{2} + \frac{24}{3} + \frac{24}{6} = 24.$$

$$\frac{6}{2} + \frac{6}{3} \text{ of } 6 + \frac{6}{6} = 9.$$

$$24 - 9 = + 15 = \text{error.}$$

Assume 20 and 10.

$$\frac{20}{2} + \frac{20}{3} + \frac{20}{6} = 20.$$

$$\frac{10}{2} + \frac{10}{3} \text{ of } 10 + \frac{10}{6} = 15.$$

$$20 - 15 = + 5 = \text{error.}$$

Errors.

$$+ 15 \times 20 = 300$$

$$+ 5 \times 24 = 120$$

$$\text{Diff.} = 10 \quad \text{diff.} = 180$$

$$180 \div 10 = 18 = \text{one number.}$$

$$30 - 18 = 12 = \text{other number.}$$

(10)

Suppose 1st horse to be worth £20.

$$20 + 50 = 70; 70 \div 2 = £35 = \text{value of 2nd horse.}$$

$$35 + 50 = 85, \text{ but it should equal } 60, \text{ i. e. } (20 \times 3).$$

$$\text{Then } 60 - 85 = - 25 = \text{error.}$$

Suppose 1st horse to be worth £60.

$$£60 + £50 = £110; £110 \div 2 = £55 = \text{worth of 2nd horse.}$$

$$55 + 50 = 105, \text{ but it should equal } 180, \text{ i. e. } (60 \times 3).$$

$$180 - 105 = + 75 = \text{error.}$$

Errors.

$$+ 75 \times 20 = 1500$$

$$- 25 \times 60 = 1500$$

$$\text{Sum} = 100$$

$$\text{Sum} = 3000$$

$$3000 \div 100 = £30 = \text{value of 1st horse.}$$

$$£30 + £50 = £80; £80 \div 2 = £40 = \text{value of 2nd horse.}$$

(1)

Suppose there were 11 beggars.

$$11 \times 4 = 44; 44 + 6 = 50 = \text{number of pence he had.}$$

$$11 \times 6 = 66; 66 - 12 = 54 = \text{ " " " }$$

$$54 - 50 = + 4 = \text{error.}$$

Suppose there were 12 beggars.

$$12 \times 4 = 48; 48 + 6 = 54 = \text{pence he had.}$$

$$12 \times 6 = 72; 72 \div 12 = 60 = \text{pence he had.}$$

$$60 - 54 = + 6 = \text{error.}$$

Errors.

$$+ 6 \times 11 = 66$$

$$+ 4 \times 12 = 48$$

$$\text{Diff.} = 2 \quad \text{diff.} = 18, \text{ and } 18 \div 2 = 9 = \text{number of beggars.}$$

EXERCISE 162.—Page 357.

(1)

Here  $P = \$713.29$ ,  $r = .045$ , and  $t = 14$ .

$$\text{Then } A = P(1+r)^t, \text{ or } \log. A = \log. P + \log. (1+r) \times t$$

$$= 2.853267 + (.019116 \times 14) = 3.120891 = \log. \text{ of } Ans.$$

$$\text{Hence amount} = \$1320.96.$$

(2)

Here  $n = 7$ ,  $r = .015$ .

$$\log. n. = .845098$$

$$\text{Then } t = \frac{\log. n.}{\log. (1+r)} = \frac{.845098}{.006466} = 130.698 \text{ payments, and}$$

$$130.698 \div 4 = 32.674 \text{ years} = 32 \text{ years 8 months 2 days.}$$

(3)

Here  $A = \$1111.11$ ,  $P = 111.11$ , and  $r = .08$ .

$$\log. A - \log. P = 3.045757 - 2.045753 = 1.000004$$

$$\text{Then } t = \frac{\log. A - \log. P}{\log. (1+r)} = \frac{1.000004}{.033424} = 29.918 \text{ years} = 29 \text{ years 11 months.}$$



(4)

Here  $A = \$3333.33$ ,  $P = \$222.22$ , and  $t = 120$ .

$$\text{Then } r = \sqrt[t]{\frac{A}{P}} - 1; \text{ or } \log. (1+r) = \frac{\log. A - \log. P}{t} =$$

$$\frac{3.522878 - 2.346783}{120} = \frac{1.176095}{120} = .0098007. \text{ Hence } 1+r$$

$$= 1.0228, r = .0228, \text{ and rate per cent.} = 2\frac{3}{4}\%.$$

(5)

Here  $n = 2$  and  $r = .07$ .

$$\text{Then } t = \frac{\log. n}{\log. (1+r)} = \frac{0.301030}{0.029384} = 10.2446 \text{ years} = 10 \text{ yrs.}$$

2 months 28 days.

(6)

Here  $A = \$100$ ,  $r = .0225$ , and  $t = 28$ .

$$\text{Then } P = \frac{A}{(1+r)^t}, \text{ or } \log. P = \log. A - \log. (1+r) \times t.$$

$$\log. P = 2 - (0.009664 \times 28) = 2 - 0.270592 = 1.729408.$$

Hence  $P = \$53.63$ .

(7)

Here  $P = \$2468.13$ ,  $r = .0375$ , and  $t = 26$ .

$$\text{Then } A = P(1+r)^t, \text{ or } \log. A = \log. P + \log. (1+r) \times t.$$

$$\log. A = 3.392368 + (0.015988 \times 26) = 3.392368 + 0.415688$$

$$= 3.808056.$$

Hence  $A = \$6427.705$ .

(8)

Here  $A = \$7137.40$ ,  $r = .0425$ , and  $t = 22$ .

$$\text{Then } P = \frac{A}{(1+r)^t}, \text{ or } \log. P = \log. A - \log. (1+r) \times t.$$

$$\log. P = 3.853540 - (0.018076 \times 22) = 3.853540 - 0.397672$$

$$= 3.455868.$$

Hence  $P = \$2856.723$ .

(16)

Here  $n = 19$ , and  $r = .0525$ .

$$\text{Then } t = \frac{\log. n}{\log. (1 + r)} = \frac{1.278754}{0.022223} = 57.5445 \text{ payments} =$$

28.7722 years = 28 years 9 months 8 days.

## EXERCISE 163.—Page 360.

(1)

Here  $r = .03$ ,  $a = 500$ ,  $A = 8365$ .

$$\text{Formula IV. } t = \frac{\sqrt{\left\{ \frac{8rA}{a} + (2-r)^2 \right\}} - (2-r)}{2r}$$

$$= \frac{\sqrt{\left\{ \frac{8 \times .03 \times 8365}{500} + (2 - .03)^2 \right\}} - (2 - .03)}{2 \times .03}$$

$$= \frac{\sqrt{\left\{ \frac{2007.6}{500} + 3.8809 \right\}} - 1.97}{.06}$$

$$= \frac{\sqrt{(4.0152 + 3.8809)} - 1.97}{.06} = \frac{\sqrt{7.8961} - 1.97}{.06}$$

$$= \frac{2.81 - 1.97}{.06} = \frac{.84}{.06} = \frac{84}{6} = 14 \text{ payments} = 7 \text{ years.}$$

(2)

Here  $a = 112.50$ ,  $r = .015$ ,  $t = 44$ .

$$\text{Formula I. } A = at \left( 1 + \frac{(t-1)r}{2} \right)$$

$$= 112.50 \times 44 \left( 1 + \frac{(44-1) \times .015}{2} \right) = 4950 \times 1.3225$$

$$= \$6546.375.$$

(3)

Here  $a = 300$ ,  $A = 1680$ , and  $t = 5$ .

$$\text{Formula III. } r = \frac{2(A - at)}{at(t-1)} = \frac{2\{1680 - (300 \times 5)\}}{300 \times 5(5-1)}$$

$$= \frac{2(1680 - 1500)}{2 \times 180} = \frac{360}{6000} = \frac{360}{6000} = .06$$

$$\therefore \text{Rate per cent} = .06 \times 100 = 6.$$

(4)

Here  $A = 2080$ ,  $r = .04$ , and  $t = 16$ .

$$\text{Formula II. } a = \frac{2A}{t\{2 + (t-1)r\}} = \frac{2 \times 2080}{16\{2 + (16-1) \cdot 04\}}$$

$$= \frac{4160}{4160} = \frac{4160}{4160} = \frac{4160}{4160} = 1$$

$= \$100 = 1$  payment or rent for half a year, hence yearly  
rent  $= \$100 \times 2 = \$200$ .

## EXERCISE 164.—Page 366.

(1)

Here  $r = .04$ , and  $v = \$3000$ .

$$\text{Then } a = vr = 3000 \times .04 = \$120.$$

(2)

Here  $a = 563$ , and  $v = 11260$ 

$$\text{Then } r = \frac{a}{v} = \frac{563}{11260} = .05, \text{ and hence rate}$$

per cent.  $= 5$ .

(3)

Here  $a = 75$ ,  $r = .05$ , and  $s = 14$ .

$$\begin{aligned} \text{Then } v &= \frac{a}{r(1+r)^s} = \frac{75}{.05 \times (1.05)^{14}} \\ &= \frac{75}{\log. 75 - \{(\log. 1.05 \times 14) + \log. .05\}} \\ &= \frac{75}{1.875061 - (0.021189 \times 14 + \log. .05)} \\ &= \frac{75}{1.875061 - (0.296646 + 2.698970.)} \\ &= \frac{75}{2.879445}. \end{aligned}$$

$\therefore v =$  nat. number corresponding to the logarithm 2.879445, which is \$757.608.

(4)

Here  $a = \$90$ ,  $r = .04$ ,  $t = 12$ ,  $s = 7$ , and  $\therefore s + t = 19$ .

$$\begin{aligned} \text{Formula VIII. } v &= \frac{a}{r} \left\{ \frac{1}{(1+r)^t} - \frac{1}{(1+r)^{s+t}} \right\} \\ &= \frac{90}{.04} \left\{ \frac{1}{(1.04)^{12}} - \frac{1}{(1.04)^{19}} \right\} = \frac{9000}{4} \left\{ \frac{1}{1.60101} - \frac{1}{2.10682} \right\} \\ &= 2250 \times (.624605 - .474649) = 2250 \times .149956 \\ &= \$337.401. \end{aligned}$$

(5)

Here  $a = 1500$ , and  $r = .05$ .

$$\begin{aligned} \text{Formula IX. } v &= \frac{a}{r} = \frac{1500}{.05} = \frac{150000}{5} = \$30000 \\ &= 20 \times 1500 \text{ or 20 years' purchase.} \end{aligned}$$

(6)

Here  $a = 22$ ,  $v = 308.6416$ , and  $r = .04$ .

$$\begin{aligned} \text{Then Formula VII. } t &= \frac{\log. a - \log. (a - vr)}{\log. (1+r)} \\ &= \frac{\log. 22 - \log. (22 - 308.6416 \times .04)}{\log. (1.04)} \\ &= \frac{1.342423 - \log. (9.65425)}{\log. (1.04)} = \frac{1.342423 - 0.984707}{0.017033} \\ &= \frac{0.357716}{0.017033} = 21 + \frac{357716}{17033} \end{aligned}$$

(7)

Here  $a = 154$ ,  $t = 19$ , and  $r = .05$ .

$$\text{Formula V. } v = \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\}$$

$$\begin{aligned} &= \frac{154}{.05} \times \left\{ 1 - \frac{1}{(1.05)^{19}} \right\} = \frac{15400}{5} \times \left\{ 1 - \frac{1}{2.5269} \right\} \\ &= 3080 \times (1 - .39574) = 3080 \times .60426 = \$1861.12 + \end{aligned}$$

(8)

Here  $A = 600$ ,  $t = 40$ , and  $r = .0375$ .

$$\begin{aligned} \text{Formula II. } a &= \frac{Ar}{(1+r)^t - 1} = \frac{600 \times .0375}{(1.0375)^{40} - 1} \\ &= \frac{22.5}{22.5} = \frac{2250000}{4.36034 - 1} = \frac{336034}{3.36034} \\ &= £6.6957 = £6 \text{ } 13\text{s. } 10\frac{1}{2}\text{d } +. \end{aligned}$$

(9)

Here  $a = 8$ ,  $A = 187.315625$ , and  $r = .03$ .

$$\log. (Ar + a) - \log. a$$

$$\begin{aligned} \text{Formula IV. } t &= \frac{\log. (1+r)}{\log. (187.315625 \times .03 + 8) - \log. 8} \\ &= \frac{\log. 1.03}{\log. (5.61946875 + 8) - \log. 8} \end{aligned}$$

$$\begin{aligned} &= \frac{\log. 1.03}{\log. 13.61946875 - \log. 8} = \frac{1.134160 - 0.903090}{\log. 1.03} \\ &= \frac{0.231070}{0.012837} = \frac{231070}{12837} = 18. \end{aligned}$$

(10)

Here  $a = 74$ ,  $r = .04$ , and  $t = 30$ 

$$\text{Formula I. } A = a \left\{ (1+r)^t - 1 \right\} = 74 \times \left\{ (1.04)^{30} - 1 \right\}$$

$$\begin{aligned} &= \frac{74}{.04} \times (3.24332 - 1) = \frac{7400}{4} \times 2.24332 = \$4150.142 \\ &= \$56.08494 \end{aligned}$$

By Table, page 302. Amount of \$1 for 30 years, at 4 per cent.

$$\begin{aligned} &= \$56.08494 \\ \text{Then } \$56.08494 \times 74 &= \$4150.28. \end{aligned}$$

Exerc

\$7580

D is to

for

=

\$6139.8

ama

A is to

\$4604.85

\$1

\$1

A and

therefore

A by hi

he can do

hour. Th

The is con

Then 192

\$179.14

\$718.9458

\$1497

EXERCISE 165—Page 367.

EXAMINATION PROBLEMS.

FIRST SERIES.

(2)

$\$7580 \times .19 = \$1440.20$ , and  $\$7580 - \$1440.20 = \$6139.80$ .  
D is to have one third as much as A, B, and C together, therefore he will have one-fourth of the whole.  $\frac{1}{4}$  of  $\$6139.80 = \$1534.95 = D's$  share.

$\$6139.80 - \$1534.95 = \$4604.85 =$  amount to be divided among A, B, and C.

B is to have  $\$90.90$  more than C.

A is to have  $\$111.11 + \$90.90 = 202.01$  " " "

$\$292.91$

$\$4604.85 - \$292.91 = \$4311.94 =$  three times C's share.

$\$4311.94 \div 3 = \$1437.31\frac{1}{3} = C's$  share.

$\$1437.31\frac{1}{3} + \$90.90 = \$1528.21\frac{1}{3} = B's$  share.

$\$1528.21\frac{1}{3} + \$111.11 = \$1639.32\frac{1}{3} = A's$  share.

(3)

A and B working together can do the work in 96 hours, therefore in one hour they will do  $\frac{1}{96}$  of it.

A by himself can do the work in 192 hours; therefore in 1 hour he can do  $\frac{1}{192}$  of it.  $\frac{1}{96} - \frac{1}{192} = \frac{1}{192} =$  part B can do in one hour. Therefore he will require as many hours to finish it as  $\frac{1}{192}$  is contained times in the whole, i. e.  $1 \div \frac{1}{192} = 192$  hours. Then  $192 \div 14 = 13\frac{4}{7}$  days.

(4)

$\pounds 179\ 14s.\ 8\frac{1}{2}d. = \$718.94\frac{1}{2} = \$718.94583.$

$\$718.94583 \div .00000048 = \$7189458333.3 \div 48 =$   
 $\$1497803812.4444.$

(5)

77		44..18..30..77..88..27
30		4..18..80
36		2..8
77	x	30 x 36 = 83160 = 1. c. m.

Q



(11)

Logarithm of 129140163 = 8.111061.  
 $8.111061 \div 17 = .477121 = \text{logarithm of } 3.$

(12)

Suppose 48.

Suppose 36

$\begin{array}{r} 18 \\ \hline 66 \\ 84 \\ \hline - 18 \end{array}$	$\begin{array}{r} 18 \\ \hline 54 \\ 63 \\ \hline - 9 \end{array}$
---	--

Errors. Assumed numbers.

$$\begin{array}{rcl} -18 & \times & 36 = 648 \\ -9 & \times & 48 = 432 \\ \hline \end{array}$$

Difference of errors = 9

9)216 = sum of products.

24

SECOND SERIES.

(13)

B is to have \$69.18 more than C.  
 A is to have \$69.18 + \$93.40 = \$162.58 " " "

\$231.76

\$897.43 - \$231.76 = \$665.67 = Amount to be divided  
 equally amongst A, B, and C.

\$665.67 ÷ 3 = \$221.89 = C's share.

\$221.89 + \$69.18 = \$291.07 = B's "

\$291.07 + \$93.40 = \$384.47 = A's "

(14)

7 lbs. wheat	= 9 lbs. rye			
5 " rye	= 8 " oats	}	$x = 9$	
13 " oats	= 21 " buckwheat		$5 = 8$	
27 " buckwheat	= 20 " barley		$13 = 21$	$x$
24 " barley	= 26 " peas		$3 \quad 27 = 20$	$4$
11 " peas	= 35 " potatoes		$3 \quad 24 = 26$	$2$
x " potatoes	= 16 " wheat		$11 = 35$	
			$x = 16$	

$$\begin{array}{rcl} 4 \times 2 \times 35 \times 16 & 4480 & \\ \text{Ans. } \frac{\quad}{3 \times 11} & = & 135\frac{1}{11} \end{array}$$



(15)

$\frac{3}{4}$  of  $4\frac{1}{2}$  of  $7\frac{1}{2}$  of  $\frac{9}{19\frac{1}{2}}$  of  $\frac{4}{5}$  of 3 oz. 4 drs. 2 scr. 5 grs. =  $\frac{3}{4}$  of  $\frac{9}{2}$  of

$\frac{3}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{3}$  of  $\frac{1}{3}$  of 1725 grs. = 10350 grs.

$\frac{6}{11}$  of  $\frac{6}{11}$  of  $2\frac{1}{2}$  of  $\frac{1}{3}$  of  $6\frac{1}{2}$  times 7 lbs. 3 oz. =  $\frac{6}{11}$  of  $\frac{1}{11}$  of  $\frac{121}{12}$  of  $\frac{1}{3}$  of  $\frac{1}{3}$  of 41760 grs. = 62640 grs.

$$10350 \div 62640 = .165229.$$

(16)

Dissimilar.

Similar.

Similar and Coterminous.

$$623 \cdot 42793 \quad = \quad 623 \cdot 42793793 \quad = \quad 623 \cdot 42793793793$$

$$93 \cdot 4267192 \quad = \quad 93 \cdot 4267192 \quad = \quad 93 \cdot 42671929292$$

$$\text{Difference} = 530 \cdot 00121864500$$

(17)

$$\$1.00 - \$0.046 = \$0.954, \text{ and } \$7493 \div 0.954 = \$7854.29.$$

(18)

36 : 20 weeks

6 : 5 days

9 : 11 hours

11 : 24 cellars

20 : 22 feet long

16 : 22 feet wide

5 : 4 feet deep

$$\begin{array}{r} 8 \qquad \qquad \qquad 2 \quad 11 \\ 18 \times 20 \times 5 \times 11 \times 24 \times 22 \times 22 \times 4 \\ \hline 88 \times 8 \times 9 \times 11 \times 20 \times 16 \times 5 \\ 8 \qquad \qquad \qquad 4 \\ \hline \end{array} \quad \therefore 18 \text{ men:}$$

$$= \frac{11 \times 22}{9} = 26\frac{2}{3}$$

(19)

$\frac{1}{4}$  of  $\frac{3}{4}$  of  $\frac{1}{4} = \frac{3}{64}$ ; and if  $\frac{3}{64}$  of a certain number =  $\frac{72}{64}$ ,  $\frac{1}{4} = \frac{1}{4}$   
and  $\frac{3}{64} = \frac{1}{32} \times 35 = 12$ .

$$((([12 \times 12]) + 31) \times 3) - 33) \times 300] \div 17] \times 9) = 81000$$

(20)

$$\begin{array}{r|l} 1176 & 480 \cdot 788 \cdot 848 \cdot 1176 \\ 32 & 20 \cdot 82 \cdot 29 \\ 145 & 5 \quad 25 \\ \hline 1176 \times 32 \times 145 = 5456640. \end{array}$$

17598

As n  
without

\$12000-

\$25000-

. =  $\frac{3}{4}$  of  $\frac{2}{3}$  of

of  $\frac{7}{11}$  of  $\frac{121}{42}$

terminous.

793793

929292

64500

854.29.

11  
 $\times 22 \times 22 \times 4$

$\times 14 \times 5$

$\frac{4}{2}$

$\frac{7}{8}, \frac{1}{8} = \frac{1}{2}$

$\cdot 17\frac{1}{2} \times 9)$

(21)

838)171347(204

1676

3747

3352

395)838(2

790

17598)46090(2

35196

10894)17598(1

10894

6704)10894(1

6704

4190)6704(1

4190

2514)4190(1

2514

1676)2514(1

1676

838)1676(2

1676

48)395(8

384

11)48(4

44

4)11(2

8

9)4(1

3

1)3

3

As no number greater than unity will divide all of them without a remainder, they have no G. C. M.

(22)

$$\begin{aligned} \$12000 \times 4 &= \$48000 \\ \$12000 + \$8000 &= \$20000 \times 2 = \$40000 \end{aligned}$$

\$88000 = product of A's stock and time.

$$\begin{aligned} \$25000 \times 5 &= \$75000 \\ \$25000 - \$10000 &= \$15000 \times 3 = \$45000 \end{aligned}$$

\$120000 = product of B's stock and time.

$$\$35000 \times 2 = \$70000$$

(Continued on next page.)

(22 continued.)

$$\frac{1}{4} \text{ of } \$35000 = \$10000. \quad \$35000 - \$10000 = \$25000 \times 4 = 100000$$

$$\frac{\$170000}{\$170000}$$

= product of O's stock and time.

$$\$88000 + \$120000 + \$170000 = \$378000 = \text{sum of the products of stocks and times.}$$

$$15000 \times 88000$$

$$\text{Then } \$378000 : \$88000 :: \$15000 : \frac{378000}{88000} = \$3492.06$$

= A's share.

$$15000 \times 170000$$

$$\$378000 : \$170000 :: \$15000 : \frac{378000}{170000} = \$6746.03$$

= O's share.

$$\$15000 - (\$3492.06 + \$6746.03) = \$4761.91 = \text{B's share.}$$

(23)

A's gain in 5 months = \$125 ∴ his gain for 9 months

$$= 1\frac{1}{4} \times \$125 = \$225$$

B's gain in 6 months = \$125 ∴ his gain for 9 months

$$= 1\frac{1}{2} \times \$125 = \$187\frac{1}{2}$$

C's gain in 9 months = \$125

$$\text{Sum} = \$537\frac{1}{2}$$

$$\text{Then } \$537\frac{1}{2} : \$225 :: \$400 : \frac{400 \times 225}{537\frac{1}{2}} = \$167\frac{1}{2} = \text{A's stock.}$$

$$\$537\frac{1}{2} : \$187\frac{1}{2} :: \$400 : \frac{400 \times 187\frac{1}{2}}{537\frac{1}{2}} = \$139\frac{1}{2} = \text{B's stock.}$$

$$\$537\frac{1}{2} : \$125 :: \$400 : \frac{400 \times 125}{537\frac{1}{2}} = \$93\frac{1}{2} = \text{C's stock.}$$

(24)

 $\frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{5}{16} = \frac{1}{3\frac{1}{3}} = \text{part of the cistern filled in one hour when the four pipes are left open.}$ 
 $\frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{5}{16} = \frac{1}{3\frac{1}{3}} = \text{part of the cistern emptied in one hour when the four are left open.}$ 
 $\frac{2}{3} - \frac{1}{3\frac{1}{3}} = \frac{1}{6} = \text{part of the cistern which remains filled after the eight pipes have been left open for one hour. And if } \frac{1}{6} \text{ of the cistern are emptied in one hour, it will take } 1 \div \frac{1}{6} = 6 \text{ hours to empty the whole of it.}$

THIRD SERIES.

(26)

As often as the first receives 4 the second receives 3, therefore as often as the first receives 6 the second receives 4½. Then  $6 + 4\frac{1}{2} + 7 = 17\frac{1}{2}$  loaves.

$$2310 \times 6$$

$$17\frac{1}{2} : 6 :: 2310 : \frac{2310 \times 6}{17\frac{1}{2}} = 792 \text{ loaves} = \text{number the first receives.}$$

$$2310 \times 4\frac{1}{2}$$

$$17\frac{1}{2} : 4\frac{1}{2} :: 2310 : \frac{2310 \times 4\frac{1}{2}}{17\frac{1}{2}} = 594 \text{ " = " second "}$$

$$2310 \times 7$$

$$17\frac{1}{2} : 7 :: 2310 : \frac{2310 \times 7}{17\frac{1}{2}} = 924 \text{ " = " third "}$$

(27)

To produce a mixture worth 8 cents a pound, we require 4 lbs. @ 12 cents, 4 @ 4 cents, 1 @ 5 cents, and 3 @ 9 cents, or 3 lbs @ 12 cents, 1 @ 4 cents, 4 @ 5 cents, and 4 @ 9 cents, lbs.lbs.lbs. lbs.lbs.lbs.

$$\begin{array}{ll} \text{Then } 4:72::4:72 \text{ lbs. @ 4 cts.} & \text{or } 3:72::1:24 \text{ lbs. @ 4 cts.} \\ 4:72::1:18 \text{ lbs. @ 5 cts.} & 3:72::4:96 \text{ lbs. @ 5 cts.} \\ 4:72::3:54 \text{ lbs. @ 9 cts.} & 3:72::4:96 \text{ lbs. @ 9 cts.} \end{array}$$

(28)

Here  $A = \$4444.44$ ,  $r = .0444$ , and  $t = 4.3\frac{1}{2}$

$$\text{Then } P = \frac{A}{1 + rt} = \frac{\$4444.44}{1 + (.0444 \times 4.3\frac{1}{2})} = \frac{\$4444.44}{1.19289\frac{1}{2}} = \$3725.764.$$

(29)

$$\$1.00 - \$0.0225 = \$0.9775. \quad \$23470 \div 0.9775 = \$24010.23.$$

(30)

Here  $A = \$7493.47$ ,  $r = .07$ , and  $t = 8$ .

$$\text{Then } P = \frac{A}{1 + t} = \frac{\$7493.47}{1 + (.07 \times 8)} = \frac{\$7493.47}{1.56} = \$4803.5064.$$

(31)

$$\$17460 \div 1.03125 = \$16930.909 = \text{sum to be invested.}$$

$$16930.909 \div 2.95 = 5739.29 \text{ yds. cloth.}$$

$$16930.909 \times .02\frac{1}{2} = \$423.27272 = \text{ad valorem duty.}$$

$$\$17460 + \$1347.90 + \$479.40 + \$169.83 + \$423.27272 =$$

$$\$19880.40272 = \text{whole cost.}$$

$$\$25000 - \$19880.40272 = \$5119.59728 = \text{whole gain.}$$

$$\text{Then } \$19880.40272 : \$100 :: \$5119.59728 : \frac{5119.59728 \times 100}{19880.40272} =$$

$$25.75 = 25\frac{3}{4} \text{ per cent.}$$

(32)

V.	III.	VIII.	XII.
134234	= 21122021	= 12701	= 3281
5	3	8	12
8	7	10	38
5	3	8	12
44	22	87	464
5	3	8	12
222	68	696	5569 den.
5	3	8	
1113	206	5569 den.	
5	3		
5569 den.	618		
	3		
	1856		
	3		
	5569 den.		

(33)

$$\frac{3}{4} \text{ of } 4\frac{1}{2} \text{ of } \frac{9\frac{1}{2}}{13} \text{ of } \frac{1}{15} \text{ of } \frac{1}{3} \text{ of } £43 \text{ 18s. 11}\frac{1}{2}\text{d. } £43 \text{ 18s. 11}\frac{1}{2}\text{d.} =$$

$$\$175.79\frac{1}{2}.$$

(Continued on next page.)

(33 continued.)

$$\frac{7}{13} \text{ of } \frac{32}{13} \text{ of } \frac{1}{15} \text{ of } \frac{1}{3} \text{ of } \$175.79\frac{1}{2} = \frac{3}{7} \text{ of } \frac{8}{2} \text{ of } \frac{15}{1} \text{ of } \frac{1}{15} \text{ of } \frac{7}{9} \text{ of } \$175.79\frac{1}{2} = \frac{1}{3} \text{ of } \$175.79\frac{1}{2} = \$263.6875.$$

$$3\frac{3}{4} \text{ of } \frac{1}{17\frac{1}{2}} \text{ of } .56 \text{ of } 1.75 \text{ of } 6\frac{1}{2} \text{ times } \$97.18 =$$

$$3\frac{3}{4} \text{ of } \frac{1}{17\frac{1}{2}} \text{ of } \frac{56}{100} \text{ of } \frac{17\frac{1}{2}}{100} \text{ of } 6\frac{1}{2} \text{ times } \$97.18; 6\frac{1}{2} \text{ times } \$97.18 = \$631.67.$$

$$\begin{array}{r} 85 \quad 2 \quad 14 \quad 7 \\ \text{--- of --- of --- of --- of } \$631.67 = \frac{49}{9 \times 25} \text{ of } \$631.67 \\ 9 \quad 35 \quad 100 \quad 100 \\ \quad 50 \quad 25 \\ \quad 25 \\ = \frac{49}{25} \text{ of } \$631.67 \\ \frac{49}{25} \text{ of } \$631.67 = \$137.5636. \end{array}$$

$$\text{Then } \$263.6875 - \$137.5636 = \$126.1239 = \text{difference.}$$

(34)

$$\begin{aligned} 1\frac{1}{3} &= 1 \div 13 \therefore \log. 1\frac{1}{3} = \log. 1 - \log. 13 = 0 - 1.113943 \\ &= -1.113943 \\ 19.5 &= 3 \times 13 \times 5 \div 10 \therefore \log. 19.5 = \log. 3 + \log. 13 \\ &\quad + \log. 5 - \log. 10. \end{aligned}$$

$$\log. 3 = 0.477121$$

$$\log. 13 = 1.113943$$

$$\log. 5 = \log. 10 - \log. 2 = 1 - 0.301030 \therefore \log. 5 = 0.698970$$

$$\text{Sum} = 2.290034$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Rem.} = 1.290034$$

$$= \log. 19.5.$$

$$1125 = 5^3 \times 3^3 \therefore \log. 1125 = (\log. 5) \times 3 + (\log. 3) \times 2.$$

$$\log. 5 = 0.698970 \times 3 = 2.096910$$

$$\log. 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Sum} = 3.051152 = \log. \text{ of } 1125.$$

(Continued on next page.)

(34 continued.)

$$28\cdot16 = 28\frac{1}{2} = \frac{13^2}{2} = 13^2 \div 2 \therefore \log. 28\cdot16 = (\log. 13) \times 2 \\ - (\log. 2 + \log. 3.)$$

$$\log. 13 = 1\cdot113943 \times 2 = 2\cdot227886$$

$$(\log. 2 + \log. 3) = (0\cdot301030 + 0\cdot477121) = 0\cdot778151$$

$$\text{Diff.} = 1\cdot449735$$

$$= \log. 28\cdot16.$$

$$65000 = 13 \times 5 \times 1000 \therefore \log. 65000 = \log. 13 + \log. 5 \\ + \log. 1000.$$

$$\log. 13 = 1\cdot113943$$

$$\log. 5 = 0\cdot698970$$

$$\log. 1000 = 3$$

$$\text{Sum} = 4\cdot812913 = \log. \text{ of } 65000.$$

$$\log. \cdot0005 = \log. 5 \text{ with characteristic changed to } -4 \\ = 4\cdot698970.$$

$$152\cdot1 = 3^2 \times 13^2 \div 10 \therefore \log. 152\cdot1 = (\log. 3) \times 2 \\ + (\log. 13) \times 2 - \log. 10.$$

$$\log. 3 = 0\cdot477121 \times 2 = 0\cdot954242$$

$$\log. 13 = 1\cdot113943 \times 2 = 2\cdot227886$$

$$\text{Sum} = 3\cdot182128$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Diff.} = 2\cdot182128 = \log. 152\cdot1$$

$$8\cdot112 = 2^4 \times 13^2 \times 3 \div 1000 \therefore \log. 8\cdot112 = (\log. 2) \times 4 \\ + (\log. 13) \times 2 + \log. 3 - \log. 1000.$$

$$\log. 2 = 0\cdot301030 \times 4 = 1\cdot204120$$

$$\log. 13 = 1\cdot113943 \times 2 = 2\cdot227886$$

$$\log. 3 = 0\cdot477121$$

$$\text{Sum} = 3\cdot909127$$

$$\text{From which take } \log. 1000 = 3$$

$$\text{Diff.} = 0\cdot909127 = \log. 8\cdot112.$$

13)  $\times 3$

27886

78151

49735

3 + log. 5

0 - 4

3)  $\times 2$

g. 152.1

g. 2)  $\times 4$

8.112.

(35)

XII.

871tet-72 (18-72)

6e4

179tet

159768

20352720

1et372e4

517428000.

4977tet-08

3e8391e4

$t^3 \times 300 = 21000$

$t \times 8 \times 30 = 1800$

$8^3 = 54$

22854

$t^3 \times 300 = 2454000$

$t^8 \times t \times 30 = 22800$

$t^3 = 84$

2476884

$t^8 t^3 \times 300 = 249961000$

$t^8 t \times 2 \times 30 = 54500$

$2^2 = 4$

2499e5504

(36)

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 5 \text{ years} = \frac{3}{2}$  of life time + 5 years = age at birth of son.  
 $\frac{3}{2} - (\frac{1}{2} + 5) = \frac{1}{2}$  of his life time - 5 years = time he lived after birth of son.

$\frac{1}{2}$  of father's life time - 5 years - 4 years = age of son =  $\frac{1}{2}$  father's age.

$\frac{1}{2}$  of father's life time - 9 years =  $\frac{1}{2}$  father's age.

$\therefore$  9 years is the difference between  $\frac{1}{2}$  and  $\frac{1}{2}$  of father's age.

$\therefore$  9 years is equal to  $\frac{1}{2}$  of father's age.

If 9 years is  $\frac{1}{2}$  of his age,  $\frac{1}{2}$  will be the  $\frac{1}{2}$  of 9 which is 3 years.

If  $\frac{1}{2}$  is 3 years,  $\frac{3}{2}$  or the whole age will be  $3 \times 28 = 84$  years.

Or by Position.

Assume 42 for father's age at death, the son's age = 21.

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 5 = \frac{3}{2} + 5$ ;  $\frac{1}{2}$  of 42 = 21 and  $21 + 5 =$

$21\frac{1}{2}$  = age of father when son was born.

$\therefore$  he lived after birth of his son  $42 - 21\frac{1}{2} = 20\frac{1}{2}$  years.

(Continued on next page.)



(38 continued.)

By the question he lived  $21 + 4 = 25$  years.The error  $25 - 20\frac{1}{2} = -4\frac{1}{2}$ .Assume 98 for father's age, then son's age  $= \frac{1}{2}$  of 98 = 49. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 5 = 1\frac{1}{2} + 5$ ;  $\frac{1}{2}$  of 98 =  $38\frac{1}{2}$ , and  $38\frac{1}{2} + 5 = 43\frac{1}{2}$   
= age of father at birth of son. $\therefore$  he lived after birth of his son  $98 - 43\frac{1}{2} = 54\frac{1}{2}$  years.But by the question he lived  $49 + 4$  years = 53 years.Then  $53 - 54\frac{1}{2} = +1\frac{1}{2}$  = error.

Errors.

$$-4\frac{1}{2} \times 98 = 441$$

$$+1\frac{1}{2} \times 42 = 63$$

$$\text{Sum} = 6 \qquad 504$$

$$504 \div 6 = 84 = \text{father's age.}$$

(37)

m.	fur.	per.	yds.	ft.	in.		fur.	per.	yds.
63	3	7	3	2	7	$\div$	7	23	3 $\frac{1}{2}$
8							40		
507							303		
40							5 $\frac{1}{2}$		
20287							1518 $\frac{1}{2}$		
5 $\frac{1}{2}$							151 $\frac{1}{2}$		
101438							1670 $\frac{1}{2}$		
10143 $\frac{1}{2}$							3		
111581 $\frac{1}{2}$							5010 $\frac{1}{2}$		
3							12		
334746 $\frac{1}{2}$							60129		
12									
4016965									

(Continued on next page.)

(37 continued.)

60129)4018965(66-80578 times

360774

409225

360774

484510

481032

347800

300845

471550

420903

506470

481032

(38)

6.3 ÷ .000000274

274)6300000000(21992700-72992700

548

820

548

2720

2468

2540

2468

740

548

1920

1918

2000

1918

820

548

2720

2468

2540

2468

740

548

1920

1918

200 remainder,

(39)

$$\frac{1}{2} \text{ yds.} : 6\frac{1}{2} \text{ yds.} :: \$19 : 4 \times 19 \times 19 = 7139 = \$5.482.$$

(40)

$$I = Prt. = \$4237.71 \times .065 \times 1.67 = \$460.0034205.$$

(41)

$$t = \frac{A - P}{Pr} = \frac{\$1000 - \$674.30}{\$674.30 \times .085} = \frac{325.70}{57.3155} = 5.68258 \text{ years} =$$

5 years 8 months 5.7288 days.

(42)

By Table, page 260, the amount of \$1 for 14 payments at 4 per cent. is \$1.73168.

$$\text{Then } \$1.73168 \times 813.71 = \$1409.0853328 = \text{Amount.}$$

$$\text{Subtract } 813.71$$

$$\text{Difference} = 595.3753328 = \text{Interest.}$$

(43)

$$\begin{array}{rcl} \$300 & \times & 0 = 0 \\ 700 & \times & 4 = 2800 \\ 750 & \times & 7 = 5250 \\ 850 & \times & 9 = 7650 \\ 400 & \times & 13 = 5200 \\ 1300 & \times & 19 = 24700 \end{array}$$

$$4300 \quad ) \quad 45600 \text{ (10 months } 18\frac{2}{3} \text{ days.}$$

$$\underline{4300}$$

$$2600$$

$$\underline{30}$$

$$78000 = \text{days}$$

$$\underline{4300}$$

$$35000$$

$$\underline{24400}$$

$$45138$$

482.

8 years =

ents at 4

days.

(44)

23 per cent. of \$4200 =  $\frac{23}{100}$  of 4200 = \$966.00, and \$4200 — \$966.00 = \$3234.00. E has half as much as A, B, C, and D together; therefore E has *one-third* of \$3234.00, which is \$1078.00.

Deducting E's share, \$1078, from \$3234, the whole sum to be divided, there remains \$2156 to be divided among A, B, C and D. Now D gets a certain amount; C gets \$42.11 more than D; B gets \$61.34 (42.11 + 19.23) more than D; and A gets \$78.44 (61.34 + 17.10) more than D. Together they get, then, *four times* D's share, together with \$42.11 + \$61.34 + \$78.44, or, in other words, four times D's share, together with \$181.89.

That is, four times D's share, together with \$181.89 is equal to \$2156.

Hence \$2156.00 — \$181.89 = \$1974.11 = four times D's share. Then \$1974.11 ÷ 4 = \$493.5275 = D's share.

Add 42.11

Sum \$535.6375 = C's share.

Add 19.23

Sum \$554.8675 = B's share.

Add 17.10

Sum \$571.9675 = A's share.

(45)

$$P = \frac{A}{1+rt} = \frac{\$3786.80}{1+1.76} = \frac{3786.80}{2.76} = \frac{378680}{276} = \$1372.02898 +$$

(46)

$$\{(37 - 2\frac{7}{10}) \times .46 \div \frac{2}{3} \text{ of } .142857\} \div 8\frac{1}{2} \text{ times } (\frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{237}{2310})$$

$$\{(.73 \times .12345 \div \frac{7}{10}) + \frac{2}{3} + 9\frac{3}{10} + 17\frac{1}{11}\} \div 27.4922077$$

$$\{(3\frac{3}{10} - 2\frac{7}{10}) \times \frac{1}{10} \div \frac{2}{3} \text{ of } \frac{1}{3}\} \div \frac{1}{12} \times (\frac{7}{10} + \frac{1}{10} + \frac{1}{10} - \frac{237}{2310})$$

$$\{(\frac{6}{11} \times \frac{13333}{10000} \div \frac{7}{10}) + \frac{2}{3} + 9\frac{3}{10} + 17\frac{1}{11}\} \div 27.4922077$$

(Continued on next page.)

$$*rt = .16 \times 11 = 1.76.$$

(46 continued.)

$$\begin{aligned}
 &= \frac{(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}) \times \frac{1}{7} \times \frac{1}{8} \times \frac{1}{9}}{\left\{ (\frac{1}{10} \times \frac{1}{11} \times \frac{1}{12} \times \frac{1}{13}) \times \frac{1}{14} + 27\frac{1}{15} \right\} \div 27\cdot4922077} \\
 &= \frac{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{7} \times \frac{1}{8} \times \frac{1}{9}}{\frac{1}{10} + 27\frac{1}{15}} \div 27\cdot4922077 = \frac{1}{27\frac{1}{15} \div 27\cdot4922077} \\
 &= \frac{1}{27\cdot4922077 \div 27\cdot4922077} = \frac{1}{1} = 1
 \end{aligned}$$

(47)

312312302 quaternary = 224690 decimal scale.

2312132 quaternary = .11678 decimal scale.

Sum = 236368

4234 quinary = 569 decimal, and  $569 \times 23011 = 13093259$ . $236368 \times 13093259 = 3094827443312$ . $555 + 444 + 333 + 222 + 111$  senary = 2553 senary = 645 decimal. $3094827443312 - 645 = 3094827442667$ .

6542 septenary = 2333 decimal.

 $3094827442667 \div 2333 = 1326544124\frac{1}{333}$  den.

X. VIII.

 $1326544124 = 11704272374$ 

X. VIII.

 $1376 = 2537$ 

X. VIII.

 $2333 = 4435$ 

X. VIII.

 $\therefore 1326544124\frac{1}{333} = 11704272374\frac{1}{333}$ .

(48)

 $\cdot 1 = \frac{1}{10}$  and  $(\frac{1}{10})^2 = \frac{1}{100} = 01$ . $\cdot 1 = \frac{1}{9}$  and  $(\frac{1}{9})^2 = \frac{1}{81} = 012345679$ .

FIFTH SERIES.

(50)

Assume 27 | 2..9..18..27..48 and 81, strike out 2, 9 and 16, since they are contained as factors in the others.

$$\text{The l. c. m.} = 27 \times 16 \times 3 = 1296.$$

(51)

$$t = \frac{\log. n}{\log. (1+r)} = \frac{\log. 7}{\log. (1.06)} = \frac{0.845098}{0.025306} = 33.395 \text{ years.}$$

(52)

$$20 \text{ miles} = 1267200 \text{ inches; and } 14 \text{ ft. } 10 \text{ in.} = 178 \text{ inches.}$$

$$1267200 \div 178 = 7119\frac{2}{3} \text{ times.}$$

(53)

$$1749600 = 2^5 \times 3^7 \times 5^2; \text{ increasing each index by unity and multiplying, we have } 6 \times 8 \times 3 = 144.$$

(54)

$$\frac{2}{3} \text{ of } \frac{96}{\frac{5}{2}} \div \frac{\frac{1}{2} \text{ of } 7}{3\frac{1}{2}} = \frac{2}{3} \times \frac{2\frac{1}{2}}{\frac{7}{2}} \div \frac{\frac{7}{2}}{3\frac{1}{2}} = \frac{2}{3} \times \frac{2\frac{1}{2}}{\frac{7}{2}} \div \frac{\frac{7}{2}}{\frac{7}{2}} = \frac{2}{3} \times \frac{2\frac{1}{2}}{\frac{7}{2}} \div \frac{7}{7} = \frac{2}{3} \times \frac{2\frac{1}{2}}{\frac{7}{2}} = \frac{2}{3} \times \frac{5}{2} = \frac{5}{3} = 1\frac{2}{3}.$$

(55)

A can do the whole work in 12 days, therefore he can do  $\frac{1}{12}$  in 1 day. A and B together can do the work in 5 days, therefore they can do  $\frac{1}{5}$  in 1 day. Therefore B can do  $\frac{1}{5} - \frac{1}{12} = \frac{7}{60}$  in 1 day, and he will require as many times 1 day to do the whole work as  $\frac{7}{60}$  is contained times in 1, i. e.  $1 \div \frac{7}{60} = \frac{60}{7} = 8\frac{4}{7}$  days.

(56)

$$P = \frac{A}{(1+r)^t}; \log. P = \log. A - \log. (1+r) \times t = \log. 8899.77 \\ - \log. (1.06) \times 22 = 3.949378 - 0.025306 \times 22 \\ = 3.949378 - 0.556732 = 3.392646, \text{ and } \log. 3.392646 \\ = \$2469.71.$$

By Table, page 260, amount of \$1 at 6 per cent. for 22 payments = 3.60354.

Then  $\$8899.77 \div 3.60354 = \$2469.73$  nearly.

(57)

Let the 1st number be 2. Then  $2 \times 2 = 4$

$$1\frac{1}{2} \times 3 = 4$$

$$10 - (2 + 1\frac{1}{2}) = 10 - 3\frac{1}{2} = 6\frac{1}{2} \times 4 = 26\frac{1}{2}, \text{ but it should equal 4.}$$

Therefore  $26\frac{1}{2} - 4 = + 22\frac{1}{2} = \text{error.}$

Let  $1\frac{1}{2}$  be the 1st number; then  $1\frac{1}{2} \times 2 = 3$

$$1 \times 3 = 3$$

$$10 - (1\frac{1}{2} + 1) = 10 - 2\frac{1}{2} = 7\frac{1}{2} \times 4 = 30, \text{ but it should} = 3.$$

Therefore  $30 - 3 = + 27 = \text{error.}$

Errors.

$$+ 27 \times 2 = 54$$

$$+ 22\frac{1}{2} \times 1\frac{1}{2} = 34$$

Diff. =  $4\frac{1}{2}$  diff. = 20, and  $20 \div 4\frac{1}{2} = 4\frac{1}{3} = \text{1st number.}$

$$4\frac{1}{3} \times 2 = 9\frac{1}{3} = \text{1st product.}$$

Second number =  $9\frac{1}{3} \div 3 = 3\frac{1}{3} \times 3 = 9\frac{1}{3} = \text{2nd product.}$

$$10 - 7\frac{1}{3} = 2\frac{1}{3} \times 4 = 9\frac{1}{3} = \text{3rd product.}$$

(58)

Suppose A has 40; then B has  $110 - 40 = 70$ , and C has  $130 - 70 = 60$ .

A and C together have  $40 + 60 = 100$ , but it should be 120.

Therefore  $100 - 120 = - 20 = \text{error.}$

Suppose A has 80; then B has  $110 - 80 = 30$ , and C has  $130 - 30 = 100$ .

A and C together have  $80 + 100 = 180$ , but they should have 120.

Therefore  $180 - 120 = + 60 = \text{error.}$

(Continued on next page.)

(58 continued.)

8899·77

× 22

392646

22 pay-

should

4.

d = 3.

ber.

L.

C.

O has

20.

as 130

re 120.

Errors.

$$\begin{array}{rcl} + 60 & \times & 40 = 2400 \\ - 20 & \times & 80 = 1600 \\ \hline \end{array}$$

$$\text{Sum} = 80$$

$$\text{Sum} = 4000$$

$$4000 \div 80 = 50 = \text{number A has.}$$

$$\text{Then B has } 110 - 50 = 60, \text{ and C has } 130 - 60 = 70.$$

$$50 + 60 + 70$$

$$\frac{\quad}{3} = 60 = \text{each man's share when equally divided.}$$

(59)

$$\begin{aligned} \text{Formula I, p. 333. } l &= a + (n - 1) d = 7 + (47 - 1) \times 4 \\ &= 7 + (46 \times 4) = 7 + 184 = 191. \end{aligned}$$

$$\begin{aligned} \text{Formula VI, p. 333. } s &= \left\{ 2a + (n - 1) d \right\} \frac{n}{2} \\ &= \left\{ 2 \times 7 + (93 - 1) \times 4 \right\} \frac{93}{2} = \left\{ 14 + (92 \times 4) \right\} \frac{93}{2} \\ &= (14 + 368) \times \frac{93}{2} = \frac{382 \times 93}{2} = 17763. \end{aligned}$$

(60)

$$t = \frac{\log. n}{\log. (1 + r)} = \frac{\log. 21}{\log. (1.07)} = \frac{1.322219}{0.029384} = 44.997 \text{ years.}$$

SIXTH SERIES.

(61)

B gets \$196·87 more than C, and A gets \$387 + \$196·87 = \$583·87 more than C, therefore together they get *three* times C's share, together with \$196·87 + \$583·87, i. e. three times C's share, together with \$780·74; but together they get \$3700.

Therefore \$3700 = three times C's share, together with \$780·74, or \$3700 - \$780·74 = \$2919·26 = three times C's share.

Hence \$2919·26 ÷ 3 = \$973·08 $\frac{2}{3}$  = C's share.

$$\begin{array}{r} \text{Add} \\ 196\cdot87 \end{array}$$

$$\begin{array}{r} \text{Sum} = \$1169\cdot95\frac{1}{3} = \text{B's share.} \\ \text{Add} \\ 387\cdot00 \end{array}$$

$$\text{Sum} = \$1556\cdot95\frac{2}{3} = \text{A's share.}$$



(62)

$$5716 = 2^2 \times 1429$$

$$1 \quad .. \quad 2 \quad .. \quad 4$$

$$1 \quad .. \quad 1429$$

$$1 \quad .. \quad 2 \quad .. \quad 4 \quad .. \quad 1429 \quad .. \quad 2858 \quad .. \quad 5716$$

(63)

$$\{(17\frac{7}{11} - 10\frac{8}{11}) - (.4 + \frac{1}{2} + .9 - \frac{1}{2})\} \div (.8378 \div \frac{1}{2} \text{ of } 31)$$

$$.6322632 \times \frac{1}{2} \text{ of } 9\frac{1}{2} \div (\frac{1}{2} \text{ of } 4\frac{1}{2} \text{ of } \frac{1}{11} \text{ of } 85\frac{1}{2} \div 101)$$

$$6\frac{3}{4} - 1 \div (8\frac{3}{4} \times \frac{1}{2})$$

$$= \frac{8\frac{3}{4} \times \frac{1}{2} \times \frac{1}{2} \div (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{11} \times \frac{1}{2} \times \frac{1}{101})}{5\frac{3}{4} \times 8\frac{3}{4} \times \frac{1}{2}}$$

$$= \frac{8\frac{3}{4} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{11} \times \frac{1}{2} \times \frac{1}{101} \times \frac{1}{101}}{\frac{23}{4} \times \frac{1}{2} \times \frac{1}{2}}$$

$$= \frac{\frac{3161}{101} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{11} \times \frac{1}{2} \times \frac{1}{101}}{14 \times 37 \quad 14 \times 37}$$

$$= \frac{\frac{5}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}}{\frac{5}{5 \times 37}} = \frac{5}{4} = 1\frac{1}{4} = 2\frac{1}{2}.$$

(64)

Each child gets 1 child's share,  $\therefore$  17 children get 17 shares.

Each woman gets *three* times a child's share,  $\therefore$  4 women get 12 shares.

Each man gets *six* times a child's share,  $\therefore$  3 men get 18 shares.

And together they get 47 times a child's share.

Therefore  $\$7200 \div 47 = \$153.19\frac{7}{11}$  = a child's share.

$$\$153.19\frac{7}{11} \times 3 = \$459.57\frac{1}{11}.$$

$$\$153.19\frac{7}{11} \times 6 = \$919.14\frac{2}{11}.$$

(65)

$25400 = 2^3 \times 5^2 \times 127$ . Adding unity to each index and multiplying the results, we get  $4 \times 3 \times 2 = 24$ .

(66)

$$\frac{1}{2} \text{ of } 4\frac{1}{2} \text{ of } \frac{97}{14} \text{ of } \frac{1}{2} \text{ of } £3 \text{ } 16s. \text{ } 11\frac{1}{2}d. = \frac{2}{3} \times \frac{9}{2} \times \frac{95 \times 14}{7 \times 11} \times \frac{1}{8}$$

$$\times \$15.39\frac{1}{2} = 6 \text{ times } \$15.39\frac{1}{2} = \$92.35.$$

$$\frac{1}{11} \text{ of } 4\frac{1}{2} \text{ of } \frac{19\frac{1}{2}}{3\frac{1}{2}} \text{ of } \frac{25}{117} \text{ of } \frac{1}{2\frac{1}{2}} \text{ of } .85 \text{ of } \frac{1}{42\frac{1}{2}} \text{ of } \$1783$$

$$\frac{1}{18}$$

$$= \frac{1}{11} \times \frac{2}{3} \times \frac{32}{12} \times \frac{25}{117} \times \frac{1}{2\frac{1}{2}} \times \frac{25}{100} \times \frac{1}{2\frac{1}{2}} \text{ of } \$1783.$$

$$= \frac{8}{11} \times \frac{28}{4} \times \frac{78}{19} \times \frac{25}{117} \times \frac{11}{23} \times \frac{85}{100} \times \frac{2}{85} \times \frac{17.83}{1}$$

$$= \$17.83 \times 4 = \$71.32. \quad \$92.35 - \$71.32 = \$21.03.$$

(67)

$$\begin{array}{l} 7 : 13 = 7 \div 13 = .538 \\ 9 : 16 = 9 \div 16 = .562 \\ 8 : 15 = 8 \div 15 = .533 \\ 10 : 19 = 10 \div 19 = .526 \end{array} \left. \vphantom{\begin{array}{l} 7 : 13 \\ 9 : 16 \\ 8 : 15 \\ 10 : 19 \end{array}} \right\} \begin{array}{l} \text{Therefore } 9 : 16 \text{ is the} \\ \text{greatest, and } 10 : 19 \text{ is} \\ \text{the least.} \end{array}$$

$$\text{Compound ratio} = \frac{7}{13} \times \frac{3}{16} \times \frac{8}{15} \times \frac{10}{19} \times \frac{21}{247} = \frac{21}{247} = 21:247.$$

(68)

$$67.432 = 67 \frac{111}{250} = \frac{16708}{250} \text{ and } 7.9030 = 7 \frac{903}{1000} = \frac{7903}{1000}$$

$$\frac{66758}{990} \div \frac{78957}{9990} = \frac{66758}{990} \times \frac{111}{78957} = \frac{7410138}{868527} = 8.5312452.$$

(69)

9 per. 9 yds. 7 ft. 120 in. = 365628 inches  
 $\frac{1}{3}$  of  $\frac{1}{3}$  of  $\frac{1}{3}$  of 35 acres 2 roods =  $\frac{1}{27}$  of 35 acres 2 roods =  $\frac{1}{27}$  of  
 222678720 inches

$$\frac{365628}{\frac{1}{27} \text{ of } 222678720} = \frac{2559396}{133607232} = 0.019156118.$$

(70)

Dissimilar.

Similar.

17.0342

17.03424242

27.06357

27.06357575

98.123456

98.123456456

829.6423

829.642342342

986.1234298

986.1234298429

9.876342

9.876342876342

813.9864234567

813.9864234567

Similar and Coterminous.

17.0342424242424242

27.0635757575757575

98.123456456456456456

829.642342342342342342

986.123429842984298429

9.876342876342876342

813.986423456745674567

4 carried

2781.849813156689829957



Height

 $CF = \sqrt{C}$  $GH = 1$  $GI = \sqrt{1}$ 

The mixt

mixt

20 g

taine

Then  $\frac{1}{2}$ 

1



## SEVENTH SERIES.

(73)

$$\begin{array}{r}
 401241 \cdot 3424 \quad (422 \cdot 88) \\
 31 \\
 \hline
 132 \quad ) \quad 412 \\
 \quad 314 \\
 \hline
 1342 \quad ) \quad 4341 \\
 \quad 3234 \\
 \hline
 13443 \quad ) \quad 110234 \\
 \quad 101434 \\
 \hline
 140012 \quad ) \quad 330024 \\
 \quad 330024 \\
 \hline
 \end{array}$$

(74)

Suppose father's age = 60, the son's age now =  $60 \div 5 = 12$ ,  
 and son's age four years ago =  $12 - 4 = 8$ . But the son's  
 age four years ago should, by the question, have been  $60 \div 7 = 8\frac{4}{7}$ .

Therefore  $8 - 8\frac{4}{7} = -\frac{4}{7} = \text{error}$ .

Suppose father's age = 35; then son's age now =  $35 \div 5 = 7$ ,  
 and age four years ago =  $7 - 4 = 3$ .

But son's age four years ago should, by question, have been  $35 \div 7 = 5$ .

Therefore  $3 - 5 = -2 = \text{error}$ .

Errors.

$$-2 \times 60 = 120$$

$$-\frac{4}{7} \times 35 = 20$$

$$\text{diff. } 1\frac{1}{2} \quad \text{diff. } = 100$$

$$100 \div 1\frac{1}{2} = 70 = \text{father's and son's age} = 70 \div 5 = 14.$$

(75)

$$\begin{array}{r} \cdot 72347 \div \cdot 0032 = \frac{72275}{99900} \div \frac{32}{9900} = \\ \frac{72275}{99900} \times \frac{11}{32} = \frac{795025}{3552} = 223.82460585 \end{array}$$

(76)

Logarithm of 97294764.372 is 7.988089  
 $7.988089 \div 11 = 0.726139$   
 Log. 0.726189 = 5.32341 = 11th root of 97294764.372.

(77)

Assume  $43\frac{1}{2}$  for the greater number

$$7\frac{1}{2} : 3\frac{1}{2} :: 43\frac{1}{2} : \frac{43\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{2}} = 21 \text{ the less ;}$$

$$43\frac{1}{2} - 21 = 22\frac{1}{2} \text{ but it should} = 30$$

$$\text{Therefore error} = 22\frac{1}{2} - 30 = -7\frac{1}{2}.$$

Assume  $72\frac{1}{2}$  for the greater number

$$7\frac{1}{2} : 3\frac{1}{2} :: 72\frac{1}{2} : \frac{72\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{2}} = 35 = \text{the less ;}$$

$$72\frac{1}{2} - 35 = 37\frac{1}{2} \text{ but it should} = 30$$

$$\text{Therefore error} = 37\frac{1}{2} - 30 = +7\frac{1}{2}.$$

Errors.

$$+ 7\frac{1}{2} \times 43\frac{1}{2} = 326\frac{1}{4}$$

$$- 7\frac{1}{2} \times 72\frac{1}{2} = 533\frac{1}{4}$$

$$\text{Sum} = 15 \quad \text{Sum} = 870$$

$$870 \div 15 = 58 \text{ greater}$$

$$7\frac{1}{2} : 3\frac{1}{2} :: 58 : \frac{58 \times 3\frac{1}{2}}{7\frac{1}{2}} = 28 \text{ less.}$$

(78)

Assume 35	15	16	18	28	62	58	8
Assume 16		15	18	4	62	9	8
Assume 9			9		31	9	
					31		

$$l. c. m. = 35 \times 16 \times 9 \times 31 = 156240.$$

(79)

Here  $a = 1$ ,  $d = 6$ ,  $n = 101$ ,

$$s = \left\{ 2a + (n-1)d \right\} \frac{n}{2} = \left\{ 2 \times 1 + (101-1) \times 6 \right\} \frac{101}{2}$$

$$= (2 + 600) \frac{101}{2} = \frac{602 \times 101}{2} = 30401.$$

(80)

$$\frac{19}{7} \times \frac{11}{58} \times \frac{35}{121} \times \frac{117}{29} \times \frac{8}{42} \times \frac{41}{3} = \frac{117 \times 4 \times 5}{7 \times 7 \times 11 \times 3} = \frac{2284}{1617} = 2284 : 1617.$$

(82)

$$\left( \left( (9\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} - 16\frac{1}{2}) \times .54 \right) \div 14 \right) \times 35 \text{ times } .142857$$

$$\left\{ .97 \times .24378 \times (1\frac{1}{4} \times 4\frac{1}{2}) \right\} \times (4\frac{1}{4} - 2\frac{1}{4})$$

$$\left( (16\frac{1}{2} - 16\frac{1}{2}) \times 6\frac{1}{2} \right) \div 1\frac{1}{2} \times 35 \times \frac{1}{4}$$

$$\frac{88 \times 33\frac{1}{2} \times 1\frac{1}{4} \times 1\frac{1}{2} \times (4\frac{1}{4} - 2\frac{1}{4})}{12\frac{1}{2} \times 1\frac{1}{4} \times 7\frac{1}{4} \times 2\frac{1}{4} \times \frac{1}{4}} = \frac{1}{12\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{4} \times 1\frac{1}{2} \times 1\frac{1}{2}} = \frac{1}{12\frac{1}{2}}$$

(83)

Suppose the *hour* hand moves over 4 minutes, then since the minute hand moves 12 times as fast, it will have travelled over 48 minutes. But in order to overtake the hour hand, the minute hand must traverse the entire circle, 60 minutes, plus the 4 minutes we have supposed the hour hand to have moved forward, i. e. 64 minutes. Then 48 should equal 64, for we should find the same number by each process;  $48 - 64 = -16$  error.

Suppose hour hand moves over 6 minutes, the minute hand moves over  $6 \times 12 = 72$  minutes. But minute hand moves over  $60 + 6 = 66$  minutes.

Then  $72 - 66 = +6$  error.

(Continued on next page.)

120 ÷ 2

ben

= 6

Log.

∴ Log.

31

∴ Log.

∴ Log.

Log. 7

1.

Log. 1

= 93.1

∴ Log. 9

x

Simple Int

Amount Co

= \$700

Int.

\$98.814 - \$

(83 continued.)

Errors.

$$\begin{array}{r} - 16 \times 6 = 96 \\ + 6 \times 4 = 24 \\ \hline \end{array}$$

Sum 22    Sum 120

$120 \div 22 = 5\frac{1}{11}$  min. = minutes passed over by the hour hand,  
hence space passed over by the minute hand =  $5\frac{1}{11} \times 12$   
=  $65\frac{5}{11}$  min. = 1 hour  $5\frac{5}{11}$  min. = time.

(84)

$$\begin{aligned} \text{Log. } 5 &= \text{log. } 10 - \text{log. } 2 = 1 - 0.301030 = 0.698970 \\ 3850000 &= 5 \times 7 \times 11 \times 10000. \end{aligned}$$

$$\begin{aligned} \therefore \text{Log. } 3850000 &= \text{log. } 5 + \text{log. } 7 + \text{log. } 11 + \text{log. } 10000 \\ &= 0.698970 + 0.845098 + 1.041393 + 4 = 6.585461. \end{aligned}$$

$$3181.81 = 31.81 \times 100 = 31\frac{81}{100} \times 100 = 31\frac{81}{100} \times 100.$$

$$\begin{aligned} \therefore \text{Log. } 3181.81 &= \text{log. } 5 + \text{log. } 7 + \text{log. } 1000 - \text{log. } 11 \\ &= 0.698970 + 0.845098 + 3 - 1.041393 = 3.502675 \\ 0.0000154 &= 2 \times 7 \times 11 \div 10000000 \end{aligned}$$

$$\begin{aligned} \therefore \text{Log. } 0.0000154 &= \text{log. } 2 + \text{log. } 7 + \text{log. } 11 - \text{log. } 10000000 \\ &= 0.301030 + 0.845098 + 1.041393 - 7 = 5.187521. \end{aligned}$$

$$\begin{aligned} \text{Log. } \frac{1}{11} &= \text{log. } 1 - (\text{log. } 7 + \text{log. } 11) = 0 - (0.845098 \\ &+ 1.041393) = 0 - 1.886491 = 2.113509. \end{aligned}$$

$$1.571428 = 1\frac{4}{7} = \frac{1}{7}.$$

$$\begin{aligned} \text{Log. } 1.571428 &= \text{log. } 11 - \text{log. } 7 = 1.041393 - 0.845098 \\ &= 0.196295 \end{aligned}$$

$$93.17 = 9317 \div 100 = 11^3 \times 7 \div 100.$$

$$\begin{aligned} \therefore \text{Log. } 9317 &= 3 \text{ times log. } 11 + \text{log. } 7 - \text{log. } 100 = 1.041393 \\ &\times 3 + 0.845098 - 2 = 1.969277. \end{aligned}$$

# EIGHTH SERIES.

(85)

$$\text{Simple Interest} = Prt = \$700 \times .045 \times 3 = \$94.50.$$

$$\begin{aligned} \text{Amount Compound Interest} &= P(1+r)^t = \$700 \times (1.045)^3 \\ &= \$700 \times 1.14116 = \$798.814 - \$700 = \$98.814 = \text{Comp} \\ &\text{Int.} \end{aligned}$$

$$\$98.814 - \$94.50 = \$4.314.$$



(86)

X's gain =  $\frac{1}{12}$ , and Z's =  $\frac{1}{12}$ ;  $\therefore$  Y's gain =  $1 - (\frac{1}{12} + \frac{1}{12})$   
 $= 1 - \frac{1}{6} = \frac{5}{6}$ .

X's gain is  $\frac{1}{12}$  for 3 months, therefore for 1 month it is  $\frac{1}{36}$ .

Y's gain is  $\frac{5}{6}$  for 9 months, " " "  $\frac{5}{2}$ .

Z's gain is  $\frac{1}{12}$  for 4 months, " " "  $\frac{1}{3}$ .

$\frac{1}{36} : \frac{1}{36} :: \$3024 : \$3024 \times \frac{1}{36} \times \frac{1}{3} = \$672 =$  X's stock.

$\frac{1}{3} : \frac{1}{6} :: \$3024 : \$3024 \times \frac{1}{6} \times \frac{1}{3} = \$1120 =$  Y's stock.

(87)

$$\frac{1}{2} \times \sqrt{1\frac{1}{2}} \div (1\frac{1}{2})^2 = \frac{1}{2} \times \sqrt{\frac{3}{2}} \div (\frac{3}{2})^2 = \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}.$$

(88)

$4^2 = 16 \times 300$	=	4800	80677568161 (4321 cubert
$4 \times 3 = 12 \times 30$	=	360	64
$3^2$	=	9	16677
		5169	15507
$43^2 = 1849 \times 300$	=	554700	1170568
$43 \times 2 = 86 \times 30$	=	2580	
$2^2$	=	4	
		557284	1114568
$432^2 = 186624 \times 300$	=	55987200	56000161
$432 \times 1 = 432 \times 30$	=	12960	
$1^2$	=	1	
		56000161	56000161

(89)

$$7 = \left\{ 8 - 1 \begin{array}{l} \nearrow 3+4 \\ \searrow 1+6 \end{array} \right\} = 7$$

4 lbs. at 8d. }  
 1 lb. at 4d. } Make a mixture of 6 lbs. at 7d.  
 1 lb. at 6d. }

$$6 : 112 :: 4 : \frac{112 \times 4}{6} = 74\frac{2}{3} \text{ at 8d.}$$

(Continued on next page.)

Assume 4  
 Since 1st -  
 And 1st +  
 + 3rd  
 And 2nd -  
 + 3rd  
 And 3rd -  
 + 2nd

Assume 48  
 Since 1st +  
 And 1st +  
 + 3rd  
 And 2nd +  
 + 3rd  
 And 3rd +  
 + 2nd

- 2  
 - 1  
 -  
 Diff. = 1  
 657\frac{1}{2}

(89 continued.)

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 4d.}$$

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 6d.}$$

(90)

Assume 40 as the sum of the three numbers.

Since  $1\text{st} + 2\text{nd} + 3\text{rd} = 40$ ,

And  $1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 6 \dots \therefore 2\text{nd} + 3\text{rd} = 12$

And  $2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{1}{3}(1\text{st} + 3\text{rd}) = 6 \dots \therefore 1\text{st} + 3\text{rd} = 9$

And  $3\text{rd} + \frac{1}{4}(1\text{st} + 2\text{nd}) = 34 \therefore \frac{1}{4}(1\text{st} + 2\text{nd}) = 6 \dots \therefore 1\text{st} + 2\text{nd} = 8$

Adding,  $2 \times (1\text{st} + 2\text{nd} + 3\text{rd}) = 29$

$\therefore 1\text{st} + 2\text{nd} + 3\text{rd} = 14\frac{1}{2}$ .

But the sum should equal 40.

Hence  $14\frac{1}{2} - 40 = -25\frac{1}{2}$ .

Assume 48 as the sum of the three numbers.

Since  $1\text{st} + 2\text{nd} + 3\text{rd} = 48$ .

And  $1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 14 \dots \therefore 2\text{nd} + 3\text{rd} = 28$

And  $2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{1}{3}(1\text{st} + 3\text{rd}) = 14 \dots \therefore 1\text{st} + 3\text{rd} = 21$

And  $3\text{rd} + \frac{1}{4}(1\text{st} + 2\text{nd}) = 34 \therefore \frac{1}{4}(1\text{st} + 2\text{nd}) = 14 \dots \therefore 1\text{st} + 2\text{nd} = 18\frac{2}{3}$

Adding,  $2 \times (1\text{st} + 2\text{nd} + 3\text{rd}) = 67\frac{1}{3}$

$\therefore 1\text{st} + 2\text{nd} + 3\text{rd} = 33\frac{2}{3}$

But the sum should equal 48.

Hence  $33\frac{2}{3} - 48 = -14\frac{2}{3} = \text{error}$ .

Errors:

$$-25\frac{1}{2} \times 48 = 1224$$

$$-14\frac{2}{3} \times 40 = 586\frac{2}{3}$$

$$\text{Diff.} = 11\frac{1}{2} \quad \text{Diff.} = 657\frac{1}{3}$$

$$657\frac{1}{3} \div 11\frac{1}{2} = 58 = \text{the sum of the three numbers,}$$

(Continued on next page.)

(90 continued.)

$$\begin{aligned}
 1st + \frac{1}{2}(2nd + 3rd) &= 34 \therefore \frac{1}{2}(2nd + 3rd) = 58 - 34 = 24 \\
 \therefore 2nd + 3rd &= 48. \\
 2nd + \frac{1}{3}(1st + 3rd) &= 34 \therefore \frac{1}{3}(1st + 3rd) = 58 - 34 = 24 \\
 \therefore 1st + 3rd &= 36. \\
 1st + 2nd + 3rd &= 58, \text{ and } 2nd + 3rd = 48 \therefore 1st = 10. \\
 1st + 2nd + 3rd &= 58, \text{ and } 1st + 3rd = 36 \therefore 2nd = 22. \\
 2nd + 3rd &= 48, \text{ and } 2nd = 22 \therefore 3rd = 26.
 \end{aligned}$$

(91)

4 means + 2 extremes = 8 terms.

$$\text{Formula IX, p. 333. } d = \frac{l - a}{n - 1} = \frac{40 - 1}{6 - 1} = \frac{39}{5} = 7\frac{4}{5}.$$

1, 8 $\frac{4}{5}$ , 16 $\frac{8}{5}$ , 24 $\frac{12}{5}$ , 32 $\frac{16}{5}$ , 40.

(92)

 $s = 1860040$ ,  $l = 1240029$ , and  $r = 3$ .

$$\text{Formula XI, p. 340. } a = rl - (r - 1)s = 1240029 \times 3 - (2 \times 1860040) = 3720087 - 3720080 = 7.$$

(93)

6 apples + 7 pears cost 33 pence  $\therefore$  2 apples + 2 $\frac{1}{2}$  pears cost 11 pence.10 apples + 8 pears cost 44 pence  $\therefore$  2 apples + 1 $\frac{1}{2}$  pears cost 8 $\frac{1}{2}$  pence.Subtract, and 2 $\frac{1}{2}$  - 1 $\frac{1}{2}$  pears cost 11d. - 8 $\frac{1}{2}$ d.That is,  $\frac{1}{2}$  of a pear costs 2 $\frac{1}{2}$ d.If  $\frac{1}{2}$  cost  $\frac{1}{2}$ d.,  $\frac{1}{4}$  will cost  $\frac{1}{4}$  of  $\frac{1}{2}$ d., which is  $\frac{1}{8}$ d.If  $\frac{1}{4}$  cost  $\frac{1}{4}$ d.,  $\frac{1}{8}$  will cost  $\frac{1}{8}$ d. = 3d.6 apples + 7 pears cost 33 pence, and 7 pears cost 21d.  $\therefore$  6 apples cost 12d. and 1 apple costs 2d.

(94)

$$\begin{aligned}
 &\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{\frac{57}{2}}{\frac{1}{2}} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\
 &= 1 \times \frac{2}{4} \times \frac{5}{2} \times \frac{19}{12} \times \frac{2}{8} \times \frac{4}{5} \times \frac{2}{4} = \frac{19}{2 \times 4 \times 3 \times 2} = 1\frac{1}{12}
 \end{aligned}$$

(95)

\$10 =  $\frac{1}{2}$  of 2nd rem. — \$20  $\therefore \frac{1}{2}$  of 2nd rem. = \$30  $\therefore$  2nd rem. = \$40.  
 \$40 =  $\frac{1}{2}$  of 1st rem. — \$30  $\therefore \frac{1}{2}$  of 1st rem. = \$70  $\therefore$  1st rem. = \$87.50.  
 \$87.50 =  $\frac{1}{2}$  of original sum — \$50  $\therefore \frac{1}{2}$  of original sum = \$137.50  
 $\therefore$  original sum = \$137.50  $\times$  2 = \$275.

(96)

$a = 60, n = 17$ , and  $d = 4$ .

Formula VI, p. 333.  $s = \left\{ 2a + (n-1)d \right\} \frac{n}{2}$ .

$$= \left\{ 2 \times 60 + (17-1) \times 4 \right\} \frac{17}{2} = (120 + 64) \times \frac{17}{2}$$

$$= \frac{184 \times 17}{2} = \$1564 = \text{sum received for 17 years.}$$

Formula I, p. 333.  $l = a + (n-1)d = 60 + (17-1) \times 4$   
 $= 60 + 64 = \$124 = \text{wages for 17th year.}$

NINTH SERIES.

(98)

£749 16s. 5½d. = £749.823958 ; £1 sterling = \$4.867  
 £749.823958  $\times$  4.867 = \$3649.3932.

(99)

$$\begin{array}{r} 2)177408 \\ \hline \end{array}$$

$$\begin{array}{r} 2)88704 \\ \hline \end{array}$$

$$\begin{array}{r} 2)44352 \\ \hline \end{array}$$

$$\begin{array}{r} 2)22176 \\ \hline \end{array}$$

$$\begin{array}{r} 2)11088 \\ \hline \end{array}$$

$$\begin{array}{r} 2)5544 \\ \hline \end{array}$$

$$\begin{array}{r} 2)2772 \\ \hline \end{array}$$

$$\begin{array}{r} 2)1386 \\ \hline \end{array}$$

$$\begin{array}{r} 3)693 \\ \hline \end{array}$$

$$\begin{array}{r} 3)231 \\ \hline \end{array}$$

$$\begin{array}{r} 7)77 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ \hline \end{array}$$

$$2^8 \times 3^2 \times 7 \times 11.$$

(100)

Formula III, page 354,  $r = \sqrt{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt{\frac{A}{P}}$

$$\text{Log. } (r + 1) = (\text{log. } A - \text{log. } P) \div t$$

$$\begin{aligned} \text{That is, log. } (r + 1) &= (\text{log. } 11111 \cdot 11 - \text{log. } 704) \div 11 \\ &= (4.045757 - 2.847573) \div 11 \\ &= 1.198184 \div 11 = 0.108925 \end{aligned}$$

Therefore  $r + 1$  = natural number corresponding to the logarithm 0.108925 which is 1.285.

Since  $r + 1 = 1.285$ ,  $r = .285$  = rate per unit and rate per cent.  $= .285 \times 100 = 28\frac{1}{2}$ .

(101)

If 9 be  $\frac{1}{3}$ ,  $\frac{1}{3}$  or the whole will equal  $9 \times 13 = 117$ .

(102)

$$3 \text{ gal.} + 4 \text{ gal.} + 7 \text{ gal.} = 14 \text{ gal.}$$

$$\text{Hence } 14 \text{ gal.} : 292 \text{ gal.} :: 3 \text{ gal.} : \frac{292 \times 3}{14} = 62\frac{1}{2} \text{ of 1st kind.}$$

$$14 \text{ gal.} : 292 \text{ gal.} :: 4 \text{ gal.} : \frac{292 \times 4}{14} = 83\frac{1}{2} \text{ gal. of 2d. "}$$

$$14 \text{ gal.} : 292 \text{ gal.} :: 7 \text{ gal.} : \frac{292 \times 7}{14} = 146 \text{ gal. of 3d. "}$$

(103)

$$£\frac{1}{2} + £\frac{1}{2} + £\frac{1}{2} + £\frac{1}{2} = £1\frac{1}{2}$$

$$\begin{aligned} \text{Then } £1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{4}{9} &= \frac{£15000}{77} \\ &= £194 \text{ 16s. } 1\frac{1}{2}\text{d.} \end{aligned}$$

$$\begin{aligned} £1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{4}{9} &= \frac{£10000}{77} \\ &= £129 \text{ 17s. } 4\frac{1}{2}\text{d.} \end{aligned}$$

$$\begin{aligned} £1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{4}{9} &= \frac{£7500}{77} \\ &= £97 \text{ 8s. } 0\frac{1}{2}\text{d.} \end{aligned}$$

$$\begin{aligned} £1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{4}{9} &= \frac{£6000}{77} \\ &= £77 \text{ 18s. } 6\frac{1}{2}\text{d.} \end{aligned}$$

EXERCISE

By Tab

Hence P

By For

100

=

-06

100

=

6

Since each

Also, 5 m

Hence the

hours is c

Then Form

-1 of

 $\sqrt{1932}$ 

6:

\* Log.

$$\frac{1}{(1.06)^{23}} =$$

(104)

By Table, page 363, present value of annuity of \$1 at 6 per cent. for 23 payments = \$12 30338.

Hence present value of \$100 = \$12.30338 × 100 = \$1230.338.

By Formula V, page 361,  $v = \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\}$

$$= \frac{100}{.06} \times \left( 1 - \frac{1}{(1.06)^{23}} \right) = \frac{10000}{6} \times (1 - 0.261795)$$

$$= \frac{10000}{6} \times 0.738205 = \frac{738205}{6} = \$1230.34$$

(105)

Since each loses 1 hour per day for 24 days, the whole hours lost = 24 × 25.

Also, 5 men working 1 hour per day for 12 days make up 5 × 12 × 1 = 60 hours.

Hence they will each have to work as many hours per day as 60 hours is contained times in 24 × 25 hours, i.e.  $\frac{24 \times 25}{60} = 10$  hours.

(106)

$$a = 5, s = 161 \text{ and } d = 6$$

Then Formula II, p. 333.  $l = -\frac{1}{2}d + \sqrt{2ds + (a - \frac{1}{2}d)^2} = -\frac{1}{2} \text{ of } 6 + \sqrt{2 \times 6 \times 161 + (5 - \frac{1}{2} \text{ of } 6)^2} = -3 + \sqrt{1932 + 4} = -3 + \sqrt{1936} = -3 + 44 = 41 \text{ years.}$

(107)

$$6^3 : 10^3 :: 1 \text{ day} : \frac{10^3 \times 1}{6^3} = \frac{1000}{216} = 4.629 \text{ days.}$$

$$\begin{aligned} \bullet \text{ Log. } \frac{1}{(1.06)^{23}} &= \log. 1 - \log. 1.06 \times 23 = 0 - 0.025306 \times 23 \\ &= 0 - 0.582033 = \bar{1}.417967 \end{aligned}$$

$\therefore \frac{1}{(1.06)^{23}} = \text{natural number corresponding to the logarithm}$

$\bar{1}.417967$ , which is 0.261795

(108)

For 12 months he was to receive £8 and a suit of clothes; for 7 months he received £2 13s. 4d. and the suit of clothes; ∴ for 5 months he would have received the difference between £8 and £2 13s. 4d., which is £5 6s. 8d.

Hence for 1 month he would have received £5 6s. 8d. ÷ 5, which is £1 1s. 4d., and hence his wages for the year would have been, in money alone, £1 1s. 4d. × 12, i.e., £12 16s. Therefore the suit of clothes was valued at £12 16s. — £8 = £4 16s.

## TENTH SERIES.

(109)

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$ ; if  $\frac{13}{12}$  of a number = 48,  $\frac{1}{12}$  will =  $48 \div 13 = 3\frac{2}{13}$ . If  $3\frac{2}{13} = \frac{1}{12}$ ,  $\frac{1}{12}$ , or the whole number =  $3\frac{2}{13} \times 12 = 44\frac{4}{13}$ .

(110)

$$6^3 : 8^3 :: 600 : \frac{600 \times 8^3}{6^3} = \frac{600 \times 512}{216} = 1422\cdot2 \text{ lbs.}$$

(See Art. 33, sec. X.)

(111)

Part of ball remaining after 1st has taken off her share =  $\frac{1}{4}$   
Then whole ball : remainder :: cube of diameter of whole : cube of diameter of remainder

$$1 : \frac{1}{4} :: 5^3 : x^3 \text{ hence } x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{31\frac{1}{4}} = \sqrt[3]{93\cdot75} = 4\cdot542$$

∴ Part taken off by 1st = 5 in. — 4·542 in. = 0·458 in.

After 2nd had taken off her portion  $\frac{1}{4}$  of the ball remained.

$$1 : \frac{1}{4} :: 5^3 : x^3, \text{ hence } x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{31\frac{1}{4}} = \sqrt[3]{93\cdot75} = 4\cdot542$$

∴ Part taken off by 2nd = 4·542 — 3·968 = 0·574 in.

After 3rd had taken off her share there remained  $\frac{1}{4}$  of the ball.

$$1 : \frac{1}{4} :: 5^3 : x^3, \text{ hence } x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{31\cdot25} = 3\cdot149 \text{ in.}$$

∴ Part taken off by 3rd = 3·968 — 3·149 = 0·819 inches

Remainder = 3·149 = part taken off by 4th.

NOTE.—  
twice as m  
be 6 or 7.

1st

2nd

es; for 7  
clothes;  
rence be-

8d. ÷ 5,  
ar would  
£12 16s.  
s. — £8

13 = 3 <sup>9</sup>/<sub>13</sub>  
4.

= 4  
: cube

= 4.542

ed.  
968 in.

ball.  
n.  
es

$$71214.43 \div 12.342 = 71214430 \div 12342$$

$$12342)71214430(5570.238552$$

62831

72734

62831

88033

87625

3070.0

2468.4

500.50

371.36

118.130

111.067

7.0520

6.2831

.65780

.62831

.028480

.024684

.003685

(112)

$$5570.238552(71.118 = \text{sq. rt.}$$

54

151) 170

151

1521) 18.23

15.21

15221) 3.0285

1.5221

152228) 1.406452

1.360051

.036401

NOTE.—Unless the quotient is carried out to *six places* of decimals, i.e., twice as many as are required in the root, the last figure in the root will be 6 or 7.

(113)

1st	{	\$60 × 48 =	\$2880 for 1 month	} = \$43280 for 1 month.
		\$800 × 43 =	34400 for 1 month	
		\$1500 × 4 =	6000 for 1 month	
		Sum =	\$43280	
2nd	{	\$600 × 48 =	\$28800 for 1 month	} = \$104400 for 1 month.
		\$1800 × 42 =	75600 for 1 month	
		Sum =	\$104400	

(Continued on next page.)



(113 continued).

$$\begin{array}{lcl}
 \text{3rd} & \left\{ \begin{array}{l} \$400 \times 48 = \$19200 \\ \$500 \times 42 = 21000 \\ \$500 \times 36 = 18000 \\ \$500 \times 30 = 15000 \\ \$500 \times 24 = 12000 \\ \$500 \times 18 = 9000 \\ \$500 \times 12 = 6000 \\ \$500 \times 6 = 3000 \\ \hline \text{Sum} = \$103200 \end{array} \right. & = \$103200 \text{ for 1 month.}
 \end{array}$$

$$\begin{array}{lcl}
 \text{4th} & \left\{ \begin{array}{l} \$900 \times 40 = \$36000 \\ \$900 \times 34 = 30600 \\ \$900 \times 28 = 25200 \\ \$900 \times 22 = 19800 \\ \$900 \times 16 = 14400 \\ \$900 \times 10 = 9000 \\ \$900 \times 4 = 3600 \\ \hline \text{Sum} = \$138600 \end{array} \right. & = \$138600 \text{ for 1 month.}
 \end{array}$$

\$43280

104400

103200

138600

4 years at \$1.25 per day

$$= \$1.25 \times 4 \times 365 = \$1825 = \text{share of 5th.}$$

\$389480 for one month.

\$20000 — \$1825 = \$18175 = sum to be divided among the four.

\$389480 : \$18175 :: \$43280 : \$2019.651 = share of 1st.

\$389480 : \$18175 :: \$104400 : \$4871.803 = " 2nd.

\$389480 : \$18175 :: \$103200 : \$4815.805 = " 3rd.

\$389480 : \$18175 :: \$138600 : \$6467.739 = " 4th.

(114)

$$\begin{aligned}
 \text{Simple Interest, formula IX, p. 248. } t &= \frac{n-1}{r} = \frac{16-1}{.05} = \frac{15}{.05} \\
 &= \frac{1500}{5} = 300 \text{ years.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Compound Interest, formula V, p. 354. } t &= \frac{\log. n}{\log. (1+r)} \\
 &= \frac{\log. 16}{\log. 1.05} = \frac{1.204120}{0.021189} = \frac{1204120}{21189} = 56.827 \text{ years.}
 \end{aligned}$$

(115)

For every \$1 the first gave, the second gave \$3, and the third \$6.  $\$1 + \$3 + \$6 = \$10$ .

Hence the 1st gave \$1, the second \$3, and the third \$6 as often as \$10 is contained times in \$9202, which is  $920\frac{1}{2}$  times.

$$\$1 \times 920\frac{1}{2} = \$920.20 = \text{payment of 1st person.}$$

$$\$3 \times 920\frac{1}{2} = \$2760.60 = \quad \quad \quad \text{2nd "}$$

$$\$6 \times 920\frac{1}{2} = \$5521.20 = \quad \quad \quad \text{3rd "}$$

(116)

$$25 + 22 = 47 = \text{whole number of men.}$$

$$165 \div 47 = 3\frac{1}{2} = \text{acres cleared by each man.}$$

$$3\frac{1}{2} \times 22 = 77\frac{1}{2} \text{ acres} = \text{acres cleared by company of 22 men.}$$

$$165 \text{ acres} - 77\frac{1}{2} \text{ acres} = 87\frac{1}{2} \text{ acres} = \text{acres cleared by company of 25 men.}$$

1st company contains 3 more men than 2nd company and receives \$86 more.

Therefore \$86 pays 3 men. Hence each man gets  $\$86 \div 3 = \$28.66\frac{2}{3}$ .

Each man clears  $3\frac{1}{2}$  acres, and receives  $\$28.66\frac{2}{3}$  for it; therefore cost of 1 acre  $= \$28.66\frac{2}{3} \div 3\frac{1}{2} = \$8.49\frac{1}{2}$ .

(117)

$$15^2 = 225; 346 - 225 = 121 = \text{square of the less.}$$

$$\text{Hence less} = \sqrt{121} = 11.$$

(118)

$$\text{Formula V, page 248, } A = P(1 + rt) = \$1200 \times 1.95 = \$2340.00.$$

(119)

$$\begin{array}{l|l} 24 : 496 \\ 9 : 11 \\ 7 : 4 \\ 465 : 337\frac{1}{2} \\ 3\frac{1}{2} : 5\frac{1}{2} \\ 2\frac{1}{2} : 3\frac{1}{2} \end{array} \quad \begin{array}{l} \\ \\ \\ \\ :: 5\frac{1}{2} : x \end{array}$$

(Continued on next page.)

(119 continued.)

$$\begin{array}{cccccccccccc}
 31 & & & & & & & & & & & & & \\
 62 & & & & & & & & & & & & & \\
 408 & 11 & 4 & & & & & & & & & & & \\
 \hline
 1 & \times & 1 & \times & 1 & \times & 2 & \times & 5 & \times & 2 & \times & 2 & \times & 24 & \times & 8 & \times & 8 & \times & 31
 \end{array}$$

$$\begin{array}{c}
 8 \quad 8 \\
 \times \frac{8}{11} \times \frac{8}{7} = 11 \times 4 \times 3 = 132 \text{ days.}
 \end{array}$$

(120)

$$\begin{array}{lcl}
 A + B + C & = & \frac{47}{60} \\
 B + C + D & = & \frac{37}{60} \\
 A + C + D & = & \frac{43}{60} \\
 A + B + D & = & \frac{45}{60} \\
 \hline
 3A + 3B + 3C + 3D & = & \frac{171}{60} \\
 \therefore A + B + C + D & = & \frac{57}{20}
 \end{array}
 \quad
 \begin{array}{lcl}
 A + B + C + D & = & \frac{47}{60} \\
 A + B + C & = & \frac{47}{60} \\
 \therefore D & = & \frac{10}{60} = \frac{1}{6} \\
 A + B + C + D & = & \frac{57}{60} \\
 B + C + D & = & \frac{37}{60} \\
 \therefore A & = & \frac{20}{60} = \frac{1}{3} \\
 A + B + C + D & = & \frac{47}{60} \\
 A + C + D & = & \frac{43}{60} \\
 \therefore B & = & \frac{4}{60} = \frac{1}{15} \\
 A + B + C + D & = & \frac{47}{60} \\
 A + B + D & = & \frac{45}{60} \\
 \therefore C & = & \frac{2}{60} = \frac{1}{30}
 \end{array}$$

$\frac{47}{60} : \frac{37}{60} :: \frac{1}{3} : \frac{1}{3} \times \frac{60}{30} \times \frac{47}{30} = \frac{47}{30} = D$ 's true share which is therefore  $= \frac{47}{30}$  of \$6213 = \$1090.

$\frac{47}{60} : \frac{37}{60} :: \frac{1}{3} : \frac{1}{3} \times \frac{60}{30} \times \frac{37}{30} = \frac{37}{30} = A$ 's true share which is therefore  $= \frac{37}{30}$  of \$6213 = \$2180.

$\frac{47}{60} : \frac{37}{60} :: \frac{1}{3} : \frac{1}{3} \times \frac{60}{30} \times \frac{37}{30} = \frac{37}{30} = B$ 's true share which is therefore  $= \frac{37}{30}$  of \$6213 = \$1635.

$\frac{47}{60} : \frac{37}{60} :: \frac{1}{3} : \frac{1}{3} \times \frac{60}{30} \times \frac{37}{30} = \frac{37}{30} = C$ 's true share which is therefore  $= \frac{37}{30}$  of \$6213 = \$1308.

ELEVENTH SERIES.

(121)

$$\begin{aligned} \cdot 7; \cdot 82 = 53; \cdot 727 = 733; \cdot 91325 = 133359 = 81310 = 15557. \\ 8 \cdot 671347 = 8^{671347} = 8^{671347} = 8^{671347} = 8^{671347}. \end{aligned}$$

(122)

$$713 \text{ unden.} = 861 \text{ den.}; 291 \text{ unden.} = 342 \text{ den.}; 3t1 \text{ unden.} = 471 \text{ den.}$$

$$\text{Then } 713 \text{ unden.} = 861 \frac{342}{3t1} \text{ den.} = 861 \frac{342}{3t1} \text{ den.}$$

$$12123 \text{ quat.} = 411 \text{ den.}; 1122 \text{ quat.} = 363 \text{ den.}; 100000 \text{ quat.} = 1024 \text{ den.}$$

$$\text{Then } 12123 \text{ quat.} = 411 \frac{363}{1024} \text{ den.}$$

(123)

$$\begin{aligned} 3\frac{3}{4} \text{ of } 2\frac{1}{2} \text{ of } 7\frac{1}{2} \text{ of } £1 &= 2\frac{1}{2} \text{ of } 1\frac{1}{2} \text{ of } 1\frac{1}{2} \text{ of } £1 \\ &= £1 \frac{180}{12} \dots\dots\dots = £56 \ 1 \ 2\frac{1}{2} \\ 9\frac{3}{4} \text{ of } 3\frac{3}{4} \text{ of } 1s. &= 2\frac{1}{2} \text{ of } 1\frac{1}{2} \text{ of } 1s. = 1\frac{1}{2} \text{ of } 1s. \dots = 1 \ 16 \ 8 \\ 8\frac{1}{2} \text{ of } 4\frac{1}{2} \text{ of } 1d. &= 3\frac{3}{4} \text{ of } 3\frac{3}{4} \text{ of } 1d. = 1\frac{1}{2} \text{ of } 1d. \dots = 0 \ 2 \ 10\frac{1}{2} \end{aligned}$$

$$\text{Sum} = £58 \ 0 \ 8\frac{1}{2}$$

$$1\frac{1}{2} \text{ of } 1\frac{1}{2} \text{ of } 3\frac{3}{4} \text{ of } 3\frac{3}{4} \text{ d.} = 1\frac{1}{2} \times 1\frac{1}{2} \times 3\frac{3}{4} \times 3\frac{3}{4} = 1\frac{1}{2} \text{ d.}$$

$$£58 \ 0s. \ 8\frac{1}{2} \text{ d.} = 22285 \text{ d.}$$

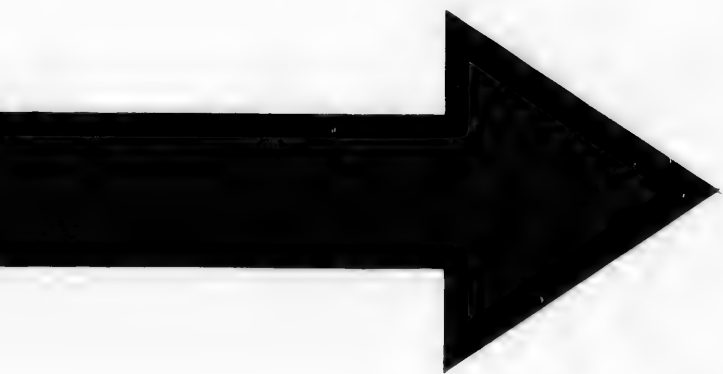
$$22285 \div 120 = 185 \frac{85}{120} \times 120 = 22285 \times \frac{1}{2} = 11142 \frac{1}{2}$$

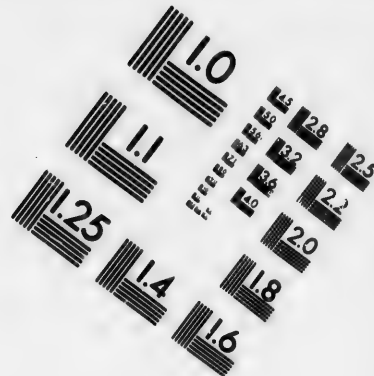
(124)

$$\begin{array}{l|l} 24 : 90 & \\ 2\frac{1}{2} : 4\frac{1}{2} & \\ 12\frac{1}{2} : 9\frac{3}{4} & :: 139\frac{1}{2} : x \\ 4\frac{7}{8} : 4\frac{1}{2} & \\ 3\frac{1}{2} : 2\frac{1}{2} & \end{array}$$

(Continued on next page.)



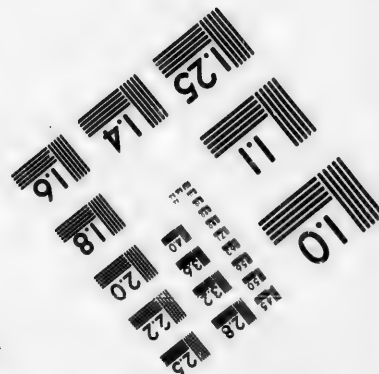
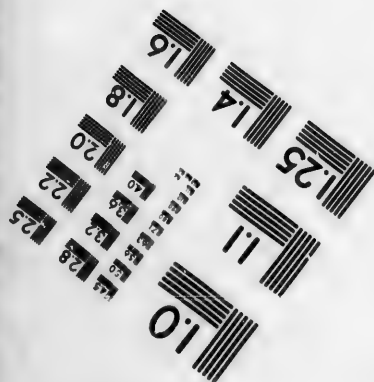
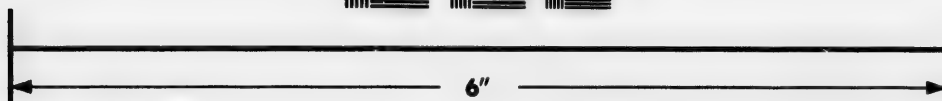




Resolution test chart showing various line patterns and numerical values:

- 1.0
- 1.1
- 1.25
- 1.4
- 1.6
- 1.8
- 2.0
- 2.2
- 2.5
- 2.8
- 3.2
- 3.6
- 4.0
- 4.5
- 5.0
- 5.6
- 6.3
- 7.1
- 8.0
- 9.0
- 10

Resolution Test Chart  
 Model No. 100  
 Made in U.S.A.



# Photographic Sciences Corporation

**23 WEST MAIN STREET  
WEBSTER, N.Y. 14580  
(716) 872-4503**





(124 continued.)

$$\begin{array}{cccccccc}
 9 & & & & & & & \\
 18 & 7 & & 3 & & 43 & & \\
 80 & 21 & 29 & 8 & 5 & 859 & 1 & 2 & 2 & 3 \\
 \hline
 1 & 5 & 8 & 2 & 2 & 4 & 24 & 5 & 25 & 84 \\
 & & & & & & 8 & & 5 & 18 \\
 \times & \frac{5}{18} & = & \frac{9 \times 7 \times 29 \times 43}{2 \times 4 \times 5 \times 4} & = & \frac{78561}{160} & = & 491\frac{11}{16}
 \end{array}$$

(125)

\$182 is  $\frac{91}{100}$  of buying price  $\therefore \$182 \div 91 = \$2 = \frac{100}{100}$  of buying price  $\therefore$  buying price  $= \$2 \times 100 = \$200$ .  
 To realize a profit of 7 per cent., he must receive \$1.07 for every \$1 the goods cost; but they cost him \$200, therefore he must sell for  $\$1.07 \times 200 = \$214$ .

(126)

$$\text{Simple Interest } t = \frac{n-1}{r} = \frac{11\frac{1}{2}-1}{.06} = \frac{10.5}{.06} = \frac{1050}{6} = 175 \text{ years.}$$

$$\text{Compound Interest } t = \frac{\log. n}{\log. (1+r)} = \frac{\log. 11\frac{1}{2}}{\log. 1.06} = \frac{1.060698}{0.025306} = \frac{1060698}{25306} = 41.914 \text{ years.}$$

(127)

An acre contains 4 roods  $= 160$  sq. perches.  
 $\therefore 160 \div 15\frac{1}{2} = 10\frac{1}{2}$  perches  $=$  length.

(128)

$$\begin{array}{l}
 35 \text{ yards} = 32 \text{ metres } \therefore 1 \text{ yd.} = \frac{32}{35} \text{ of a metre.} \\
 69\frac{1}{2} \text{ miles} = 69\frac{1}{2} \times \frac{1760}{1} \text{ yards} = 69\frac{1}{2} \times \frac{1760}{1} \times \frac{32}{35} \text{ metres} \\
 = \frac{217}{22} \times \frac{1760}{1} \times \frac{32}{35} = 217 \times 16 \times 32 = 111104 \text{ metres.}
 \end{array}$$

(129)

$r$  means + 2 extremes = 9 terms.

Formula XIII, p. 340.  $r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{19683}{3}\right)^{\frac{1}{8}} = (6561)^{\frac{1}{8}} = 3$

Hence means are 9, 27, 81, 243, 729, 2187, and 6561.

(130)

Formula XXI, p. 344.  $s = \frac{a}{1-r} = \frac{7}{1-\frac{1}{2}} = \frac{7}{\frac{1}{2}} = \frac{7}{1} \times \frac{2}{1} = 14$

(131)

Part remaining after 1st has received his share =  $\frac{1}{4}$ .

$1 : \frac{1}{4} :: 60^2 : x^2$ ; whence  $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900 \times 3}$   
 $= 30\sqrt{3} = 1.732 \times 30 = 51.96$  inches.

Hence 1st ground off  $60 - 51.96 = 8.04$  inches.

Part remaining after 2nd had taken off his share =  $\frac{1}{4}$ .

$1 : \frac{1}{4} :: 60^2 : x^2$ ; whence  $x = \sqrt{3600 \times \frac{1}{4}} = 30\sqrt{2}$   
 $= 1.4142 \times 30 = 42.426$ .

Hence 2nd ground off  $51.96 - 42.426 = 9.534$  inches.

Part remaining after the 3rd had taken off his share =  $\frac{1}{4}$ .

$1 : \frac{1}{4} :: 60^2 : x^2$ ; whence  $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900} = 30$  inches.

Hence 3rd ground off  $42.426 - 30$  inches = 12.426 inches,  
 and the 4th ground off remaining 30 inches.

(132)

1 guinea = 21s.

1 half guinea = 10½s.

1 crown = 5s.

1 half crown = 2½s.

1 shilling = 1s.

Sixpence = ½s.

Sum = 40½s.

100 guineas = 2100 shillings.

$2100 \div 40\frac{1}{2} = 51$  times and remainder, 69 half-shillings.

69 half-shil. = 34s. = £17 = 17s.

## TWELFTH SERIES.

(133)

$$\frac{3}{11} \text{ of } \frac{2}{9} \text{ of } \frac{4}{17} = \frac{8}{561}; \frac{2\frac{1}{2}}{4\frac{1}{2}} \text{ of } \frac{2}{5} = \frac{10}{17} \text{ of } \frac{2}{5} = \frac{4}{17}$$

$$\frac{8}{561} : \frac{4}{17} :: \$12\frac{4}{17} : \$12\frac{4}{17} \times \frac{4}{17} \times \frac{561}{8} = \frac{200}{88} \times \frac{4}{17} \times \frac{88}{4} = \$200.$$

(134)

By Formula III, page 354,  $r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}}$

$$\therefore \text{Log. } (1 + r) = (\text{log. } A - \text{log. } P) \div t$$

$$= (\text{log. } 1679.40 - \text{log. } 700.90) \div 5$$

$$= (3.225154 - 2.845656) \div 5.$$

$$= 0.379498 \div 5 = 0.075899.$$

$\therefore 1 + r = \text{nat. num. corresponding to the logarithm } 0.075899$   
which is 1.19,  $\therefore r = .19 = \text{rate per unit, and hence rate per cent.} = 19.$

(135)

Having paid 10 per cent. he had 90 per cent. remaining.

$$\frac{90}{100} \text{ or } \frac{9}{10} \text{ of his salary} = \$1250, \therefore \frac{1}{10} = \frac{1250}{9} = \$138\frac{2}{9}.$$

$$\text{If } \$138\frac{2}{9} = \frac{1}{10}, \text{ the whole} = \$138\frac{2}{9} \times 10 = \$1388\frac{8}{9}.$$

(136)

21 children receive 21 times a child's share

21 women " 42 " "

21 men " 63 " "

Together they receive 126 " "

$$£3 \text{ } 13\text{s. } 6\text{d.} \div 126 = 7\text{d.} = \text{a child's share.}$$

$$7\text{d.} \times 2 = 1\text{s. } 2\text{d.} = \text{a woman's share.}$$

$$7\text{d.} + 1\text{s. } 2\text{d.} = 1\text{s. } 9\text{d.} = \text{a man's share.}$$

(137)

A gets 1 time A's share

B " 1 " A's "

C " 2 " A's "

D " 4 " A's "

Together they get 8 times A's share.

$\$200 \div 8 = \$25 = \text{A's share}; \$25 = \text{B's share.}$

$\$25 + \$25 = \$50 = \text{C's share}; \$25 + \$25 + \$50 = \$100 = \text{D's share.}$

(138)

$\sqrt[3]{\frac{1}{2}} = \frac{1}{2} \sqrt[3]{\frac{1}{8}} = \frac{1}{2} \text{ of } 2.62074 = .87358$

$\sqrt[3]{\frac{1}{3}} = \frac{1}{3} \sqrt[3]{\frac{1}{6}} = \frac{1}{3} \text{ of } 2.44948 = .81649$

Difference = .05709

(139)

$\frac{387}{92807}$  when each term is divided by 121, becomes  $\frac{3}{767}$ .

$17\frac{1}{2} + \frac{1}{8} + 144\frac{1}{4} = 161 + \frac{5}{2} + \frac{1}{8} + \frac{1}{4} = 161 + \frac{11}{8} + \frac{1}{4} = 162\frac{3}{4}$

$+ \frac{1}{2} = 161 + \frac{49}{20} = 161 + 1\frac{9}{20} = 162\frac{29}{20} = 162\frac{3}{4}$

$2\frac{1}{2} - \frac{1}{8} = 2\frac{4}{8} - \frac{1}{8} = 1\frac{3}{8} = 1\frac{3}{8}$

$\begin{array}{cccccc} 3 & 6 & 4 & 15 & 21 & 54 \\ \text{of} & \text{of} & \text{of} & \text{of} & \text{of} & \\ 4 & 7 & 15 & 11 & 23 & 253 \end{array}$

$6347 \div 2\frac{1}{2} = 2539\frac{4}{5} \div \frac{1}{4} = 2539\frac{4}{5} \times \frac{4}{1} = 2308.$

(140)

884736 (96 = cube root.

729

155736

$9^3 = 81 \times 300 = 24300$

$9 \times 6 = 54 \times 30 = 1620$

$6^3 = 36$

25956

155736

$95951\frac{1}{2} = 95951.2576.$

(Continued on next page.)

(140 continued.)

$$\begin{array}{r} 95951 \cdot 2576 \\ 9 \end{array} \quad (309 \cdot 76 = \text{square root.})$$

$$\begin{array}{r} 609) \quad 5951 \\ \underline{5481} \end{array} \quad \begin{array}{r} 309 \cdot 76 \\ 1 \end{array} \quad (17 \cdot 6 = 17\frac{1}{2} = \text{fourth root.})$$

$$\begin{array}{r} 618 \cdot 7) 470 \cdot 25 \\ \underline{433 \cdot 09} \end{array} \quad \begin{array}{r} 27) 209 \\ \underline{189} \end{array}$$

$$\begin{array}{r} 619 \cdot 46) 37 \cdot 1676 \\ \underline{37 \cdot 1676} \end{array} \quad \begin{array}{r} 34 \cdot 6) 2076 \\ \underline{2076} \end{array}$$

(141)

250

300

400

500

$$1450:250::\$520:\frac{\$520 \times 250}{1450} = \$89\frac{1}{2} = \text{contrib. on 1st village.}$$

$$1450:300::\$520:\frac{\$520 \times 300}{1450} = \$107\frac{1}{2} = \text{ " 2nd "}$$

$$1450:400::\$520:\frac{\$520 \times 400}{1450} = \$143\frac{1}{2} = \text{ " 3rd "}$$

$$1450:500::\$520:\frac{\$520 \times 500}{1450} = \$179\frac{2}{3} = \text{ " 4th "}$$

(142)

By Table on p. 362, the amount of \$1 for 34 payments at 3 per cent. = \$57.73018.

$$\$57.73018 \times 260 = \$15009.84.$$

By Formula I, page 361,  $A = \frac{a \{ (1+r)^t - 1 \}}{r}$

$$= \frac{a}{r} \{ (1+r)^t - 1 \} = \frac{260}{.03} \{ (1.03)^{34} - 1 \}$$

$$= \frac{26000}{3} \times (2.731855 - 1) = \frac{26000 \times 1.731855}{3} = \$15009.41$$

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(143)

irth root.

$$\text{By Formula IX, p. 333, } d = \frac{l-a}{n-1} = \frac{79-2}{6-1} = \frac{77}{5} = 15\frac{2}{5}.$$

Hence the series is 2,  $17\frac{2}{5}$ ,  $32\frac{4}{5}$ ,  $48\frac{2}{5}$ ,  $63\frac{4}{5}$ , and 79.

$$\text{Formula I, p. 333. } l = a + (n-1)d = 3 + (9-1) \times 4 = 3 + (8 \times 4) = 3 + 32 = 35.$$

$$\begin{aligned} \text{Formula VI, p. 333. } s &= \left\{ 2a + (n-1)d \right\} \frac{n}{2} \\ &= \left\{ 2 \times 3 + (207-1) \times 4 \right\} \frac{207}{2} = \left\{ 6 + (206 \times 4) \right\} \frac{207}{2} \\ &= (6 + 824) \times \frac{207}{2} = \frac{830 \times 207}{2} = 85905. \end{aligned}$$

village.

(144)

B travels 4 miles per-day faster than A, and will therefore gain the circumference of the island in  $7\frac{3}{4} = 18\frac{1}{2}$  days.

C travels 10 miles per day faster than A, and will therefore gain the whole circumference of the island in  $7\frac{3}{8} = 7\frac{3}{8}$  days.

Now B cannot be with A except at the end of  $18\frac{1}{2}$  days or twice  $18\frac{1}{2}$  days, or three times  $18\frac{1}{2}$  days, or some other multiple of  $18\frac{1}{2}$  days.

Similarly C cannot be with A except at the end of  $7\frac{3}{8}$  days, or of some other multiple of  $7\frac{3}{8}$  days.

Therefore C and B will both be with A for the first time after the lapse of a number of days expressed by the least common multiple of  $18\frac{1}{2}$  and  $7\frac{3}{8}$ .

The greatest common factor of  $18\frac{1}{2}$  and  $7\frac{3}{8}$  is  $3\frac{1}{8}$ .

Hence the l. c. m. of  $7\frac{3}{8}$  and  $18\frac{1}{2}$  is  $\frac{7\frac{3}{8} \times 18\frac{1}{2}}{3\frac{1}{8}} = 36\frac{3}{4} = \text{number of days when A, B, and C will first be together.}$

## ARITHMETICAL RECREATIONS.

1. The third of 6 = 2, and the fourth of 20 = 5.  
Then if 2 becomes 3, what should 5 become? Evidently  
7½. *Ans.*

or

$$\left. \begin{array}{l} 6 : 20 \\ \frac{1}{3} : \frac{1}{4} \end{array} \right\} :: 3 : x = \frac{3 \times 20 \times \frac{1}{4}}{6 \times \frac{1}{3}} = 7\frac{1}{2}.$$

2. The half of 5 = 2½; then if 7 becomes 2½, what will 11 become?

$$\frac{2\frac{1}{2} \times 11}{7} = 4\frac{1}{4}. \text{ Lastly, what part of 9 is } 4\frac{1}{4}?$$

$$\frac{4\frac{1}{4}}{9} = \frac{\frac{17}{4}}{9} = \frac{17}{36}. \text{ *Ans.*}$$

or

$$\left. \begin{array}{l} 9 : 5 \\ 7 : 11 \end{array} \right\} :: \frac{1}{4} : x = \frac{\frac{1}{4} \times 5 \times 11}{9 \times 7} = \frac{\frac{55}{4}}{63} = \frac{55}{252}. \text{ *Ans.*}$$

3. 99½.

4. ½ of 2d. = 1d. Then 1d. is what part of 3d.? *Ans.* ⅓.  
5. 1½d. for a herring and a half is at the rate of 1d. per herring; hence 11 herrings will cost 11d.

6. 12 apples = 21 pears = 7 cents.

If 12 apples cost 7 cents, what will 100 apples cost?

$$12 : 100 :: 7 : \frac{100 \times 7}{12} = 58\frac{1}{3} \text{ cents.}$$

7. If 5 is ⅓ of a certain number, ⅓ will be ⅓ of 5, which is ⅓.

If ⅓ is ⅓ of a certain number, the whole number will be

$$\frac{1}{3} \times 7 = \frac{7}{3} = 2\frac{1}{3}. \text{ *Ans.*}$$

8. The hurdles are arranged so as to form a rectangular enclosure having 49 hurdles on each side and one on each end. Two additional hurdles will give two hurdles to each end, and will thus double the size of the enclosure.

9. The mode of dividing the plot may be learned from the following figure:—

Evidently

st will 11

Ans.

Ans. 3.

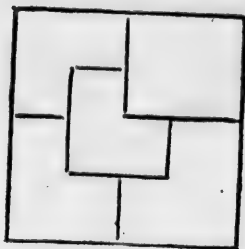
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10. 333

11. XIII; rub out the lower half, and there remains the expression VIII = 8.

12. 1st Step: Fill the 3-gallon cask and empty it into the 5-gallon cask.

2nd Step: Again fill the 3-gallon cask out of the 8-gallon cask.

3rd Step: Fill up the 5-gallon cask out of the 3-gallon cask. This will leave one gallon in the latter.

4th Step: Empty the 5-gallon cask into the 8-gallon cask.

5th Step: Pour the one gallon out of the 3-gallon cask into the 5-gallon cask.

6th Step: Fill the 3-gallon cask out of the 8-gallon cask, and empty it into the 5-gallon cask.

The following diagrams show this more clearly:

1st Step.

2nd Step.



3rd Step.

4th Step.



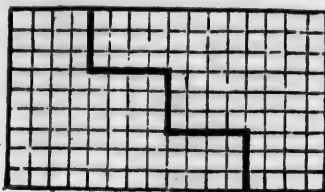
5th Step.

6th Step.





13. The heavy line in the accompanying figure shows how the board is to be cut.



8	1	6
3	5	7
4	9	2

15. Weigh out 7 lbs. as often as possible and there will remain 2 lbs.; add two four pounds and one seven pounds to this, and the sum will be 17 lbs., the share of one.  
Weigh 7 lbs. as often as possible out of the remaining 34 lbs. and there will remain 6 lbs., to which add 7 lbs. and 4 lbs., and the sum will be 17 lbs., the share of the second. The remaining 17 lbs. will be the share of the third.
16. The hurdles are, in the first case, placed 12 on a side and one on each end, and then they inclose a space represented by 13 squares whose area is, by the question, 40 square yards. If two hurdles be taken away there will remain 24, and if these be placed in the form of a square, each side containing 6 hurdles, they will enclose a space represented by 36 squares of the same size as the former. Hence they now inclose three times as much space as before, *i. e.* three times 40 square yards, or 120 square yards.
17. He takes the goose to the remote bank and leaves it there, returning, he next carries over the fox, which he leaves, but takes the goose back with him. He now leaves the goose on the first bank, and carries over the oats which he allows to remain on the remote bank with the fox, and returns for the goose.
18. The following diagrams exhibit the solution of this problem:

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problem:

EXERCISE 133.)

KEY.

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I.	II.	III.	IV.
3 3 3	4 1 4	2 5 2	1 7 1
3 P 3	1 P 1	5 P 5	7 P 7
3 3 3	4 1 4	2 5 2	1 7 1
24	20	28	24

V.	VI.
0 9 0	5 0 4
9 P 9	0 P 0
0 9 0	4 0 5
26	18

19. XII; rub out the lower half, and VII remains;  
20.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

RULE FOR FILLING MAGIC SQUARES OF ODD NUMBER OF CELLS.

Begin in centre cell of top horizontal row by placing 1 in it; ascend diagonally to the right, and where this carries us beyond the square, transport the next number to the cell at the remote end of the vertical or horizontal band to which it belongs. When in ascending we come to a cell already filled, we place the number in the cell next below the cell last filled. The following is a square of 7 cells in a side filled after this method:

(Continued on next page.)

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

21. Half-a-dozen dozen =  $6 \times 12 = 72$ .

Six dozen dozen =  $6 \times 12 \times 12 = 864$ .

$864 - 72 = 792$ . *Ans.*

22. The following shows the mode of performing this.

It will be observed that the two side counters are merely moved one counter higher when the other two are taken away.

23. This problem admits of the following two solutions:

#### 1st SOLUTION.

Persons. Full bottles. Hf.-full bottles. Empty bottles.

1st 2 3 2

2nd 2 3 2

3rd 3 1 3

7 7 7

Each person has  $3\frac{1}{2}$  bottles of wine and 7 bottles.

#### 2nd SOLUTION.

1st 3 1 3

2nd 3 1 3

3rd 1 5 1

7 7 7

Each person, as before, has 7 bottles and  $3\frac{1}{2}$  bottles of wine.

24. There were in all 8 bottles of wine, of which each drank  $\frac{1}{4}$ , which is 2. The third person, therefore, drank  $\frac{1}{4}$  of a bottle belonging to him who had but 3 bottles, and  $\frac{1}{4}$  of a bottle belonging to him who owned the 5 bottles. Hence the latter should have seven times as much of the money as the former, or, in other words, the latter gets 7 shillings, and the former 1 shilling.

25. This problem is merely to find some number between 50 and 100 which is exactly divisible by 2 and by 3, but which divided by 5 leaves a remainder 3.

The only numbers between 50 and 100 that are divisible by both 2 and 3, are 54, 60, 66, 72, 78, 84, 90, and 96, and by inspection the only one of these which gives a remainder 3 when divided by 5 is 78; therefore the basket contained 78 eggs.

26. Ans. 1 lb., 3 lbs., 9 lbs., and 27 lbs.  
For 1 lb. = 1 lb.; 2 lbs. = 3 lbs. — 1 lb., i. e. 3 lbs. in one scale and 1 lb. in the other; 3 lbs. = 3 lbs.; 4 lbs. = 3 lbs. + 1 lb.; 5 lbs. = 9 lbs. — (3 lbs. + 1 lb.); 6 lbs. = 9 lbs. — 3 lbs.; 7 lbs. = 9 lbs. + 1 lb. — 3 lbs.; 8 lbs. = 9 lbs. — 1 lb.; 9 lbs. = 9 lbs.; 10 lbs. = 9 + 1 lb.; 11 lbs. = 9 lbs. + 3 lbs. — 1 lb.; 12 lbs. = 9 lbs. + 3 lbs.; 13 lbs. = 9 lbs. + 3 lbs. + 1 lb.; 14 lbs. = 27 lbs. — (9 lbs. + 3 lbs. + 1 lb.); 15 lbs. = 27 lbs. — (9 lbs. + 3 lbs.); 16 lbs. = 27 lbs. + 1 lb. — (9 lbs. + 3 lbs.); 17 lbs. = 27 lbs. — (9 lbs. + 1 lb.); 18 lbs. = 27 lbs. — 9 lbs.; &c., &c.

27. In order to fill seven out of the eight points, it is merely requisite to remember that the second counter must be carried to the point from which the first started, the third to the point from which the second started, &c.

Thus if the first counter is carried from 1 to 4 and there deposited, the second must be taken from 6 to 1 and there deposited; the third from 3 to 6; the fourth from 8 to 3; the fifth from 5 to 8; the sixth from 2 to 5; and the seventh either from 7 to 2 or from 2 to 7.

28. The mouth fills the reservoir in 6 hours, therefore it fills  $\frac{1}{6}$  in 1 hour; the right eye fills it in 36 hours, therefore it fills

bottles.

of wine.

$\frac{1}{12}$  in 1 hour; the left eye fills it in 72 hours, therefore it fills  $\frac{1}{72}$  in 1 hour; the foot fills it in 96 hours, therefore it fills  $\frac{1}{96}$  in 1 hour. Hence together they fill  $\frac{1}{6} + \frac{1}{72} + \frac{1}{96} + \frac{1}{12} = \frac{1}{3}$  in 1 hour, and to fill the reservoir they require  $1 \div \frac{1}{3} = 3$  hours = 4 hours 43 min. 16 $\frac{2}{3}$  sec.

29. The person who thinks of the numbers must proceed as follows: He must multiply the 1st by 2 and add 5 to the product; he must next multiply this sum by 5 and add the second number to the product; he must next multiply this result by 10 and add the third number to the product; lastly, he must subtract 250 and name the remainder.

The three digits of the remainder will be the three numbers thought of, and will be in the order in which they were thought of.

The reason is obvious: let  $a = 1st$ ,  $b = 2nd$ , and  $c = 3rd$  number thought of.

$$a \times 2 + 5 = 2a + 5.$$

$$(2a + 5) \times 5 + b = 10a + b + 25.$$

$$(10a + b + 25) \times 10 + c = 100a + 10b + c + 250.$$

$$(100a + 10b + c + 250) - 250 = 100a + 10b + c = a \text{ in hundreds' place, } b \text{ in tens' place, and } c \text{ in units' place.}$$

30. Since each man possesses 63 square rods of land more than his son, we must form three pairs of numbers, such that the difference of their squares shall be 63.

The difference of the squares of two numbers is equal to their sum multiplied by their difference, and hence 63 must be divided into two factors in three distinct ways, thus:

$$63 = 63 \times 1 = 21 \times 3 = 9 \times 7.$$

If sum = 63 and difference = 1, the numbers are 32 and 31.

If sum = 21 and difference = 3, the numbers are 12 and 9.

If sum = 9 and difference = 7, the numbers are 8 and 1.

Hence the squares of Jones, Brown, and Smith, are respectively 32 rods, 12 rods, and 8 rods on the side, and the son's squares are respectively 31, 9, and 1 yards on the side.

Jones' piece was 23 rods longer on each side than Tom's, and since the difference between 32 and 9 is 23, we may conclude that Jones' square was 32 rods to the side, and Tom's 9 rods on a side.

Brown's piece was 11 rods longer on a side than Harry's, and since if the above numbers 12 and 1 have 11 for their difference, we may conclude that Brown's piece was 12 rods on a side, and Harry's piece 1 rod.

Hence Tom was Brown's son, Harry was Smith's son, and Ned was Jones' son.

31. The mode of arranging the crew may be remembered by attention to the vowels in the following line.

*Populeam virgam mater regina ferebat.*

The vowels refer to the crew as follows,  $a = 1$ ,  $e = 2$ ,  $i = 3$ ,  $o = 4$ , and  $u = 5$ .

We begin with 4 whites because the first vowel is  $o$ , next  $u = 5$  blacks, next  $e = 2$  whites, next  $a = 1$  black, next  $i = 3$  whites, next  $a = 1$  black, next  $a = 1$  white, next  $e = 2$  blacks, next  $e = 2$  whites, next  $i = 3$  blacks, &c., as follows,  $o$  standing for a white and  $+$  for a black.

oooo+++++oo+oo+o++++o++++o+

32. You select the multiplier or the multiplicand, such that the sum of its digits shall be exactly divisible by nine. Hence upon the principle of the proof by casting out the nines, the product has the sum of its digits exactly divisible by nine. By subtracting the sum of the digits of the remainder from the next higher multiple of 9 you determine the digit crossed out.

Thus suppose you select 117, and he takes for multiplicand 21613. Then  $21613 \times 117 = 2528721$ . Now, suppose he crosses out the 7; upon reading you the remaining digits 252821, you find that their sum  $= 20$ , which taken from 27 the next higher multiple of 9 leaves 7 the digit he crossed out.

If he crosses out a 0 or a 9, you cannot determine which, but in all other cases you can tell the exact figure.

33. You write the second, fourth, sixth, &c. lines in such a manner as to make the sum of the first pair, the sum of the second pair, &c. an exact number of 9's. Then having settled the number of pairs, you get the answer by multiplying by that number a row of 9's containing as many digits as there are to be figures in the line.

Thus suppose you agree to write 5 lines each, and that each line is to contain 5 digits, or not more than 5 digits. Then  $99999 \times 5 = 499995$  will be the answer. This is shown as follows:

Hence Tom was Brown's son, Harry was Smith's son, and Ned was Jones's son.

Suppose he writes	41113	}	= 99999
You write	58886		
Suppose he writes	61451	}	= 99999
You write	38548		
Suppose he writes	6500	}	= 99999
You write	93499		
Suppose he writes	1	}	= 99999
You write	99998		
Suppose he writes	99999	}	= 99999
You write	00000		
Sum	499995		

32. You select the multiplier or the multiplicand, such that the sum of its digits shall be exactly divisible by nine. Hence upon the principle of the proof by casting out the nines, the product has the sum of its digits exactly divisible by nine. By subtracting the sum of the digits of the remainder from the next higher multiple of 9 you determine the digit crossed out.

Thus suppose you select 21613 and he takes for multiplicand 21613. Then  $21613 \times 117 = 2528231$ . Now, suppose he crosses out the 7; upon reading you the remaining digits 252821, you find that their sum = 20, which taken from 27 the next higher multiple of 9 leaves 7 the digit he crossed out.

If he crosses out a 0 or a 9, you cannot determine which, but in all other cases you can tell the exact figure.

33. You write the second, fourth, sixth, &c. lines in each a manner as to make the sum of the first pair, the sum of the second pair, &c. an exact number of 9's. Then having selected the number of pairs, you get the answer by multiplying by that number a row of 9's containing as many digits as there are to be figures in the line.



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